Performance Analysis of Pilot Aided OFDMA Systems for Mesh Networks

Jung-Hyun Kim, Jihyung Kim, Kwang Jae Lim, and Dong Seung Kwon
Wireless System Research Department, Internet Research Division
Electronics and Telecommunications Research Institute (ETRI)
Daejeon, Korea
{jh.kim06, savant21, kjlim, dskwon}@etri.re.kr

Abstract—This paper presents a systematic approach for analyzing the bit error probability of pilot aided orthogonal frequency-division multiple access (OFDMA) systems. A comparative analysis with the conventional pilot pattern schemes, overlapped pilot scheme and interlaced pilot scheme, is developed. The results obtained here can be directly applied to evaluate the performance of OFDMA systems in mesh networks as well.

Keywords-mesh networks; channel estimation; OFDMA;

I. INTRODUCTION

In recent years, orthogonal frequency division multiple access (OFDMA) has been widely adopted for many contemporary wireless systems such as wireless LANs, digital video broadcasting for handheld terminals (DVB-H) [1], Worldwide Interoperability for Microwave Access (WiMAX) [2], and second generation terrestrial digital video broadcasting (DVB-T2) [3] due to its flexibility on resource allocation and robustness to multipath fading channel.

Channel estimation plays an important part in an OFDMA system because it is used to coherently decode the transmission signal and to combine the diversity. The effect of channel estimation for OFDM systems is considered recently for examples in [4], [5], and [6]. For the channel estimation, a known signal so-called pilot is usually employed. The pilot symbols are uniformly inserted into the transmission data stream. These pilot symbols are transmitted through the channel to convey the channel information. Channel estimator at the receiver obtains the channel information from these corrupted pilot symbols and determines the channel response of data symbol region by interpolating the channel response between samples obtained using pilot symbols.

For a single user case, it is obvious that more pilot symbols lead to better performance but with sacrificing in the symbol rate. Therefore the number of pilot symbols is a trade-off between channel estimation accuracy and bandwidth efficiency. However, for multi-user case, it is not obvious that more pilot symbols outperform less pilot symbols due to interference between users. If pilot symbols are corrupted, data fail to be demodulated irrespective of correcting processes such as despreading and decoding. Pilot power boost-up does not help in this case because signal to interference ratio remains the same. The solution is to make the pilot symbols from different users not to collide each other because pilot symbols are relatively stronger than spread data symbols. The channel estimation performance is improved by reducing the number of pilot symbol collisions. However better channel estimation performance does not always guarantee better data detection performance. The interference of pilot symbol region and the interference of data symbol region are trade-off each other. With this point, we focus on how to design pilot pattern for mesh networks where serious interference exists.

In this paper, our objective is to present an initial approach to design pilot patterns for mesh networks by analyzing the performance of OFDMA system using traditional pilot schemes, overlapped pilot scheme and interlaced pilot scheme, for multi-user channel. The analysis and results can be directly extended to the design of pilot patterns for mesh networks.

The rest of the paper has been organized as follows. Section II contains the system model and pilot aided channel estimation for OFDMA system. Section III provides bit error probability (BEP) analysis for uncoded OFDMA and coded OFDMA systems. Simulation results are in Section IV. Finally, Section V summarizes our main results.

II. SYSTEM MODEL

A typical OFDM system is reviewed as follows. In the discrete time domain, the transmitted $l$th OFDM signal is expressed by

$$x_l[n] = \sum_{k=0}^{N-1} X[k] e^{j2\pi kn/N}$$

(1)

where $X[k]$ is a data signal and $n$ is the time index and $k$ and $N$ are subcarrier index and total number of subcarriers respectively.

The received OFDM signal can be written as

$$Y_l[k] = \sum_{n=0}^{N-1} \{ h_l[n] \otimes x_l[n] + z_l[n] \} e^{-j2\pi kn/N}$$

$$= \sum_{n=0}^{N-1} \left( \sum_{m=0}^{\infty} h_l[m] x_l[n-m] + z_l[n] \right) e^{-j2\pi kn/N}$$
We consider the least-squares (LS) estimate for the channel estimation. The $l$th estimated channel response can be obtained as follows:

$$
\hat{H}_l[p] = \frac{Y_l[p]}{X_l[p]} = \frac{Z_l[p]}{X_l[p]} = H_l[p] + V_l[p] \quad (4)
$$

where the subscript $p$ denotes the index of pilot subcarrier, $X_l[p]$ is the $p$th pilot signal at the $l$th OFDM symbol and $Y_l[p]$ is the received symbol corresponding to pilot signal $X_l[p]$ and $V_l[p]$ is the channel estimation error term.

For the data location, the channel response can be estimated by taking interpolation between the pilot’s channel estimate. There are several forms to interpolate: uniform, spline interpolation, and 2D Wiener interpolation etc. Here, the linear interpolation is used. In the linear interpolation, the data channel estimate is given by

$$
\hat{H}_l[d] = \left(1 - \frac{s}{S}\right) \hat{H}_l[p] + \frac{s}{S} \hat{H}_l[p+1] \quad (5)
$$

where $d$ denotes the index of data subcarrier, $S$ is the interval between pilot subcarriers and $s$ is the distance between the $p$th pilot subcarrier and the $d$th data subcarrier.

Thus, the transmitted data signal at the $d$th data subcarrier and the $l$th OFDM symbol can be estimated by

$$
\hat{X}_l[d] = \frac{Y_l[d]}{\hat{H}_l[d]} \quad (6)
$$

For two schemes, overlapped pilot scheme and interlaced pilot scheme, the above formula can be re-written as follows: Overlapped pilot scheme:

$$
\hat{X}_l[d_1] = \frac{Y_l[d_1]}{\hat{H}_l[d_1]} = \frac{Z_l[d_1]}{\hat{H}_l[d_1]} + \frac{V_l[d_1]X_l[d_1]}{\hat{H}_l[d_1]} \quad (7)
$$

$$
\hat{Z}_l[d_1] = I_l[d_1] + N_l[d_1] \quad (8)
$$

$$
I_l[d_1] = \sum_{i \in \kappa} H_l^{(i)}[d_1]X_l^{(i)}[d_1] \quad (9)
$$

where $I_l[d] = \frac{S}{k}$ is the interference of $d$th data signal and $l$th OFDM symbol, $H_l^{(i)}[d]$ and $X_l^{(i)}[d]$ are the channel response and the data signal of $i$th user respectively and $\kappa$ is the set of neighboring users.

Interlaced pilot scheme:

$$
\hat{X}_l[d_2] = \frac{Y_l[d_2]}{\hat{H}_l[d_2]} = \frac{Z_l[d_2]}{\hat{H}_l[d_2]} + \frac{V_l[d_2]X_l[d_2]}{\hat{H}_l[d_2]} \quad (10)
$$

$$
\hat{Z}_l[d_2] = I_l[d_2] + N_l[d_2] \quad (11)
$$

$$
I_l[d_2] = \sum_{i \in \kappa, i \neq j} H_l^{(i)}[d_2]X_l^{(i)}[d_2] + H_l^{(j)}[p]X_l^{(j)}[p] \quad (12)
$$

$$
\hat{X}_l[d_2] = \frac{Y_l[d_2]}{\hat{H}_l[d_2]} = \frac{Z_l[d_2]}{\hat{H}_l[d_2]} + \frac{V_l[d_2]X_l[d_2]}{\hat{H}_l[d_2]} \quad (13)
$$

$$
\hat{Z}_l[d_2] = I_l[d_2] + N_l[d_2] \quad (14)
$$

$$
I_l[d_2] = \sum_{i \in \kappa} H_l^{(i)}[d_2]X_l^{(i)}[d_2] \quad (15)
$$

A. Pilot Aided Channel Estimation

For the channel estimation, a known symbol so-called pilot signal is usually employed. Before the transmission, pilot signals are uniformly inserted into the data stream. Upon receiving the corrupted pilot signals at the receiver, the channel impulse response at pilot locations is estimated. The channel impulse response at data locations can then be obtained through interpolation with the channel impulse response estimated at pilot locations.
where \( j \) is the user index that \( j \)th user’s pilot signal is interfering.

For the interlaced pilot scheme, we can divide the data signals into two parts. The first part is the region that one of neighbor user’s pilot signal and another neighbor users’ data signals are interfering. The second part is the region that all of neighbor users’ data signals are interfering. The first part gets more interference than second part because the pilot power is boosted for accurate channel estimation. At the above equations, we denote \( d_1 \) as the data subcarrier index in the first part and \( d_2 \) as the data subcarrier index in the second part.

III. PERFORMANCE ANALYSIS OF OFDMA SYSTEM
A. Uncoded OFDMA System

In this paper, we show the BEP performance analysis of the QPSK because the performance of other constellations, 16-QAM and 64-QAM, can be derived in a similar manner. A QPSK symbol can be written as \( X = X_I + iX_Q = |X|e^{i\theta} \), where \( X_I, X_Q \in \{1, -1\} \).

\[
\hat{X} = \frac{HX + Z}{H} = \frac{1}{|H|} \left( |H|e^{i\theta - \angle H}X + |Z|e^{i\theta - \angle H} \right) = \frac{1}{|H|} \left( |H|e^{i\psi}X + Z' \right) = \frac{1}{r_2} \left( r_1 X e^{i\psi} + Z' \right) = \frac{1}{r_2} X |\cos(\theta + \psi) + Z'| + \frac{1}{r_2} X |\sin(\theta + \psi) + Z'| = \hat{X}_I + i\hat{X}_Q
\]

where \( r_1 = |H|, r_2 = |\hat{H}|, \) and \( Z' = Z'+iZ_Q \) is a zero-mean complex Gaussian random variable with the variance equal to that of \( Z \). If the transmitted data signal is \( X = -1 + i1 \), the conditional BEPs are given by

\[
P_b(E|r_1, r_2, \psi, X = -1 + i1) = \frac{1}{2} P_{b,t}(E|r_1, r_2, \psi, X = -1 + i1) + \frac{1}{2} P_{b,Q}(E|r_1, r_2, \psi, X = -1 + i1) = \frac{1}{2} Pr(\hat{X}_I > 0 | X_I = -1) + \frac{1}{2} Pr(\hat{X}_Q < 0 | X_Q = +1)
\]

\[
= \frac{1}{4} Pr(N_I > r_1 | X |\cos(\psi + \theta)) + \frac{1}{4} Pr(-N_Q > r_1 | X |\sin(\psi + \theta))
\]

For the above equation, each term on the last line is either of the form

\[
Pr[\pm N_I > a |X|\cos(\psi + \theta)] = Q \left( \frac{a |X|\cos(\psi + \theta)}{\sqrt{\sigma_N^2 + \sigma_f^2}} \right) \quad (18)
\]

\[
Pr[\pm N_Q > a |X|\sin(\psi + \theta)] = Q \left( \frac{a |X|\sin(\psi + \theta)}{\sqrt{\sigma_N^2 + \sigma_f^2}} \right) \quad (19)
\]

where \( Q(x) \triangleq 1/\sqrt{2\pi} \int_x^{\infty} e^{-t^2/2} dt, \ a = \pm 1, \ |X| = \sqrt{X_I^2 + X_Q^2} = \sqrt{2}, \ \sigma_N^2 = var(N_I) = var(N_Q) = (N_0/2) \) is the noise variance and \( \sigma_f^2 \) is the interference variance.

We can obtain the overall conditional BEP is

\[
P_b(E|r_1, r_2, \psi) = \sum_{X \in \chi} \frac{1}{4} P_b(E|r_1, r_2, \psi, X)
\]

\[
= \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a |X|\cos(\psi + \theta)}{\sqrt{\sigma_N^2 + \sigma_f^2}} \right) + \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a |X|\sin(\psi + \theta)}{\sqrt{\sigma_N^2 + \sigma_f^2}} \right) \quad (20)
\]

where \( \chi \) is the set of QPSK constellations.

Similarly with [4], the joint probability density function (pdf) of \((r_1, r_2, \psi)\) is

\[
p(r_1, r_2, \psi) = \frac{r_1 r_2}{2\pi \sigma_1^2 \sigma_2^2 (1 - \rho^2)} \exp \left\{ - \frac{1}{2(1 - \rho^2)} \left[ \frac{r_1^2}{\sigma_1^2} + \frac{r_2^2}{\sigma_2^2} - 2 \frac{r_1 r_2}{\sigma_1 \sigma_2} (\rho_1 \cos \psi - \rho_2 \sin \psi) \right] \right\} \quad (21)
\]

where \( \sigma_1^2 \triangleq 1/2 E[|H|^2], \sigma_2^2 \triangleq 1/2 E[|\hat{H}|^2], \rho \triangleq \sqrt{\rho_1^2 + \rho_2^2}, \rho_1 \triangleq \frac{\mu_1}{\sigma_1 \sigma_2}, \) and \( \rho_2 \triangleq \frac{\mu_2}{\sigma_1 \sigma_2} \) with \( \mu_1 = \frac{1}{2} Re \left\{ E[|H|H^*] \right\} \) and \( \mu_2 = \frac{1}{2} Im \left\{ E[|H|H^*] \right\} \).

The BEP is given by

\[
P_b(E) = \int_0^\infty \int_0^\infty \int_{-\pi}^\pi P_b(E|r_1, r_2, \psi) \cdot p(r_1, r_2, \psi) d\psi dr_1 dr_2.
\]

Substituting (20) and (21) into (22), we obtain (23).

Assuming that the channel response is stationary during one packet duration and transmitted symbols are mutually uncorrelated, the instantaneous effective signal to interference and noise ratio (SINR) per bit of \( k \)th subcarrier is
\[ P_b(E) = \frac{1}{2} \left[ 1 - \frac{1}{2} \frac{\rho_1 + \rho_2}{\sqrt{\gamma_b}} \frac{(\rho_1 + \rho_2)^2}{2} \right] \]

obtained as

\[ \pi_b[k] = \frac{E[|H[k]X[k]|^2]}{E[|Z[k]|^2 + |V[k]|^2|X[k]|^2]} \]

where \( V[k] = H[k] - H[k] \) is the channel estimation error at \( k \)th subcarrier and \( K \) denotes the number of bits represented by one symbol. For example, \( K = 1 \) for BPSK and \( K = 2 \) for QPSK.

For a reasonably good estimate, \( H \approx H \) then the average SINR per bit is approximated by

\[ \pi_b \approx \frac{E[|HX|^2]}{K(E[|Z|^2])}. \]

From the above equation 25, the average SINR per bit of QPSK is expressed as

\[ \pi_b = \frac{2\sigma^2}{2(\sigma_N^2 + \sigma_I^2)} \]

where \( \sigma^2 = \frac{1}{2} E[|H|^2] \), \( \sigma_N^2 \) is the variance of noise and \( \sigma_I^2 \) is the variance of interference.

For two cases, the overlapped pilot case and the interlaced pilot case, above equation (20) and (22) can be re-written as follows:

Overlapped pilot scheme:

\[ P_b(E|r_1, r_2, \psi) = \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a|X|r_1 \cos(\psi + \theta)}{\sigma_N^2 + \sum_{u \in \kappa} \sigma_{D,u}^2} \right) \]

\[ + \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a|X|r_1 \sin(\psi + \theta)}{\sigma_N^2 + \sum_{u \in \kappa} \sigma_{D,u}^2} \right) \]

\[ P_b(E) = \int_0^\infty \int_0^\infty \int_{-\pi}^\pi P_b(E|r_1, r_2, \psi) \cdot p(r_1, r_2, \psi) d\psi d\psi_1 d\psi_2 \]

where \( \kappa \) is the set of neighbor users and \( \sigma_{D,u}^2 \) is the variance of interference by \( u \)th neighbor user data signal.

Interlaced pilot scheme:

\[ P_{b,1}(E|r_1, r_2, \psi) = \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a|X|r_1 \cos(\psi + \theta)}{\sigma_N^2 + \sum_{u \in \kappa} \sigma_{D,u}^2} \right) \]

\[ + \sum_{X \in \chi} \frac{1}{4} Q \left( \frac{a|X|r_1 \sin(\psi + \theta)}{\sigma_N^2 + \sum_{u \in \kappa} \sigma_{D,u}^2} \right) \]

\[ P_{b,2}(E|r_1, r_2, \psi) = \int_0^\infty \int_0^\infty \int_{-\pi}^\pi P_{b,2}(E|r_1, r_2, \psi) \cdot p(r_1, r_2, \psi) d\psi d\psi_1 d\psi_2 \]

where the subscript \( j \) is the user index that \( j \)th user’s pilot is interference and \( \sigma_{P,j}^2 \) is the variance of interference by \( j \)th neighbor user pilot signal, \( \epsilon = \epsilon_D/\epsilon_F, \epsilon_D(\epsilon_F) \) is the number of signals affected by data(pilot) signals of another users as interference.

B. Coded OFDMA System

Error correcting coding is an essential part of OFDMA systems for wireless communications. OFDMA in a fading environment is almost always used with coding to improve its performance and as such is often referred to as Coded OFDMA or COFDMA. Just as we can introduce time diversity through coding and interleaving in a flat-fading single-carrier system, we can introduce frequency diversity through coding and interleaving across subcarriers in an OFDMA system. With coding and interleaving across subcarriers, the strong subcarriers help the weak ones. Thus overall data detection performance is dependent on the ratio of strong part and weak part.
To compare the performance of COFDMA systems using overlapped pilot scheme and interlaced pilot scheme, we consider the union bound derived in [7], [8]. We use turbo code and BPSK signaling on the Rayleigh fading channel.

Then the union bound on the bit-error rate is given by

$$P_b(E) \leq \sum_{i=1}^{k} \frac{k \choose i} P(i + 2\mu_i)$$

and the probability of an error by maximum likelihood decoding with codeword Hamming weight $\omega$ is expressed by

$$P(\omega) = \frac{1}{2} \left( 1 - \sqrt{\frac{R\sigma_b^2}{1 + R\sigma_b^2}} \right) \left( \frac{1}{1 + R\sigma_b^2} \right)^{\omega - 1}$$

where $\mu_i = r\rho_i$ with

$$\rho_i = \frac{1}{2} \left[ 1 - \frac{1 - 2i/k}{k} \cdot \frac{1 - (1 - 2i/k)^{\kappa/R}}{1 - (1 - 2i/k)^{\kappa/R}} \right]$$

or for large $k(k \to \infty)$:

$$\rho_i = \frac{1}{2} \left[ 1 - \frac{1 - \exp(-2i/R)}{2i/R} \right]$$

where $R$ is the code rate, $k$ is the number of information bits, $r$ is the number of redundant bits of component codes, and $\kappa$ is the time varying factor defined in [7].

For the overlapped and interlaced pilot schemes, the average SINR per bit defined by (24) becomes

Overlapped pilot scheme:

$$\bar{\gamma}_b = \frac{2\sigma_1^2}{2\left( \sigma_N^2 + \sigma_{\gamma,j}^2 + \sum_{u \in \kappa} \sigma_{D,u}^2 + \sigma_V^2\sigma_{\gamma,i}^2 \right)}$$

where $\sigma_1^2$ is the variance of channel estimation error term and $\sigma_{\gamma,j}^2$ is the variance of own data signal, $\kappa$ is the set of neighbor users and $\sigma_{D,u}^2$ is the variance of interference by $u$th neighbor user data signal.

Interlaced pilot scheme:

$$\bar{\gamma}_{b,1} = \frac{2\sigma_1^2}{2\left( \sigma_N^2 + \sigma_{\gamma,j}^2 + \sum_{u \in \kappa} \sigma_{D,u}^2 + \sigma_V^2\sigma_{\gamma,i}^2 \right)}$$

$$\bar{\gamma}_{b,2} = \frac{2\sigma_1^2}{2\left( \sigma_N^2 + \sum_{u \in \kappa} \sigma_{D,u}^2 + \sigma_V^2\sigma_{\gamma,i}^2 \right)}$$

$$P_b(E) = \frac{P_{b,1}(E) + \epsilon P_{b,2}(E)}{1 + \epsilon}$$

where $\sigma_{\gamma,j}^2$ is the variance of interference by $j$th neighbor user pilot signal, and $\epsilon = \epsilon_D/\epsilon_P$, $\epsilon_D(\epsilon_P)$ is the number of symbols that affected by data(pilot) symbols of another users as an interference.

**IV. SIMULATION RESULTS**

For the simulation, we used turbo code of Fig. 2 in [9] and punctured with code rate 1/2. The LS channel estimation method is used on the Rayleigh fading channel. All simulations are performed in MATLAB program.

Figs. 2 and 3 show the BEP performance of overlapped pilot scheme and interlaced pilot scheme, and uncoded BPSK in [10]. These figures indicate that interlaced pilot scheme outperforms overlapped with some serious interference environments (not always). We can recheck this from Fig. 4.
V. CONCLUSION AND FUTURE WORK

In this paper, we analyze the performance of overlapped pilot scheme and interlaced pilot scheme for channel estimation. This comparison is of special interest since the pilot pattern affects the performance of OFDMA systems. Simulation results in terms of BEP corroborate our theoretical analysis.

We notice that interlaced pilot pattern is more suitable for the multi-user networks like mesh network in which serious interference exists. Various pilot design is possible for mesh networks and we expect that our systematic approach and simulation results obtained here can be directly applied to evaluate the performance of pilot aided OFDMA systems for mesh networks.

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