

The Mathematical Modeling of Road Transport in Context of Critical Infrastructure Protection

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Abstract—The failure of critical infrastructure is becoming a more and more debated topic in the society. There is a need to plan and implement a functional and secure critical infrastructure. Road transportation has significant influence over the correct functioning of critical infrastructure. A case of disruption of security and functionality leads to distortion between other elements. This process causes a disturbance or paralysis of critical infrastructure. The first part of the article focuses on transport critical infrastructure and its importance for maintaining vital societal function. Road transportation has also significant influence over the correct functioning of critical infrastructure. Next, we present different approaches to dynamic modeling of the impacts of road transportation. The presented approaches provide some basic knowledge to implement dynamic modeling into practice. The conclusion of the article focuses on the design of dynamic modeling in road transportation, based on a deterministic approach. Conclusion outputs are seen as a fundamental baseline for selected processes of security project Resilience 2015.

Keywords—critical infrastructure; crisis situation; extraordinary event; mathematical models of road transport; road critical infrastructure.

I. INTRODUCTION

The paper describes the use of mathematical models for modeling impacts on critical road infrastructure [2]. Critical road infrastructure has an effect on our lives. Even a simple accident can significantly disturb or even paralyze the continuity of road transportation [3]. For this reason, it is necessary to pay attention to the critical infrastructure. This article is primarily focused on the identification and designation of critical infrastructure [1]. Another aspect addressed in the paper is the mathematical modeling in road transport. When applying mathematical models to road transportation, one needs to timely minimize the negative impacts on society and on other critical infrastructure sectors [5].

II. CRITICAL INFRASTRUCTURE

Critical infrastructure is mostly known as an element or a system whose functional degradation leads to a significant impact on national security. National security considers the basic needs of the population of a country, its health and economy.

Individual elements of critical infrastructure have linkages to each other which guarantee the correct functioning of individual elements. These linkages ensure the interdependence of its various sectors and elements.

The elements we call critical infrastructure in particular. These elements are determined by the cross-cutting and sector specific criteria if the element of critical infrastructure is part of the European Critical Infrastructure. In this case, it is considered as an element of European critical infrastructure [14].

A. The Cross-cutting Criteria

In order to assess the severity of the impact of disruption to a critical infrastructure element, we consider the elements from the following perspectives:

- (a) casualties criterion (assessed in terms of the potential number of fatalities or injuries);
- (b) economic effects criterion (assessed in terms of the significance of economic loss and/or degradation of products or services; including potential environmental effects);
- (c) public effects criterion (assessed in terms of the impact on public confidence, physical suffering and disruption of daily life; including the loss of essential services).

B. The Sectoral Criteria

The sectoral criteria to identify critical infrastructure elements are set by Government Decree no. 432/2010 Coll.[14], on criteria for determining the elements of critical infrastructure.

The basic classification of sector specific criteria to identify critical infrastructure elements are: [14]

- Energy.
- Water management.
- Food and agriculture.
- Health.
- Transport.
- Communication and information systems.
- Financial market and currency.
- Emergency services.
- Public administration.

III. CRITICAL ROAD INFRASTRUCTURE

The individual elements of the critical road infrastructure are important for the state not only for passengers, but also for materials. These elements include rail, water, air or road transport. The individual elements are also at risks that can affect them through any linkage to crisis or emergency situations. These situations can impair or limit the functionality of this state. In case of limited functionality, there is a disruption of its security. In our case, we focus on the road critical infrastructure which could be described as the most important element in the transport [6].

The current state of road infrastructure can be analyzed from two perspectives. The first view is regarding road safety. The second view shows us linkages with other elements of critical infrastructure.

Traffic safety is influenced by two inputs. The first entry is homeland security, which is aimed at reducing accidents and creating traffic rules. External security includes anti-terrorism, vandalism, the elements etc.

We can conclude that road transport is the most vulnerable and the most significant element of the critical infrastructure.

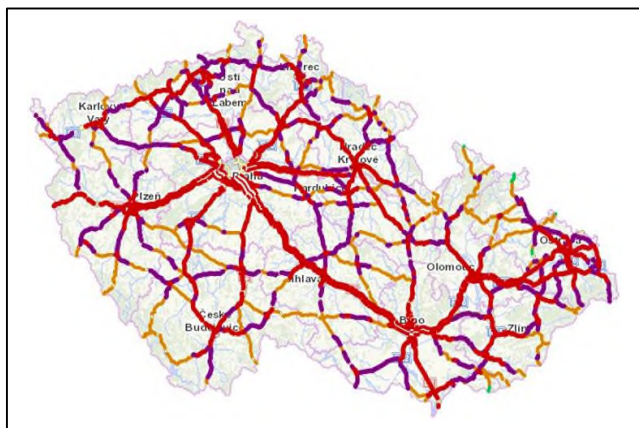


Figure 2. Intensity of Transport in Czech Republic 2015. [13]

When looking at Figure 2, we can see that the riskiest places in the Czech Republic are the main arteries of motorways and Class I roads. The biggest risk for the Czech Republic is without any doubt the D1 motorway. Looking at Figure 2, it is possible to draw attention to the utilization of major roads. They are overloaded by a large number of vehicles.

IV. MATHEMATICAL MODELS OF ROAD TRANSPORT

The goal of mathematical modeling in road transport is what the most trusted model vehicle movements and their interaction in traffic. One of the criteria is the failure of random factors and variables. These models can then be divided into stochastic and deterministic. The difference between these two models is substantial. Stochastic models operate with the probability of certain events and take into account the random effects, while deterministic models with random effects are not counted. Deterministic model is act

upon strict mathematical, statistical and logical relationships. These relations predetermine the behavior [9] [10].

A very important role in modeling road transport is the request for range of investigated network. This fact is closely related to the requirement for input data. When modeling large transport networks, we can generalize some detail or possibly neglected. For small transport networks, it cannot be afforded such a procedure, it is necessary to give weight to each transport movement. In this model we divide the macroscopic and microscopic or their combination. When you merge macroscopic and microscopic models, there is a new model that we call mesoscopic. With this model it is possible to meet only rarely. [9], [10]

A. Makroskopic Model

The most commonly are used to simulate large-scale communication networks. These models are mainly used for prognostic purposes.

$$q = v * k \quad (1)$$

Where q = intensity of transmission services [$\text{ks}\cdot\text{h}^{-1}$]; v = the current speed of vehicle [$\text{km}\cdot\text{h}^{-1}$]; k = density of vehicles in the stream [$\text{ks}\cdot\text{km}^{-1}$].

Greenshields model is among the simplest and oldest linear model. It is based on measuring the speed and intensity when the help of these data we can calculate the density. Fundamental assumption is the linear dependency of velocity on the density. The following formula expresses this dependency: [5]

$$v(k) = v_{max} * \left(1 - \frac{k}{k_{max}}\right) \quad (2)$$

where v_{max} = maximum speed [$\text{km}\cdot\text{h}^{-1}$]; k_{max} = density congestion [$\text{ks}\cdot\text{km}^{-1}$].

In areas with a low density, this model acts as unrealistic. This behavior is caused by insufficient speed and density is then symmetric parabolic dependence on the density of intensity. At lower densities it leads to maximum intensity.

B. Microscopic model

They are based on modeling of individual vehicles driving along the road when there is a consideration as the communication parameters and the vehicle and the driver behavior. During a traffic simulation we meet mostly these models.

Input parameters are achieved vehicles vehicle speed, engine power, size, acceleration and deceleration. Other essential parameters for input features are network users and their interaction. The input data are never accurate, mainly due to the uniqueness of each participant and the vehicle. The determination of acceleration depending on the environmental conditions is essential. Generally, we can express the acceleration in microscopic models as follows:

$$a = f(v, \Delta v, \Delta x) \quad (3)$$

where a = acceleration [$m \cdot s^{-2}$]; v = speed [$m \cdot s^{-1}$]; Δv = relative speed relative to the preceding vehicle [$m \cdot s^{-1}$]; Δx = distance from the preceding vehicle [m].

Consequently, the examination of the traffic flow, we filter out two basic ways of influencing the vehicle between them. The first approach is developed based on observance of a safe distance between vehicles, and when changing the speed of the vehicle prior. The second approach is based on the distance between vehicles. [12], [4]

Weidman's model – The model is a reaction of the driver, which is carried out at a certain distance and the difference in speed between the vehicles. This difference estimated driver only relatively. Using the model, we are looking threshold level drivers that are running in its decisions. These limits are divided into four categories:

- Free movement.
- Approximation.
- Monitor.
- Emergency braking.

These four categories are shown in Figure 3, which you can see below:

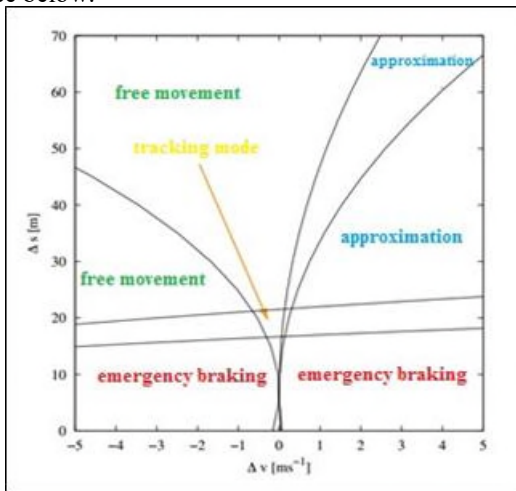


Figure 3. Wiedemann Model. [11]

In the free movement the drivers try to or do reach maximum speed. In cases where an increase of density on the road is, there occurs a state where the vehicle is traveling at a higher speed than the preceding vehicle for this vehicle approaching. Once there is the approaching to the vehicle prior to the vehicle driver tries to adjust not only the speed of the preceding vehicle but also its driving style. This leads to copying of previous vehicle, not only in terms of speed but also in view of maintaining a constant distance between vehicles. At this point, the most important element is the reaction time of the driver, which can lead to two different states. The first condition can be described as the best possible, and that is the driver can react quickly and reduce the existing distance between vehicles. The second condition leads to a bad reaction that could cause a collision.

Gipps model – This model can be also known as non-crash. It is based on the speed limit, when as the result of this action is no crash. This model was the first realistic model. The positive aspect is its ability to reproduce the characteristics of real traffic flow without necessity introducing parameters. These properties are not related to the driver. The driver is limited to maximum accelerations and decelerations that are in full speed distance between vehicles and the relative speed. Therefore, the vehicle will never exceed the maximum speed and acceleration should in the free flow of traffic drop to zero. On the basis of the vehicle speed in the traffic flow calibration has been experimental data expressed by the following formula:

$$v_n(t + \tau) \leq v_n(t) + 2.5a_n\tau * \left(1 - \frac{v_n(t)}{V_n}\right) * \sqrt{\left(0.025 + \frac{v_n(t)}{V_n}\right)} \quad (4)$$

Where a_n = maximum vehicle acceleration [$m \cdot s^{-2}$]; τ = the reaction time of the driver [s]; V_n = target vehicle speed corresponding free traffic without restrictions [$m \cdot s^{-1}$]; $v_n(t)$ = vehicle speed at time t [$m \cdot s^{-1}$] [11]

IDM model – The biggest advantage of the above mentioned model is the number of input parameters. These source parameters are intuitive character. From this perspective, it saves a great deal of time because they do not have to work on long analysis to obtain the input data. The basic equation of this model is expressed dependence acceleration:

$$a_i(s_i, v_i, \Delta v_i) = a_{i0} \left[1 - \left(\frac{v}{v_{i0}}\right)^\delta - \left(\frac{s_{opt}(v_i, \Delta v_i)}{s_i}\right)^2 \right] \quad (5)$$

Where a_i = acceleration of the vehicle [$m \cdot s^{-2}$]; a_{i0} = comfortable acceleration [$m \cdot s^{-2}$]; v_{i0} = target vehicle speed [$m \cdot s^{-1}$]; v = vehicle speed [$m \cdot s^{-1}$]; s_{opt} = optimum distance vehicles [m]; s_i = immediate distance from the preceding vehicle [m]; δ = acceleration factor of realism of the reference vehicle.[11]

Usefulness of the models has in every sector its pros and cons. In the community of mathematicians and physicists dealing with this issue it has become the most recognized IDM model also termed as "Intelligent Driver Model". Gipps and Wiedemann models and are most prevalent in simulation software. [7], [8]

V. THE USE OF DYNAMIC MODELING OF THE IMPACTS OF ROAD TRANSPORT

Macroscopic and microscopic models have their uses for modeling in road transport. The output element of these models is the evaluation of vehicle movement and their interaction in the road. We can say that it is modern approaches. On this basis, we developed a group of authors on the subject of dynamic system modeling in road transport. This system is based on a deterministic approach. The basis of this system is the analysis section where it disposes critical element and detour routes on the basis of the macroscopic model. The brain of the system is the algorithm

that evaluates the performance parameters of the sections, the degree of intensity transmittance pass routes in the transport and effects of operating parameters on alternate roads.

In Figure 4. below we can see the progress of the algorithm for dynamic system modeling the impact of road transport:

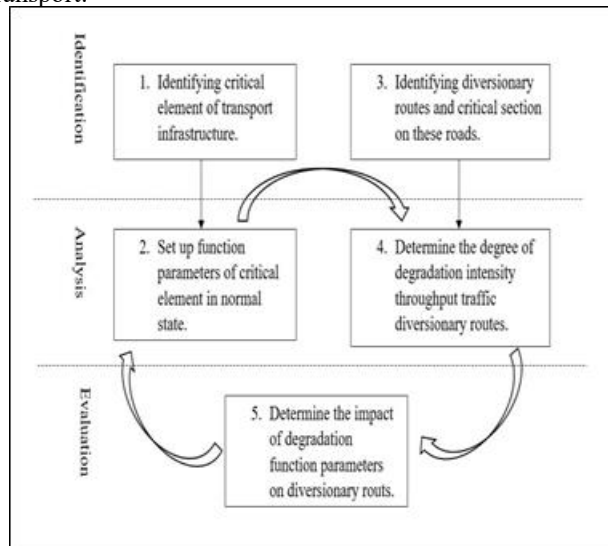


Figure 4. Algorithm of Dynamic Modeling of the Impacts of Road Transport. [1]

Step 1. Identifying critical element in road infrastructure - In case of rejection of a critical element of road transport is the first step no doubt his identification. The identification is based on a directive of the European Union, which leads to their identification and subsequent designation by sectoral and cross-cutting criteria. To be able to fulfill the cross-cutting criteria, you need to model the results of the impact that these values be able to compare the values of crosscutting. This hypothesis is the initial step due to confirm or refute criticism.

Step 2. Determination of critical element functional parameters in a normal state - After identifying follows assessment of functional parameters critical element in a normal state. This means that a certain intensity and traffic throughput as the latest phase in this step set these parameters along the entire route, where this is a critical element.

Step 3. Finding detour routes and critical sections of these routes - At a time when there is lose of a critical element we need to be prepared for alternative roundabout routes. Certainly we should not forget the possible risk sections also on alternative routes so that we are ready for the emergence of other events.

Step 4. Determine the degree of intensity throughput traffic on the detour routes. Determination of functional parameters should be done not only on the main road but also on alternative routes. Functional parameters

means to determine the intensity of transmission traffic across alternative route.

Step 5. Determination of the impact on functional parameters on alternate roads. Finally, to represent a number of casualties and economic loss. The final statement gives us lost gross domestic product (GDP) and additional operating costs.

VI. CONCLUSION

Road transport is for every state a significant element of the critical infrastructure. In the case of disruption or failure of this element can lead to other serious threats to some typical elements of critical infrastructure. This fact can achieve a negative impact not only on society. From this perspective, we must not neglect prevention and quality response to this potential threat. Above presented tool is built on the foundations of existing instruments. This tool is not only suitable for real-time analysis but also a far better overview in the art. The aim of dynamic modeling is effective to minimize the expected impact. Road transport is important for any company in many ways and therefore it is necessary to pursue this issue more intensive than before. Increasing the security of critical infrastructure element significantly will affect the security and other elements that are connected with a linkage.

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