Appliance Scheduling Optimization for Demand Response

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***Abstract*—The paper studies the challenge of the electricity consumption management in smart grids. It focuses on different impacts of demand response running in the smart grid engaging consumers to participate. The main responsibility of the demand response system is scheduling the operation of appliances of consumers in order to achieve a network-wide optimized performance. Each participating electricity consumer, who owns a set of home appliances, provides the desired expectation of his/her power consumption scenario to the demand response system. It is accompanied with time limits on the flexibility of controllable appliances for shifting their operational time from peak to off-peak periods. The appliance scheduling optimization for demand response is modeled as an optimization problem. It concentrates on reducing the total electricity bills and CO_2 emissions as well as flattening the aggregated peak demand at the same time. This paper categorizes the appliances based on shiftability and interruptibility characteristics. It uses information of dwellings to determine an effective appliance scheduling strategy. This strategy gets influenced by grid constraints imposed by distribution system operators. The simulations confirm that scheduling appliances of 100 consumers yields a significant achievement in the peak demand reduction while averagely satisfying the comfort level of consumers.**

***Keywords*–Smart grid, demand response, appliance scheduling, knapsack problem, dynamic programming, multi-objective optimization.**

NOMENCLATURE

Constants

PDT	Peak Demand Threshold
PPD	Peak Power Demand
A_i	Number of appliances in D_i
$a_{i,j}$	Appliance j in dwelling i
D_i	Dwelling i
G	Number of generations
N	Number of dwellings
p_c	Crossover propability
p_m	Mutation propability
$p_{i,j}$	Priority of appliance $a_{i,j}$
Q	Population size
T	Number of time intervals
$DF_{i,j}$	Deadline flexibility of appliance $a_{i,j}$
$TPD_{i,j}$	Total power demand of appliance $a_{i,j}$

Indices

i	Index of dwellings
j	Index of appliances
t	Index of time intervals

Variables

$x_{i,j}^t$	Decision variable of selecting $PD_{i,j}^t$
CO_2E^t	Amount of CO_2 emission at time interval t
EP^t	Electricity price at time interval t
$PD_{i,j}^t$	Power demand request of appliance $a_{i,j}$ at time interval t
$RP_{i,j}^t$	Number of remaining power requests of appliance $a_{i,j}$ at time interval t

I. INTRODUCTION

The smart grid has emerged as a novel infrastructure aiming to transform the existing power system into a reliable and consumer-centric one. It forms a distributed energy delivery network using the electricity and information streams simultaneously. This network possesses a self-healing characteristic toward facing unforeseen electricity outage circumstances. Its reliability and stability are based on intelligent controllers, in which they try to establish bilateral communication channels between consumers and Distribution System Operators (DSOs). The demand side management service provides an opportunity to energy actors for an active participation in counterbalancing the *demand response*. It helps to find the most reliable and effective energy solutions in real-time. This paper extends the work presented in [1]. Here, the key contributions include the extended mathematical formulation and description of the demand response system along with a presentation of an extensive simulation performance analysis.

Demand response is one of the most challenging issues in demand side management, which is responsible for providing effective and comprehensive energy solutions [2]. From the consumers' point of view, demand response attempts to motivate them to modify their electricity usage patterns, in response to potential grid incentives. In contrast to this point of view, DSOs intend to equilibrate demands with responses to reduce peak power demands as much as possible [3]. These purposes can be achieved through both *curtailing the power demand* and *controlling the activation time of electricity usages*. However, a mutual challenge behind these procedures is how to motivate consumers to modify their power demand profiles [4][5].

One of the most pragmatic incentives for consumers to modify their consumption behavior is electricity prices. Although demand response includes efforts to change the electricity usage of consumers with respect to the alterations in the electricity prices, however, reducing the peak demand and CO_2 emission also help to decrease the greenhouse gas emissions [6]. This reduction results in a co-optimization approach of power demand cost and CO_2 emission. In some peak hours, the demand response system has to shift some *power demand requests* from diverse dwellings to another time interval. This shifting can occur several consecutive/separate times over a day. Obviously, this leads to some changes in the daily power consumption of consumers. This causes a problem named *dissatisfaction of consumers*. As a result, maximizing the satisfaction of consumers is an essential objective as well.

Consumers are also interested to reduce their electricity cost while contributing to CO_2 emission reduction program. From the DSOs' point of view, they aim to shave the peak period, which results in flattening the aggregated power demands over time.

Figure 1 shows a conceptual view of various communications in the grid. Each dwelling has a specific scenario for its own appliances. This scenario includes the desired timetable of using appliances in a day. First, appliances are classified based on the *shiftability* feature [7]. Second, shiftable appliances are categorized by the *interruptibility* feature. These classifications permit consumers to give a priority to appliances, which is important for their starting time. Once the consumer chooses to operate an appliance in demand response ready mode, the consumer offers flexibility to the grid and provides an opportunity to the demand response system for reducing the peak demand.

This paper proposes a local power scheduling algorithm attempting to schedule power demand requests of appliances. Here, local means receiving the power demand requests with a specific time resolution and scheduling them accordingly. As its principal novelty, the algorithm runs concurrently and need not know the whole operating period of appliances. The scheduler intends to schedule power demand requests optimally once they arrive. At each time interval, its main responsibility is to allow some appliances to operate and shift the operating cycle of the remaining appliances to the future. This shifting is enabled by utilizing Peak Demand Thresholds (PDTs) imposed by DSOs. The scheduling algorithm attempts to keep the aggregated power consumptions below PDTs continuously.

This rest is organized as follows: Section II overviews the related work. Section III presents the system model. Section IV proposes the power scheduling algorithm. Section V discusses the simulation setup and analysis. Finally, Section VI concludes the paper and provides the possible future extensions.

II. Related Work

A considerable amount of literature is published on smart grids due to concerns on the inefficient structure of the current electrical grid in responding to the growing demand for electricity [8][9]. Farhangi [8] investigated the differential impacts of transforming the current electrical grid to a complex system of systems, named the smart grid while Fang *et al.* [9] surveyed the enabling technologies for data communications in the smart grid. With the advent of smart grids, new solutions are becoming available. To support these, demand response programs endeavor to change the electricity usage patterns of consumers in response to electricity prices or other signals. These programs are considered as reliable solutions to improve the energy efficiency and reduce the peak demand [10]. To reach these goals necessitates demand response service providers investing on proposing functional and potential power scheduling services to the smart grid.

Most of the current research on the power scheduling problem focuses on scheduling power demand requests of appliances of consumers wrapping as a single-objective framework while relying on historical data and forecasting services [11][12]. Agnetis *et al.* [11] defined the problem of optimally scheduling a limited number of manageable appliances of only one dwelling solving with a high computational

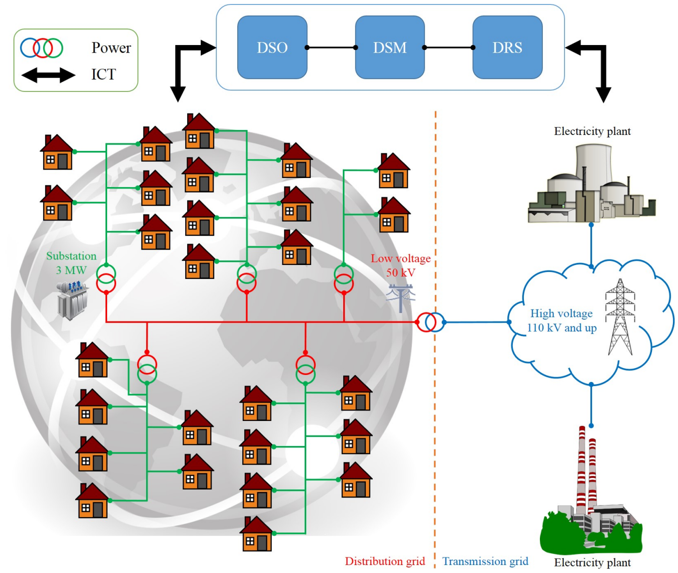

Figure 1. Conceptual view of various communications in the grid

algorithm based on the mixed integer linear programming. O'Brien [12] proposed a greedy algorithm for automatically scheduling the shiftable appliances with completely predetermined power profiles while missing to take any grid stability constraint into account.

Nevertheless, far too little attention has been paid by smart grid researchers to design a system model where power scheduling is done near real-time. Jacobsen *et al.* [1] found this gap and developed a simple but efficient smart appliance power scheduling mechanism based on the peak demand reduction strategy. Consecutively, Azar *et al.* [13] followed a design methodology that efficiently utilized a time-independent PDT policy for decreasing the aggregated peak demand considering the appliance reception minimization method. It successfully flattened the aggregated power consumption based on a centralized demand response system.

This paper advances the state of the art in formulating a demand response service where appliances send their power demand requests with a specific time resolution accompanying the consumer's time-limit flexibilities. The DSO schedules the incoming power demand requests according to the customers' and its objectives. It attempts to keep the aggregated power demands below PDTs over time.

III. System Model

This section clarifies the proposed system model, as Figure 2 illustrates its conceptual view. Consumers play a major role in this system model since they provide their desired electricity consumption scenarios and corresponding flexibilities to the demand response system. In addition, DSOs impose some grid stability constraints to maintain the electrical grid, such as PDT. Electricity prices of a typical day with the corresponding CO_2 emission data are another system input. The demand response system will receive these input data and then, executes the scheduling algorithm attempting to schedule appliances of dwellings with respect to the objectives and constraints settled in the demand response system.

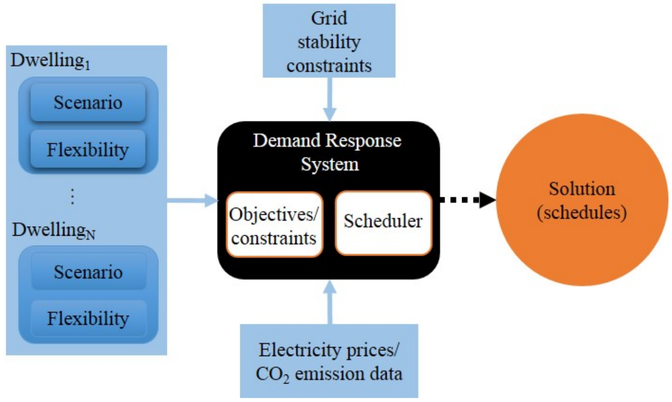

Figure 2. System model of the appliance power scheduling

A. Consumers: Appliance Point of View

This paper assumes there are $N \in \mathbb{N}$ dwellings connected to a feeder in the electrical grid. Each dwelling D_i, where $i \in \{1, 2, \ldots, N\}$, possesses $A_i \in \mathbb{N}$ appliances. Each appliance $a_{i,j}$, where $j \in \{1, 2, \ldots, A_i\}$, is a driver of residential power demands. To guarantee the full operation of appliances, the demand response system should check whether appliances have completed their responsibilities during the day or not. Therefore, Equation (1) shows this hard constraint.

$$\sum_{t=1}^{T} \left(PD_{i,j}^{t} \times x_{i,j}^{t} \right) = TPD_{i,j}, \tag{1}$$

where $PD_{i,j}^{t} \in \mathbb{R}^{*}$ (watts) is the power demand of appliance $a_{i,j}$ at time interval t. Notation $x_{i,j}^{t} \in \{0, 1\}$ is the decision variable of the optimization problem. $x_{i,j}^{t} = 1$ allows appliance $a_{i,j}$ to operate at time interval t while $x_{i,j}^{t} = 0$ shifts its operation to the future. Furthermore, $TPD_{i,j} \in \mathbb{R}^{+}$ (watts) is the total power demands of the appliance.

Appliances are classified according to some smart features named shiftability and interruptibility [7][13]. Shiftability means giving permission to the demand response system to shift the power demand requests of shiftable appliances to later time intervals. However, some appliances cannot be shifted, for instance the refrigerator. These appliances are members of non-shiftable appliances. Afterwards, shiftable appliances are divided into two groups based on the interruptibility feature. The electric vehicle is a typical example of an appliance exhibiting this feature. The demand response system can both shift and interrupt the duty cycle of charging the electric vehicle. Nevertheless, those appliances, which can be shifted, but are infeasible to be interrupted are called uninterruptible appliances (e.g., dishwasher). Their whole operating duty cycle can be shifted to another time interval. However, they should not be interrupted because of the continuity in their cycle. Equation (2) formulates this hard constraint, which is valid at each time interval:

$$\begin{aligned} &\text{Non-shiftable appliances} \rightarrow x_{i,j}^{t} = 1, \\ &\text{Uninterruptible appliances} \rightarrow \begin{cases} x_{i,j}^{t} = 1 & \text{if } x_{i,j}^{t-1} = 1, \\ x_{i,j}^{t} \in \{0,1\} & \text{otherwise}, \end{cases} \\ &\text{Interruptible appliances} \rightarrow x_{i,j}^{t} \in \{0,1\}. \end{aligned} \tag{2}$$

At each time interval t, the demand response system is signaled with power demand requests of appliances. Once it receives a power demand request from a non-shiftable appliance, it is allowed to operate. If the request belongs to an uninterruptible appliance first it should check whether the relevant appliance has been allowed to start its work at the previous time interval. If so, the system cannot interrupt and shift it to another time interval. Otherwise, it is possible to shift it, if needed. Finally, if an interruptible appliance sends a power demand request at any interval, it is possible to either allow or shift it.

In real world, consumers sometimes give priorities to use their appliances based on their preferences. For instance, the stove has higher priority compared to the laundry machine. There are two kinds of priority preference named *static* and *dynamic*. The former denotes time-independent priorities of appliances, where the pairwise comparison between each two appliances is constant with respect to some criteria such as emergent usage, welfare, or electricity cost. Each consumer can set $0 < p_{i,j} \leq 1$ as the priority of using appliance $a_{i,j}$ over the day. As a result, if the demand response system confronts a circumstance, when it should decide to select one appliance among two or more, then, the appliance, which has the highest priority will be selected [14]. Finally, as a brief description of the dynamic priority, sometimes consumers change the priorities of their appliances as time moves on. For instance, one consumer gives a priority to his/her dishwasher in the morning. In the afternoon, he/she changes its priority since the washing machine is needed to operate at the same time. Therefore, dishwasher's priority is decreased. Nevertheless, for simplifying the model, the dynamic priority constraint is not considered in this paper.

Consumers participating in demand response programs provide some flexibilities to the demand response system for operating their appliances. Let us assume one consumer is interested to plug in his/her Nissan Altra electric vehicle at 18:00. The charging cycle will typically take five hours [15]. Nonetheless, he/she is flexible to receive the electric vehicle in the finished state at most at 08:00 the next day. Therefore, the flexibility that the consumer offers to operate his/her electric vehicle is 14 hours. We name this concept as a *deadline* flexibility, which is a time-oriented constraint. This kind of flexibility helps the demand response system to shift some appliances, which relatively consume more than others, to the future. The demand response system should consider the remaining power demand flexibility (with given time limits) before shifting them. Equation (3) describes this constraint:

$$RP_{i,j}^{t} \leq \left(DF_{i,j} - t \right), \tag{3}$$

where $RP_{i,j}^{t} \in \mathbb{Z}^{*}$ relates to the number of remaining power demand requests of appliance $a_{i,j}$ from time interval t until the end of its duty cycle. Moreover, $DF_{i,j}$ (e.g., UTC) denotes the deadline flexibility of this appliance. The demand response system satisfies this constraint while it receives the power demand requests continuously. If the remaining power demand of an appliance is still less than its provided time limit flexibility, the demand response system can decide to allow it to start/continue in this time interval or to shift it to another time interval. To shift a power demand request, it is essential to ensure the satisfaction of all constraints.

Considering the aforementioned descriptions, each dwelling D_i has a specific scenario showing how the consumer intends to operate the appliances. Table I lists a sample scenario of operating the appliances in a typical dwelling. As described previously, deadline flexibility in using appliances means a firm deadline for finishing the related activity. For example, the consumer provides two hours of flexibility to the demand response system for charging the electric vehicle. More in details, it receives the first power demand request for charging the electric vehicle at the defined time. The demand response system has an opportunity to deliver the charged electric vehicle later in time by utilizing the provided deadline flexibility. It is possible to both shift and interrupt the charging process during the defined time period since the electric vehicle is a member of the interruptible appliances. Here, the priorities are time-independent (static). It is worthwhile emphasizing that the priority is applied to only shiftable appliances. Hence, the refrigerator and lighting will not undergo any scheduling procedure. They will receive an infinite priority since they are members of non-shiftable appliances.

B. Distribution System Operator: Grid Constraint Point of View

Currently, electricity producers generate more electricity since they are experiencing an insufficiency of electricity generation capacity because of the power demands by consumers. However, it can be avoided using demand shaping schemes. DSOs currently apply a threshold policy, in order to shave the peak, which results in shaping the demand profiles over time [1]. From an electricity grid point of view, the upper limit of the PDT may be enforced by the DSO by the installation of fuses and other safety-related measures such as protective relays. These devices may be dimensioned differently and the subscription fee for a dwelling often depends on the installed capacity. As a complement, adaptive schemes can be deployed as a control loop between a DSO-controlled generator side and individual dwellings [16]. Let

$$\sum_{i=1}^{N}\sum_{j=1}^{A_i}\big(PD_{i,j}^{t} \times x_{i,j}^{t}\big) \leq PDT, \tag{4}$$

where $PDT \in \mathbb{R}^{+}$ (watts) is a constant and time-independent power demand threshold, in which the demand response system attempts to keep the amount of allowed power demand requests below it. Nevertheless, Equation (4) sometimes cannot be satisfied owing to the provided deadline flexibilities and uninterruptibility feature of some appliances. Therefore, the demand response system will consider this constraint for power demand requests, in which the corresponding appliances: 1) still have time to start operating or 2) have not started yet. For the former the demand response system can still use the provided flexibility while for the latter it can shift the starting time of the appliance to the later time intervals. It is worth noting that priorities of appliances could be also considered in Equation (4).

C. Demand Response System: Objective Point of View

While the demand response system receives power demand requests of appliances, it cannot globally optimize the objectives since they are received at specific time intervals continuously. As a result, all objectives are based on a local controlling strategy, as follows.

TABLE I. A SIMPLIFIED EXAMPLE OF A DWELLING' SCENARIO

Start	End	Activity description	Deadline flexibility	Priority
00:00	24:00	Using the refrigerator	24:00	Infinite
08:00	24:00	Turning the lights on	00:30	Infinite
08:05	09:50	Putting the dishes into the dishwasher	10:30	0.2158
13:00	14:15	Putting the laundry into the washing machine	17:00	0.1063
17:25	18:15	Putting the washed laundry into the laundry dryer	22:00	0.1499
11:30	22:40	Using the computer	23:30	0.2649
19:50	22:00	Watching the TV	24:00	0.1293
20:00	22:00	Charging the electric vehicle	24:00	0.1338

1) Minimizing the Electricity Cost: Equation (5) formulates the willingness of the demand response system to minimize the electricity cost of consumers at each time interval. Here, $EP^t \in \mathbb{Z}^*$ (DKK per watts per hour) is the electricity price at each time interval.

$$f(x) = \min\sum_{i=1}^{N}\sum_{j=1}^{A_i}\big(PD_{i,j}^{t} \times x_{i,j}^{t} \times EP^{t}\big). \tag{5}$$

2) Minimizing the CO_2 Emission: Equation (6) shows the interest for reducing the CO_2 emission of dwellings at each time interval by applying the decision variable $x_{i,j}^{t}$ for all power demand requests. Here, $CO_2E^t \in \mathbb{R}^*$ (grams per watts per hour) is the amount of CO_2 emission at each time interval.

$$g(x) = \min\sum_{i=1}^{N}\sum_{j=1}^{A_i}\big(PD_{i,j}^{t} \times x_{i,j}^{t} \times CO_2E^{t}\big). \tag{6}$$

3) Maximizing the Comfort Level of Consumers: Equation (7) formulates how the demand response system is interested to maximize the comfort level of consumers over time. Comfort level indicates the consumers' desire to have their activities being done as they exactly expect from their scenarios. In fact, appliances aim to get permission to run their operations at each time interval as much as possible.

$$h(x) = \max\sum_{i=1}^{N}\sum_{j=1}^{A_i}\big(x_{i,j}^{t} \times p_{i,j}\big). \tag{7}$$

In conclusion, the demand response system considers the appliance power scheduling optimization as a mixed-integer linear programming problem including Equations (5) to (7) as its objective functions subject to Equations (1) to (4) as the relevant constraints. Next section will describe how the proposed scheduling algorithm attempts to solve this optimization problem applying diverse approaches.

IV. SCHEDULING ALGORITHM

Algorithm 1 presents the pseudo-code of the power scheduling algorithm. Considering the system model shown in Figure 2, the demand response system executes the scheduling algorithm to produce a specific schedule for appliances of dwellings based on the objectives and constraints, described in Section III. It receives power demand requests at specific time intervals. Apart from the PDT, the scheduler allows the non-shiftable power demand requests to start or to continue their

Algorithm 1: Power scheduling

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Input  : The scenarios, power profiles, classification of appliances, PDT.
Output: Schedule of appliances of all dwellings.
Preprocessing the input data;
while receiving the power demand requests over time do
    Allow the non-shiftable appliances to start or to continue;
    Update PDT;
    if there are uninterruptible appliances, which have started previously then
        Allow them to continue;
        Update PDT;
    end
    if there are appliances, which cannot be shifted due to their deadline flexibility constraint then
        Allow them to start or to continue;
        Update PDT;
    end
    if there are some remaining power demand request then
        if their total consumption is less than the remaining PDT then
            Allow them to start or to continue;
            Update PDT;
        else
            Refer to the single/multi-objective Knapsack procedure;
            Allow the output power demand requests of the Knapsack procedure to start or to continue;
            Shift the remaining power demand requests to the next time interval;
        end
    end
end
```

duties. Furthermore, if there is an uninterruptible appliance, which has started at the previous time interval, it should be allowed to continue. Finally, if there is a power demand request, where shifting it to the next time interval violates its provided deadline flexibility, then, the same action of allowing it to start takes place. After finishing these procedures, the algorithm will check whether the total power demand of the remaining requests is below the remaining PDT (capacity) or not. If so, all will be permitted to start or to continue their procedure. Otherwise, the algorithm refers to the Knapsack procedure to select some requests from the remaining power demand requests to enable them to start or to continue, and shift the unselected requests to the next time interval.

Two challenging circumstances can occur during the scheduling, and handling them confirms the robustness of the scheduling algorithm. If there is a sudden drop in the electric power, indeed no appliance can send any power demand request. Therefore, the scheduling algorithm waits until the appliance sends its new power demand request. Furthermore, if all appliances in all dwellings are configured as non-shiftable with high priorities, the scheduling algorithm will allow all of them to operate, when they send their power demand requests. This is based on respecting the consumers who do not provide any flexibility to non-shiftable appliances. However, this is considered to be an infeasible and greedy setup.

A. The Knapsack Problem

The Knapsack problem is one of the traditional problems of computer science in combinatorial optimization literature [17]. Given F items, the Knapsack tries to pack the items to obtain the maximum total value. Each item gets a weight and value. The maximum weight that the Knapsack can tolerate is limited by a fixed capacity W. This problem has two versions: "0-1" and "fractional". In the former, items are indivisible meaning it is possible to either take an item or not. In contrast, in the fractional version, items are divisible and, therefore, the Knapsack can take any fraction of an item.

This paper gets the benefit from the first version since the remaining power demand requests are similar to the indivisible items in "0-1" Knapsack problem. The "0-1" Knapsack problem is NP-Complete since the time complexity of solving it in a brute-force approach is $O(2^F)$. Time complexity measures the time that an algorithm takes as a function of the size of its input. Applying brute-force approach means calculating the fitness of 2^M solutions to locate the optimal one. The power scheduling problem is reducible to this version since the demand response system should decide to allow those indivisible power demand requests, which optimize the objective(s) and satisfy the constraints simultaneously. Therefore, the discussing problem is also NP-Complete. Hereinafter, we the scheduler needs to refer to the Knapsack problem, we name it the Knapsack procedure.

Indeed, the Knapsack procedure requires not only to decide, which power demand requests have to be processed now and delay the others afterwards, but should also consider the starting (ending) times of the latter. The latter is reflected in the flexibility that consumers provide.

Table II defines the equivalent parameters of the Knapsack and power scheduling optimization problems according to various objectives. As described previously, the Knapsack procedure receives the remaining power demand requests, which their total power demand is indeed more than the remaining capacity. It calculates the fitness of produced feasible solutions, in which each solution includes some power demand requests.

TABLE II. EQUIVALENT PARAMETERS OF THE KNAPSACK PROCEDURE AND POWER SCHEDULING OPTIMIZATION PROBLEM

	Values (items)	Objective(s)	Weights	Capacity
Single-objective	Electricity cost of power demand requests	Minimizing the total electricity costs	Power demand requests	PDT
	CO_2 emission of power demand requests	Minimizing the total CO_2 emissions	Power demand requests	PDT
	Priority of power demand requests	Maximizing the total allowed power demand requests	Power demand requests	PDT
Multi-objective	Electricity cost and priority of power demand requests	Minimizing the total electricity costs and maximizing the total number of allowed power demand requests	Power demand requests	PDT
	CO_2 emission and priority of power demand requests	Minimizing the total CO_2 emission and maximizing the total number of allowed power demand requests	Power demand requests	PDT

As a result, the solution to this problem is a subset of received power demand requests, which should be allowed to start or to continue in this time interval. Then, there will most likely be some remaining power demand requests, which cannot successfully start or continue. These power demand requests should be shifted to the future.

Depending on the number of objectives chosen by the demand response system, different approaches can be used to run the Knapsack procedure. On the one hand, if the demand response system decides to run the scheduling with one objective, the scheduling problem turns into a single-objective optimization problem. This is equal to run the single-objective "0-1" Knapsack procedure with dynamic programming at each time interval (if needed) [14]. On the contrary, if at least two objectives are chosen, the scheduling algorithm corresponds to a multi-objective optimization problem, which has to be solved with relevant techniques [18]. It is worth noting that these approaches are used at each time interval, if needed. The following describes them.

1) Dynamic Programming: We utilize a dynamic programming approach to solve single-objective power scheduling problem. As Figure 3 demonstrates its principles, this approach first characterizes the structure of an optimal solution. Then, it decomposes the problem into smaller problems. Meanwhile, it finds a relationship between the structure of the optimal solution of the original problem and solutions of the smaller problems. It recursively expresses the solution of the original problem in terms of optimal solutions to smaller problems, which supports the optimality.

To this end, it follows a bottom-up computation approach. The value of an optimal solution is computed in a bottom-up manner using a table structure. This table is repeatedly filled to use in each iteration [19]. The structure of an optimal solution to the power scheduling problem is a subset of the remaining power demand requests, which optimizes the relevant objective. Algorithm 2 declares the dynamic programming method for running the single-objective Knapsack procedure. The time complexity of approaching the Knapsack procedure using dynamic programming is $O(M \times PDT)$.

2) Multi-Objective Optimization: Multi-Objective Optimization (MOO) is an area of multiple criteria decision-making, where mathematical optimization problems involving more than one objective function should be optimized simultaneously [20]. Optimal decisions are taken in the presence of trade-offs between two or more conflicting objectives. Solving a MOO problem necessitates computing all or a representative set of Pareto-optimal solutions. In this paper, a Pareto solution comprises a subset of remaining power demand requests. When decision-making is emphasized, the objective of solving a MOO problem is to support a decision-maker in

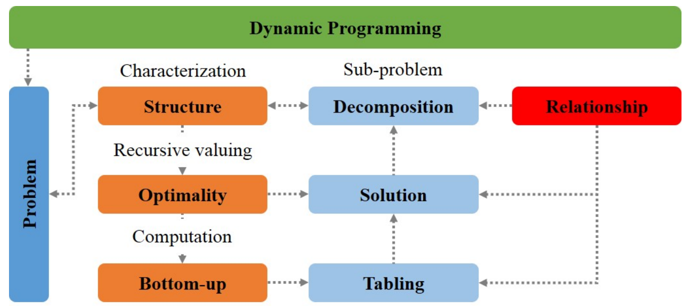

Figure 3. Principles of the dynamic programming approach

Algorithm 2: Approaching the Knapsack procedure: Dynamic programming

Input : power demand requests, PDT.
Output: The optimal solution at the current time interval.

Set F as the number of input power demand requests;
Create a $(F+1) \times (PDT+1)$ table named V;
if *the objective is minimization* **then**
  Set $V[0, 0 : PDT+1]$=Inf;
else
  Set $V[0, 0 : PDT+1]$=0;
end
for $i = 1$ *to* F **do**
  for $j = 1$ *to* PDT **do**
    if $PD[i] \leq j$ **then**
      if *the objective is minimization* **then**
        $V[i, j] = \min(V[i-1, j], PD[i] + V[i-1, j-PD[i]])$;
      else
        $V[i, j] = \max(V[i-1, j], PD[i] + V[i-1, j-PD[i]])$;
      end
    else
      $V[i, j] = V[i-1, j]$;
    end
  end
end
Return the $V[F, PDT]$ as the final solution;

finding the most preferred Pareto-optimal solution. Here, the decision-maker is the demand response system, which should decide to allow only a subset of the remaining power demand requests to optimize the objectives and satisfy the constraints at each time interval accordingly. The objective functions are in

Algorithm 3: Approaching the Knapsack procedure; Multi-objective evolutionary algorithm

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Input : Remaining power demand requests, PDT, population size (Q), number of generations (G), crossover (p_c) and
        mutation (p_m) probabilities.
Output: A near-optimal solution at the current time interval.
Randomly produce initial solutions and combine them as the parent population;
Evaluate the parent population based on the objective functions;
Calculate the Pareto-fronts and the crowding distance of solutions inside the parent population;
c = 1;
while c ≤ G do
    Apply the selection operator on the parent population and forward to the crossover operator;
    Apply the crossover operator on the received solutions with a probability of p_c and forward to the mutation operator;
    Apply the mutation operator on the received solutions with a probability of p_m and put them into the offspring
    population;
    Evaluate the offspring population based on the objectives;
    Combine the parent and offspring populations into a temporary population;
    Calculate the Pareto-fronts and crowding distances of solutions inside the temporary population;
    Select solutions from the Pareto-fronts orderly while replacing them with solutions in the parent population until
    reaching Q;
end
Return a Pareto-solution from the first Pareto-front as a near-optimal solution;
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conflict, when there exist an infinite number of Pareto-optimal solutions. A Pareto-optimal solution does not improve for one objective unless it satisfies others. The main goal in MOO problems is to find a finite Pareto-front in the objective space including a finite number of diverse Pareto-solutions.

Evolutionary Algorithms (EAs) are one of the most well-known meta-heuristic search mechanisms utilized for the MOO problems since their structure is free of search space and objective capacities [21]. EAs form a subset of evolutionary computation, in which they generally involve techniques and implementing mechanisms inspired by biological evolutions such as reproduction, mutation, recombination, natural selection, and survival of the fittest. The main advantage of EAs, when applied to solve MOO problems, is the fact that they typically generate sets of solutions, allowing computation of the entire Pareto-front. Currently, most Multi-Objective Evolutionary Algorithms (MOEAs) apply Pareto-based ranking schemes such as the Non-Dominated Sorting Genetic Algorithm-II (NSGA-II) [22]. Algorithm 3 describes the procedure of running the multi-objective Knapsack procedure using the NSGA-II. The time complexity of approaching the Knapsack procedure using the NSGA-II is $O(G \times M \times Q^2)$, where G is the number of generations, M is the number of objectives, and Q is the population size.

The NSGA-II randomly generates an initial Pareto-population, and then, applies some evolutionary procedures such as tournament selection with crossover and mutation operators. Next, it generates an offspring population from parents in each generation. It classifies the temporary population, as the combination of parent and offspring populations, based on the dominance principle to some fronts f_1, f_2, f_3 and so on. A solution Sol_1 dominates a solution Sol_2, if Sol_1 is better than Sol_2 in some objectives and perhaps equal to others. All the solutions, which lie in one specific front are non-dominant. In addition, for each solution Sol_a in f_k, there exists a solution Sol_b in $f_{k'}$ such that Sol_b dominates Sol_a, where $k' < k$. In the last step, the NSGA-II fills the next generation's population starting from the first front and continuing with solutions in the next fronts. Since the size of the combined population is twice the new one, all fronts, which could be unable to accommodate are removed. However, it needs to handle the last allowed fronts, in which some of its solutions are possibly considered in the new population. In this situation, the NSGA-II uses a niching strategy to choose solutions of the last allowed fronts, which lie in the least crowded regions of the solution space. To this end, it finds the distance between each solution and its nearest left and right neighbors in the last allowed fronts for each dimension in the objective hyperspace. Finally, it sums up such distances for each solution as the largest hypercube around it, which is empty from other solutions. The largest hypercube shows a solution with the least crowd. Figure 4 elaborates a conceptual view of Pareto-fronts and Pareto-solutions with corresponding crowding distances.

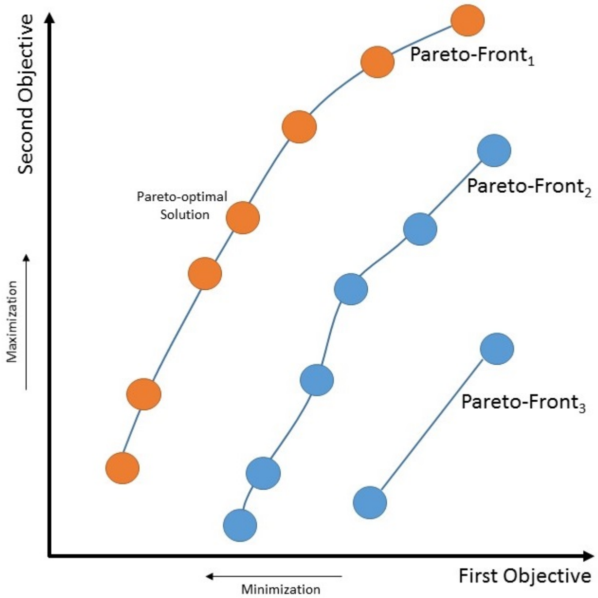
V. Simulation Setup and Analysis

This section first describes the simulation setup and subsequently, analyzes the results.

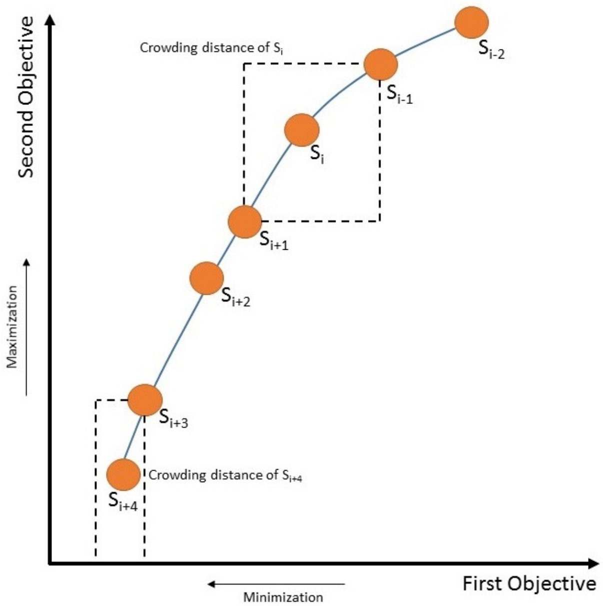
A. Simulation Setup

This work has been implemented with Matlab R2014b on a personal computer with an Intel Core i7-2.0 GHz CPU and 6 GB memory. Power profiles of all appliances are captured from the TraceBase open repository, which comprises a collection of real power traces of electrical appliances [23]. The electricity prices in the Danish day-ahead market, known as Elspot market, are provided by Nord Pool Spot with an hourly resolution on the day before the power delivery [24]. CO_2 emission intensity prognosis data are also provided in an hourly resolution by the Danish transmission system operator [25]. It is significant to note that the demand response system is set to receive the power demand requests at five-minute time intervals until finishing all activities. At each hour, it receives the power demand requests 12 times. As a result, T has been set to 24×12. $N = 100$ dwellings are assumed to provide their power demand requests over time.

A precise scenario for each dwelling is created randomly based on power profiles of appliances. Corresponding power



(a). Pareto-fronts and solutions



(b). Crowding distance of the Pareto-solutions

Figure 4. Conceptual view of Pareto-fronts and Pareto-solutions with corresponding crowding distances

demand requests are established in each scenario. To streamline the model, each appliance is operated only one time. Regarding flexibilities, we generate a random flexibility value for each appliance. A lower bound for each flexibility value is the following time interval from the moment, at which the operating cycles should finish without scheduling. An upper bound for each flexibility value is the end of the day.

It is considered that priorities are generated randomly. Figure 5 shows the aggregated power demand of the appliances of one dwelling in a typical day. Figure 6 shows the aggregated power demands of 100 dwellings. Peak power demand occurs at 20:30, which is 293 kW. Therefore, in order to allow all requested power demands at each time interval without shifting or interrupting any of them, the PDT should be at least 293 kW since it has been indicated that the PDT is constant and time-independent. However, the demand response system desires to flatten the aggregated demand by shifting power demand requests from on-peak periods to off-peak times. Therefore, it modifies the PDT to enable the shifting and interruption.

TABLE III. SIMULATION CASE STUDIES INSPIRED FROM TABLE II

	Objective(s)
Case study 1	1) Minimizing the electricity cost
Case study 2	1) Maximizing the comfort level of consumers
Case study 3	1) Minimizing the electricity cost 2) Maximizing the comfort level of consumers

As described earlier, the MOEA includes some evolutionary parameters. As a selection operator, this paper utilizes the tournament selection. Linear crossover and exchange mutation are also utilized as the exploitation parameters. Their probabilities are set to p_c=80% and p_m=20%, respectively. Finally, the population size (Q) and the number of generations (G) are both adjusted to 100.

B. Simulation Analysis

This section analyzes the results obtained based on three simulation case studies, as Table III lists. The first case study is single-objective and aims to minimize the electricity cost as its objective function (see Equation (5)). The second case study is also single-objective and attempts to only maximize the comfort level of consumers (see Equation (7)). Finally, the third case study is multi-objective and intends to both minimize the total electricity cost and maximize the comfort level of consumers. We omit to show a case study including minimization of the CO_2 emission as an objective function since it would be similar to minimizing the electricity cost. The results will be analyzed based on variations of the PDT as follows:

$$PDT = \{(10\% \sim 100\%) \times PPD\}, \quad (8)$$

where $PPD \in \mathbb{R}^+$ (watts) denotes the peak power demand. It is equal to 293 kW (see Figure 6). We change the PDT from 10% to 100% to analyze the obtained results. Hereinafter, when PDT is equal to R%, where $10\% \leq R \leq 100\%$, it means PDT=R×PPD. We examine the effects of these variations on:

- Computation time of running the algorithm over time;
- Number of referrals to the Knapsack procedure;
- Computation time of the total number of referrals to the Knapsack procedure;
- Total electricity costs of the dwellings in a day;
- Deviation between the reception and delivery times of appliances;
- Aggregated power demands of the scheduled scenarios in a day.

Figure 7 analyzes the computation of running the scheduling algorithms based on different case studies. In Figure 7(a), according to Algorithm 1, non-shiftable power demand requests will be allowed to start or to continue apart from the assigned PDT. Considering computation time, when PDT is equal to 10%, the remaining capacity for allowing the remaining power demand requests is very low or even below zero.

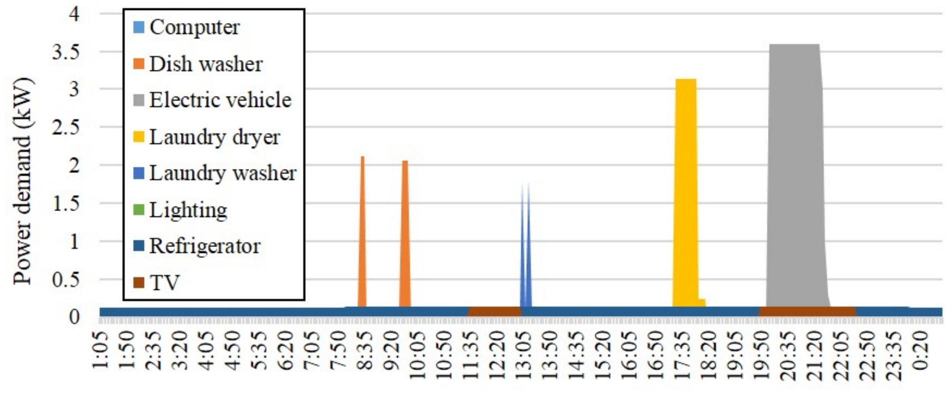


Figure 5. Aggregated power demand of appliances used in Table I

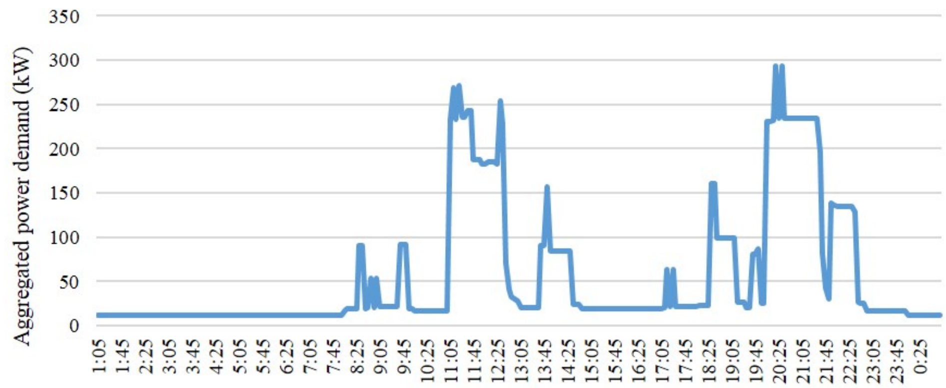
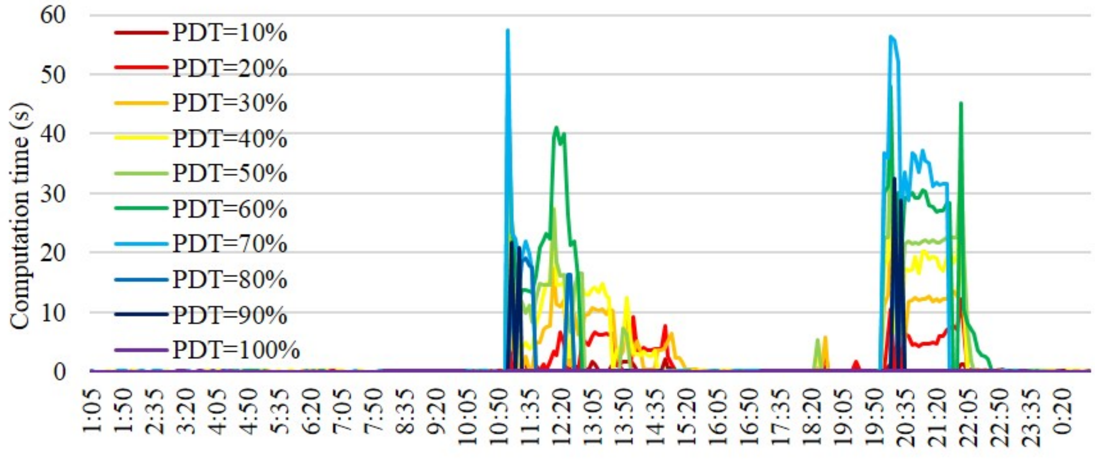


Figure 6. Aggregated power demands of 100 dwellings based on randomly generated scenarios in a typical day

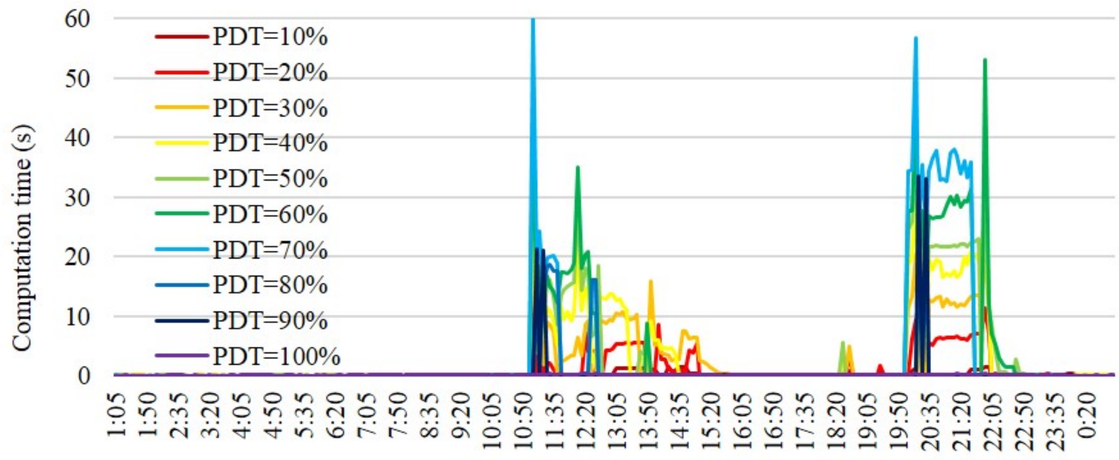
The reason is that the algorithm should satisfy Equations (1) to (4). Therefore, it is not possible to run the Knapsack procedure since the minimum consumption of the remaining power demand requests is greater than the remaining capacity. In the next intervals, the system, apart from the remaining capacity, should allow some power demand requests to start or to continue, for which shifting or interrupting them is not possible due to their deadline flexibility constraints. As a result, the number of remaining power demand requests as inputs to the Knapsack procedure will be few and, therefore, computation time will be lowered accordingly. Nevertheless, when PDT increases, the Knapsack procedure will allow more power demand requests to start or to continue at each interval. Some of these allowed power demand requests are members of the uninterruptible set. Therefore, at the next intervals, the system has to allow the corresponding appliances to continue their operation apart from the PDT. The demand response system will confront more remaining power demand requests compared to lower assigned PDT in later time intervals. This will increase the complexity and computation time of running the Knapsack procedure.

We experience more complexity and higher computation time, when assigned PDT increases. Nevertheless, the number of intervals, in which the Knapsack procedure should run decreases. Having some uninterruptible appliances and time limit flexibility constraints make this decreasing. If the system allows an uninterruptible power demand request to start at a certain time interval, it will be unable to interrupt it in the following intervals. Therefore, it will have to shift more power demand requests since the remaining capacity has decreased. These shifted power demand requests will be accumulated and, finally, the Knapsack procedure will face several remaining requests. When PDT is 90%, we observe a noticeable decrease in computation time compared to previous figures. The reason is the reduced amount of the Knapsack procedure's inputs. Since the aggregated power demands of the remaining power demand requests are less than the remaining capacity at most of the time intervals, it is not necessary to run the Knapsack procedure. Obviously, there is no need to run the Knapsack procedure at any of the time intervals, when the threshold is equal to 100%.

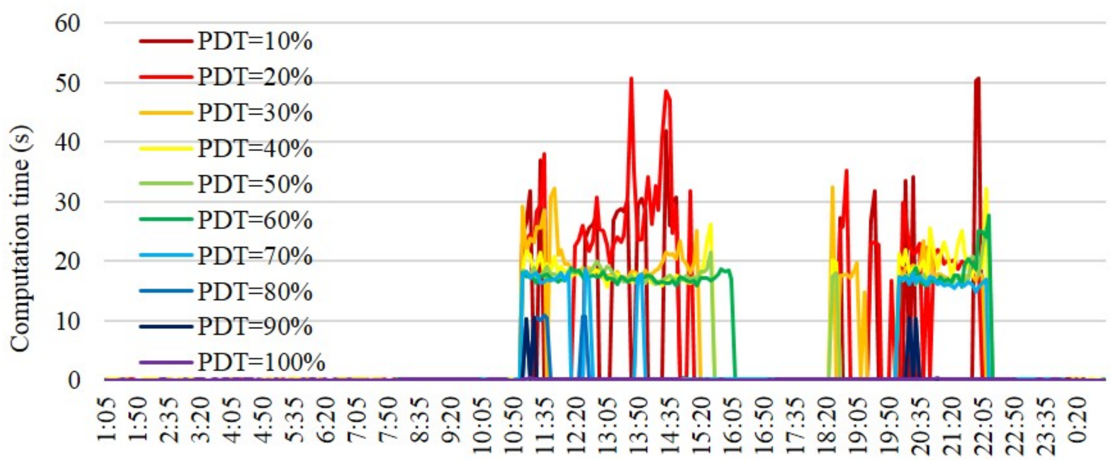
Figure 7(b) demonstrates the same analysis based on the second case study. The description of this figure is almost the same as Figure 7(a). However, there are some minor differences, which are linked to the differences in the nature of the objectives. The main reason is underlining the intention of consumers to pay for the highest comfort as little as possible. The computation time of running the third case study is illustrated in Figure 7(c). In contrast to Figures 7(a) and 7(b), here, the computation time is completely different. The main reason is the repetitive manner of the MOEA in finding the non-dominated near-optimal solution at each time interval. As described previously, there is no exact solution for multi-objective problems. Therefore, the near-optimal solutions ob-



(a). Computation time of running the scheduling algorithm based on the first case study



(b). Computation time of running the scheduling algorithm based on the second case study



(c). Computation time of running the scheduling algorithm based on the third case study

Figure 7. Computation time of running the scheduling algorithms based on different case studies

tained from running the algorithm at each time interval, affect the computation time of subsequent intervals. Computation times for the next intervals may change due to the randomized nature of finding near-optimal solutions. If all scenarios and relevant information are known before scheduling, it will be possible to limit the computation time. However, in this situation, when the system receives the power demand requests with a specific time resolution, it is not possible to do it since there is no future prediction or even forecasted data to learn before scheduling.

According to Figure 7(c), the computation time decreases, when PDT is 50% or more. The total power demand of remaining requests at 22:00 is a bit more than the remaining capacity. Also, most of the corresponding appliances are members of the uninterruptible appliances. Therefore, the Knapsack procedure's output comprises most of them. The demand response system should allow them to continue their duties at the next time intervals apart from the remaining capacity. This decreases the computation time at the next time intervals since the number of inputs to the Knapsack procedure decreases. As the final note, in this analysis, only 35% of the CPU speed and 400 MB of memory have been employed by the local power algorithm in all three case studies in the worst case.

Figure 8 analyzes the number of referrals to the Knapsack procedure in Algorithm 1. Figure 9 studies the corresponding computation time, when PDT changes. According to Figure 8, the number of referrals to the Knapsack procedure in the first two case studies is different, when PDT is equal to 10%. The reasons are first the reductive nature of Equation (5) and second the remarkable difference between the assigned PDT and the power demand of the remaining requests. When the threshold changes to at least 20%, uninterruptible power demand requests will roughly be allowed to start or to continue their work at the time they desire. Therefore, the number of inputs to the Knapsack procedure will decrease and the total number of referrals to the Knapsack procedure in the first two case studies will be almost the same. Now, due to the multi-objective nature of the third case study, the total number of referrals will also be more than previous case studies since the outcome solutions of the Knapsack procedure at each time interval are near-optimal.

According to Figure 9, the computation time of the total referrals to the Knapsack procedure increases when the number of referrals rises. However, this fact is applicable to only the first two case studies. The computation time of running the multi-objective algorithm is decreased when the number of referrals to the Knapsack procedure increases. Similar to the provided descriptions to Figure 7(c), this algorithm does not seek to obtain the optimal solution of the problem. As a result, the near-optimal solutions contain a mix of interruptible and uninterruptible power demand requests. Intuitively, the uninterruptible power demand requests will not be shifted to the next intervals and, therefore, the number of Knapsack procedure's inputs will decrease.

As the next analysis, Figure 10 displays the differences between the total electricity costs in the three case studies based on the variations in PDT. With respect to Figure 7, computation time increases nearly linearly when PDT changes. The total electricity cost is the same since the total number of interruptions decrease when the threshold increases. Thus, appliances start operating roughly at their desired time. This causes the peak times to remain over the time (see Figure 6). Nevertheless, with decreasing the PDT, some of the power demand requests should be shifted to the low price intervals, which result in decreasing the total electricity costs. As can be easily seen, the electricity cost is reduced for 1%, when PDTs are equivalent to 10% and 100%. Having almost low fluctuating Danish electricity prices make this very low reduction.

The third case study performs better in terms of electricity cost reduction. This is due to having a multi-objective problem. For instance, in the second case study, the algorithms tries to find an optimal solution at each time interval. An optimal solution should include the maximum number of possible power demand requests. However, this is different in the third case study since objectives are conflicting. Therefore, the solution's size is smaller, which causes the requests being shifted to lower electricity price periods.

Table IV analyzes the actual required PDT and the differences at peak time intervals when the assigned PDT changes based on Equation (8). The variation rates of PDT required for scheduling the power demand requests in first two case studies are almost the same. If we compare the maximum needed PDT in the first case study with the second one when assigned PDT rises, we observe that the gradients of maximum needed PDTs are almost similar to one another. Nevertheless, the decreasing gradient of PDT, when the system applies the third case study, is lower than the other case studies. The time interval, at which the peak demand happens, is equivalent in the first two case studies. This time interval is different in the third case study due to its multi-objective nature.

According to Equation (7), consumers desire to receive their appliances in the completed status at the time they expect. This expected time for each appliance is the sum of the time periods provided in the scenarios and the corresponding additional deadline flexibility period. However, it is not possible to satisfy all consumers due to some restrictions such as PDT. The average deviation between reception and delivery times of each appliance of each dwelling for all case studies is pictured in Figure 11. These waiting times do not result in a violation of the deadline flexibility constraint. Assigning 60% is beneficial to minimize the deviation between delivery and reception times of each appliance in the first two case studies. Consumers have to wait to receive their charged electric vehicle almost 30 minutes when PDT is 60%. For the multi-objective case study, if PDT is 80%, consumers should wait averagely almost 20 minutes for receiving their charged electric vehicle. It is worth emphasizing that these waiting times are in addition to the time it takes to actually charge the EV.

As the final analysis, Figures 12 demonstrates the aggregated consumption of the scenarios after applying the scheduling algorithm. The demand response system endeavors to flatten the aggregated power consumption over the day. According to Figure 12(a), it shows the best condition of aggregated power demand when PDT is equal to 60% (17 kW). If the system does not apply any scheduling algorithm on the received power demand requests, i.e., PDT is 100%, the total maximum consumption will be approximately 293 kW. It proves that the demand response system can reduce the peak demand by 40%. This fact is also applicable to the second case study shown in Figure 12(b). Finally, it is worthy to note that since the complexity of the multi-objective case study is high, it needs a high PDT. Figure 12(c) pictures the

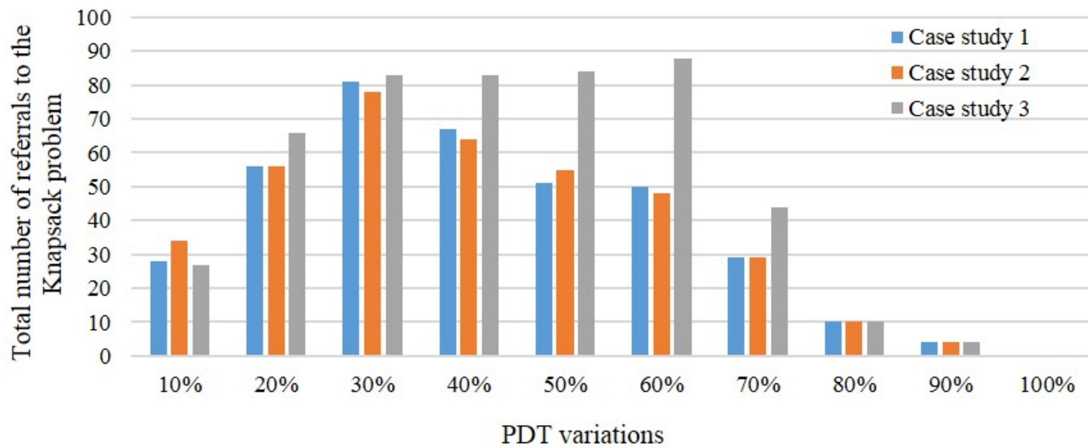


Figure 8. Total number of referrals to the Knapsack procedure in Algorithm 1

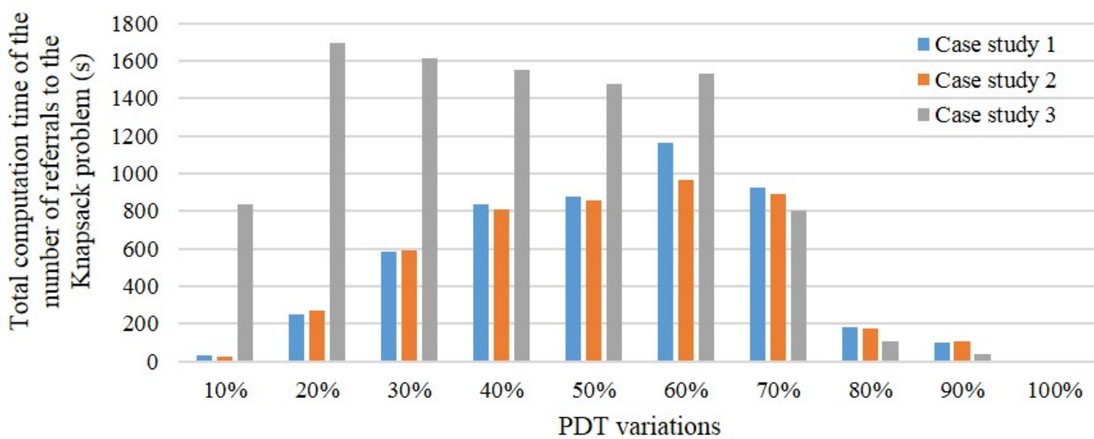


Figure 9. The computation time of referrals to the Knapsack procedure in Algorithm 1

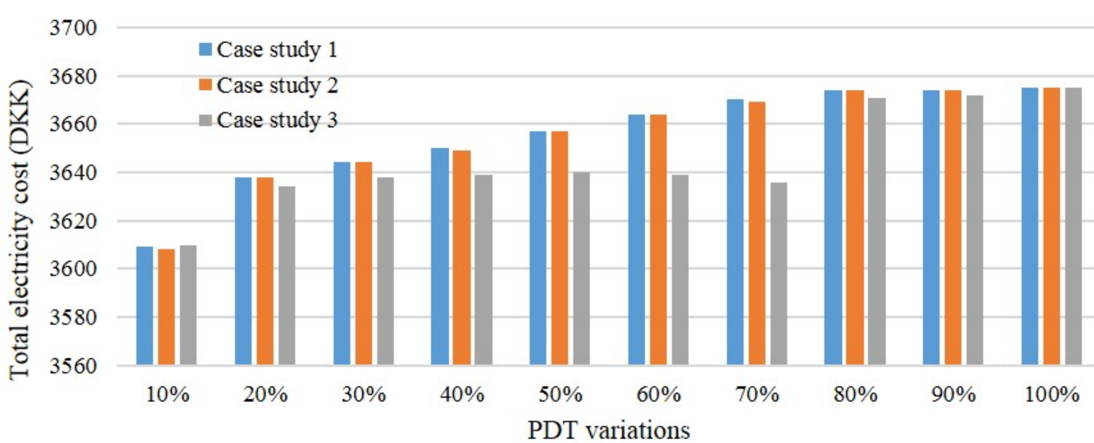
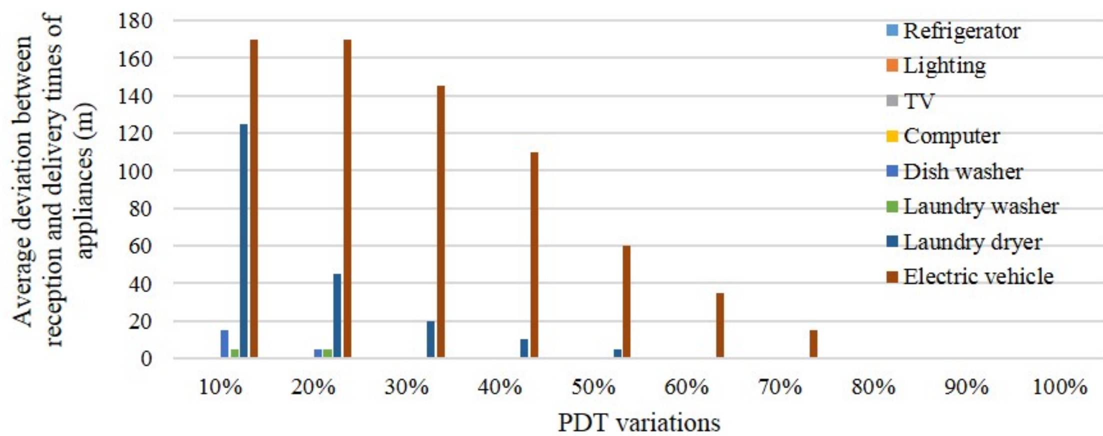
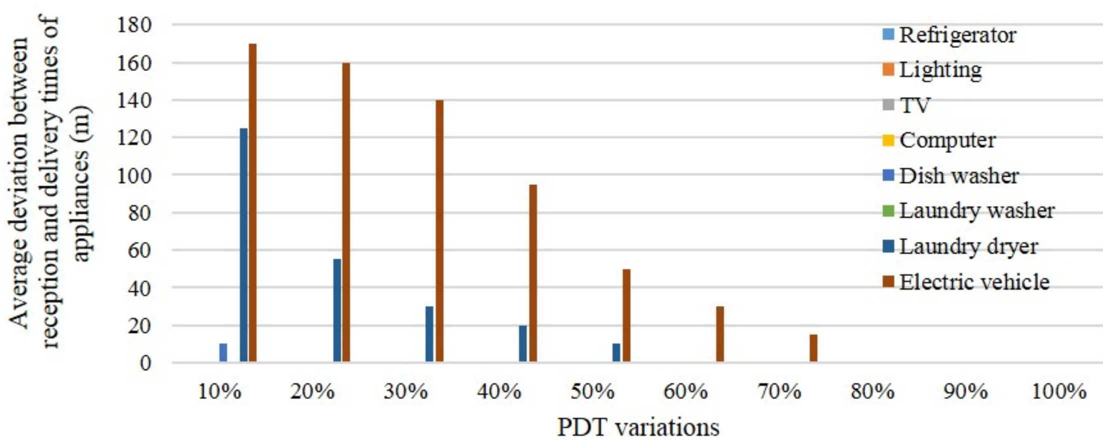


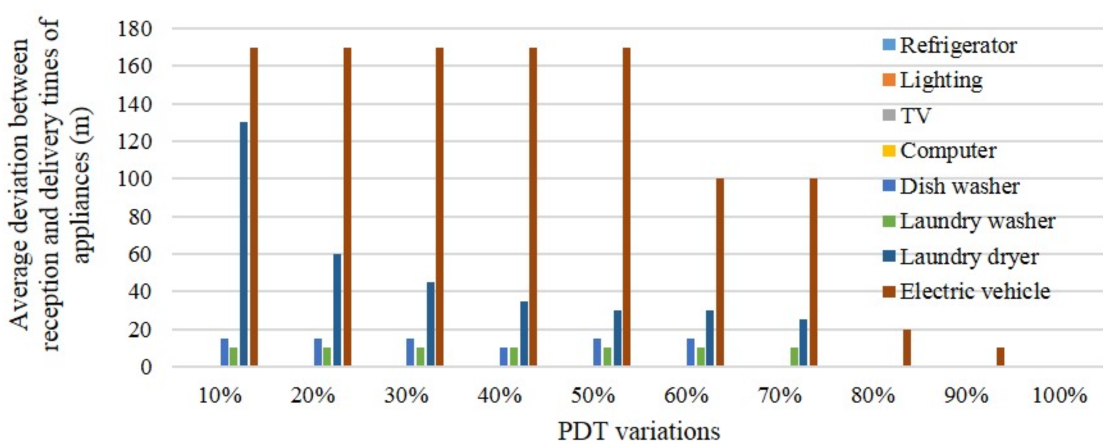
Figure 10. Total electricity costs of dwellings in a day based on three case studies



(a). Deviation time between appliance delivery and reception times based on the first case study



(b). Deviation time between appliance delivery and reception times based on the second case study

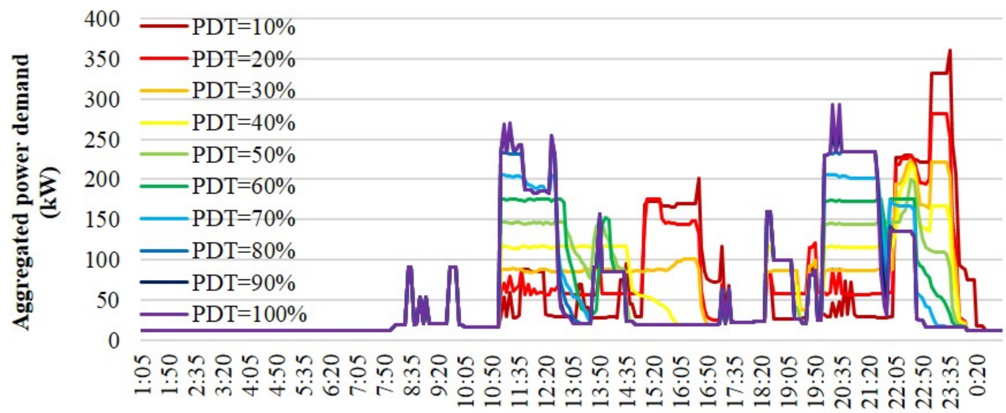


(c). Deviation time between appliance delivery and reception times based on the third case study

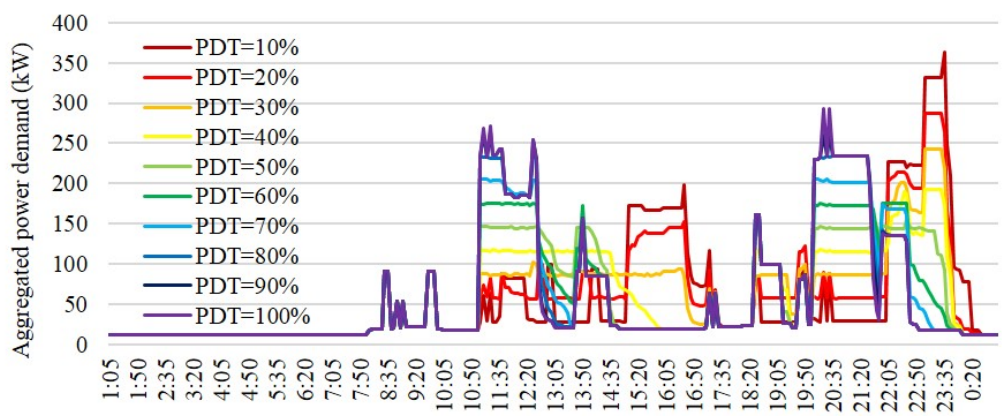
Figure 11. Deviation time between appliance delivery and reception times based on different case studies

TABLE IV. MAXIMUM NEEDED PDT AND CORRESPONDING PEAK TIME INTERVAL WHEN THE ASSIGNED PDT CHANGES

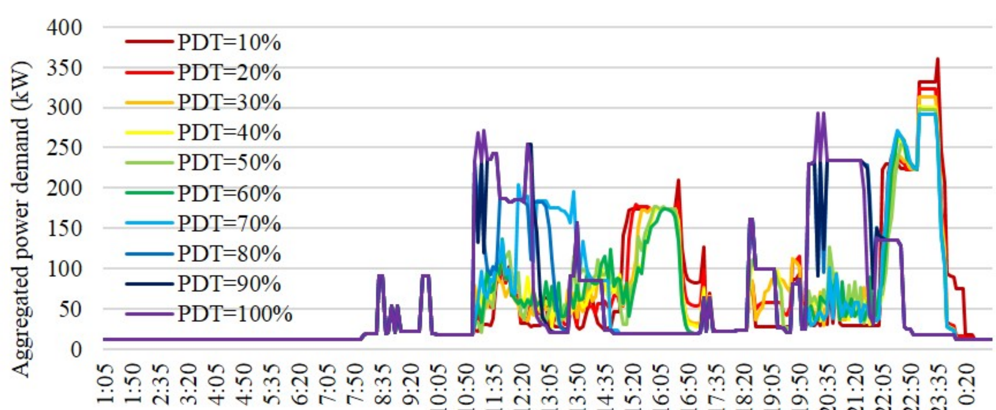
		Assigned PDT									
		10% 293 kW	20% 586 kW	30% 879 kW	40% 117 kW	50% 146 kW	60% 175 kW	70% 205 kW	80% 234 kW	90% 263 kW	100% 293 kW
Maximum needed PDT	Case study 1	360 kW	282 kW	222 kW	218 kW	200 kW	175 kW	205 kW	234 kW	263 kW	293 kW
	Case study 2	363 kW	287 kW	242 kW	192 kW	155 kW	175 kW	205 kW	234 kW	263 kW	293210
	Case study 3	360 kW	322 kW	213 kW	300 kW	298 kW	291 kW	291 kW	234 kW	254 kW	293 kW
Peak time interval	Case study 1	23:35	23:05	22:30	22:30	22:30	11:05	20:20	20:35	11:20	20:30
	Case study 2	23:35	23:05	23:05	23:05	22:35	11:20	11:05	20:35	11:20	20:30
	Case study 3	23:35	23:05	23:05	23:05	23:05	23"05	23:05	20:35	12:30	20:30



(a). Aggregated power demand of 100 dwellings based on the first case study



(b). Aggregated power demand of 100 dwellings based on the second case study



(c). Aggregated power demand of 100 dwellings based on the third case study

Figure 12. Aggregated power demand of 100 dwellings based on different case studies

aggregated consumption of 100 dwellings when the system applies the third case study. In this figure, the demand response system will receive the minimum aggregated power demand, when PDT is equal to 80%. In this status, the maximum power demand is 234 kW and the achievement is 20%.

VI. Conclusion and Future Work

This paper developed a demand response system. It received power demand requests of appliances continuously and scheduled them accordingly. Appliances are classified based on the shiftability and interruptibility features. The well-known "0-1" Knapsack procedure has been applied to the scheduling problem, when it is necessary to choose some requests to allow them to start or to continue their duties at the current time interval and shift the remaining to the future time intervals. The objectives of the proposed scheduling algorithm are minimizing the total electricity costs and CO_2 emission intensities coupling with maximizing the satisfaction of consumers. In addition, as constraints, the system attempts to keep the total power demands under a constant and time-independent power demand threshold provided by distribution system operators at each time interval. Consumers may provide time limits of flexibilities of electricity powers to the demand response system. These time limit flexibilities of power demand requests vary among appliances. It helps the system to find an optimal or near-optimal solution (based on the approach used) to decide when to shift or to interrupt power demand requests. The results were analyzed based on changing the thresholds. It was confirmed that applying this kind of threshold led to a reduction in the total electricity costs, a change in the daily behavior of consumers in a beneficial way, and additionally, a flattened aggregated power demand.

An investigation of reformulating the current power scheduling algorithm to a hierarchical scheduling algorithm to run in each dwelling is a promising future work. It would be also interesting to investigate the sensitivity of the scheduling algorithm to the stochasticity of power profiles. In practice, the adaptation of power demand thresholds can be accomplished by implementing a control loop between the demand response system and a gateway deployed in each dwelling.

Acknowledgment

The work in this document has been funded by European Union Seventh Framework program (FP7/2007-2013) under grant agreement n° 619560 (SEMIAH).

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