On the Impact of Residual Inter-Subchannel Interference for the Single-Carrier Block Transmission

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Abstract—In the fast time-varying channel, there exists interference among subchannels for the orthogonal frequency-division multiplexing (OFDM) system. The problem of inter-subchannel interference (ICI) also appears in the single-carrier (SC) block transmission with frequency-domain equalization (FDE). There are several ICI reduction algorithms for the OFDM system, and some of these algorithms can be applied in the SC-FDE system. On the other hand, unlike the scenario of carrier-frequency offset, the ICI caused by the time-varying mobile channel cannot be completely removed, and the residual ICI exists after the ICI reduction. In this paper, we analyze the effect of residual ICI. We show that the impacts of the residual ICI on the OFDM system and the SC-FDE system are quite different. For the SC-FDE system, the residual ICI incurs the “error rising” when we detect the time-domain symbols after the frequency-domain equalization. The error-rising effect is worse than the error-floor effect. To avoid the error-rising effect caused by the residual ICI, one can apply the maximum-likelihood (ML) detection, or its complexity-reduced schemes, like the K-Best algorithm. The simulation results verify our analysis.

Keywords—OFDM, ICI, SC-FDE

I. INTRODUCTION

In modern wireless communications, the orthogonal frequency-division multiplexing (OFDM) system is widely applied for its low-complexity frequency-domain equalization (FDE) in the multipath fading channel. The intersymbol interference (ISI) can be removed with a proper cyclic prefix (CP). On the other hand, one can also implement the single-carrier (SC) block transmission with FDE by adding a CP ahead of each transmitted block of symbols. Compared with traditional SC systems, the SC-FDE system attains higher spectral efficiency. The SC-FDE system also avoids the effect of high peak-to-average power ratio (PAPR) in the OFDM system.

In the high-mobility channel, both the OFDM system and the SC block transmission suffer the inter-subchannel interference (ICI). Compared with the ICI caused by the carrier-frequency offset, the ICI caused by the channel mobility is more difficult to handle. Since for the high-mobility channel, the received signal is composed of the components from different angles around the receiver, and these components have different Doppler frequency shifts. Among the ICI cancellation or reduction algorithms [1]-[9] for the OFDM system, the ICI self-reduction algorithms [3][4][6][7][9] are of much lower complexity, and the ICI reduction is independent of the signal-to-noise ratio (SNR). We can also apply some of these algorithms for the SC block transmission in the high-mobility channel. The algorithm in [6] extends periodically the time-domain (TD) signal to add the diversity of FD symbols, and the receiver makes efficient combination of the diversity to reduce the ICI. The algorithm of [9] further provides more efficient combining coefficients to improve the performance. Both the algorithms in [6][9] can be directly applied in the SC block transmission.

Since we cannot completely remove the ICI incurred by the fast time-varying mobile channel, there exits residual ICI after the ICI reduction in the receiver. However, as we will show in this paper, the impacts of the residual ICI on the OFDM and on the SC-FDE systems are quite different. For the OFDM system, the residual ICI on a subchannel only affects the FD symbols on this subchannel. If the fading of a subchannel at some time slot is deep, then the residual ICI may lead to incorrect detection of the symbol on this subchannel, but it does not affect the detection of symbols on other subchannels. However, for the SC-FDE system, the transmitted symbols are in time domain. When the fading of a subchannel is deep, the residual ICI in this subchannel may be enhanced in the process of FD equalization, and this enhanced interference affects all TD symbols after the IFFT. By analysis we show that the residual ICI leads to the “error-rising” effect on the error rate when the SNR increases for the SC-FDE system. The simulation results also verify our analysis. We can apply the maximum-likelihood (ML) detection or its complexity-reduced algorithms, such that the error-rising effect can be removed, as shown by the simulation. In our simulation, we apply an efficient ICI self-reduction algorithm that is a modified version of the algorithm in [9]. Instead of taking direct periodical extension the original symbols as both [6] and [9], the algorithm first reduces the number of symbols within a block, followed by the periodical extension to the original block size.

The rest of this paper is organized as follows. Section II introduces the signal model. The effect of residual ICI is analyzed in Section III. In Section IV, we discuss the use of ML detection and the related complexity-reduced schemes. Some simulation results are shown Section V. Finally, Section VI concludes this paper.
II. SIGNAL MODEL

Fig. 1 shows a precoded OFDM (POFDM) system. A block of transmitted symbols is denoted by

$$b = (x_0, x_1, \ldots, x_{N-1})^T$$

where $N$ is the block size. When we set $P = I$, we have the typical OFDM system; while for $P = F$, it is the SC block transmission system and $x = b$. The associated baseband signal can be expressed as

$$x(t) = \sum_{k=0}^{N-1} x_k \cdot g \left( t - \frac{kT}{N} \right), \quad -T_g \leq t < T,$$  

(2)

where $T$ denotes the original duration of transmitting a block of symbols in (1), and $T_g$ is the duration of guard interval. In (2), we denote by $g(t)$ the associated periodic pulse $[10]$ with the period $T$. The period of $g(t)$ is $T$, and during $(-T_g, 0)$ we transmit the periodic extension of $g(t)$ (or CP). The ideal interpolation pulse $g(t)$ in (2) can implement the exact multi-carrier modulation when combined with the IFFT $[10]$.

At the receiver, after removing the CP, we can write the received signal as

$$y(t) = \sum_{\ell=0}^{L_p-1} h^{(\ell)}(t) \cdot x(t - \tau^{(\ell)}) + w(t)$$

(3)

for $0 \leq t < T$, where $L_p$ is the number of paths, and we denote by $h^{(\ell)}(t)$ and $\tau^{(\ell)}$ the channel impulse response and delay, respectively, of the $\ell$th path. In (3), the noise component of $y(t)$ is denoted by $w(t)$. We assume that $\tau^{(\ell)} \leq T_g$ for $0 \leq \ell < L_p$, such that there is no ISI.

The received signal $y(t)$ is sampled at $t=kT/N$ for $0 \leq k < N$, and these $N$ samples are denoted by $y = (y_0, y_1, \ldots, y_{N-1})^T$, where $y_k = y(kT/N)$. Then $y$ is transformed into frequency domain by the FFT for equalization. The FFT of $y$ is denoted by $y_F = (Y_0, Y_1, \ldots, Y_{N-1})^T$. We can write $y_m$ as

$$y_m = H_{m,m}x_m + \sum_{m' \neq m} H_{m,m'}x_{m'} + W_m$$

(4)

for $0 \leq m < N$. In (4), we observe that on the $m$th sub-channel, there exists interference from other subchannels, due to time variation of the multipath channel. The interference from the $m'$th subchannel on the $m$th subchannel depends on $H_{m,m'}$, which can be expressed as

$$H_{m,m'} = \frac{1}{N} \sum_{\ell=1}^{L_p-1} e^{-j2\pi \frac{m-m'}{N}} \sum_{k=0}^{N-1} h^{(\ell)}(kT/N),$$

(5)

for $0 \leq m, m' < N$, where $h^{(\ell)}(kT/N)$, and $W_m$’s are the noise components of $y_m$’s. For time-invariant or slowly time-varying channels, each $h^{(\ell)}$ is constant during $0 \leq k < N$, or $h^{(\ell)} = h^{(l)}$, which is independent of $k$. In this scenario, by (5) we observe that $H_{m,m'}$’s are zero for $m \neq m'$, since

$$\sum_{k=0}^{N-1} h^{(\ell)}(k)e^{-j2\pi \frac{(m-m')k}{N}} = h^{(\ell)} \sum_{k=0}^{N-1} e^{-j2\pi \frac{(m-m')k}{N}} = 0$$

(6)

if $m \neq m'$, and there is no ICI in (4).

In the above, for the SC-FDE system, although the transmitted symbols are in time domain, it also suffers the effect of ICI. For the typical OFDM system, in which one can readily apply either the block type or the comb type pilot schemes in Fig. 2 for the CR estimation. However, for the SC block transmission, one can only readily apply the block type pilot arrangement. Since the transmitted symbols in (1) are in time domain, if we apply the comb type pilot scheme, then when inserting FD pilot symbols, we need to increase the sizes of $x_P$, $x$, $y$, and $y_F$ in Fig. 1. Then we also need to increase the sizes of $F^{-1}$ for IFFT in the transmitter and $F$ for FFT in the receiver. This makes $P$ and $F$ have different sizes.

On the other hand, for the block type pilot arrangement, consider a group of FD symbols along time slots in the process of joint CR estimation, as indicated by the red line in Fig 2(a). If we apply the ICI self-reduction schemes in [6][9], the TD periodic extension increases the relative time variation of channel along time slots, and degrades the performance of CR estimation. Therefore, we also give a modified ICI self-reduction algorithm in Appendix A based on [9]. For this modified algorithm, we do not apply direct TD periodic extension, and the block duration is the same as the original block duration.

III. THE RESIDUAL ICI

One can apply some ICI reduction algorithms to reduce the effect of ICI. However, unlike the case of carrier-frequency offset, we cannot completely remove the ICI in the time-varying multipath channel. Therefore, there exists residual ICI after the ICI reduction. We can express the signal after the ICI reduction as

$$\hat{Y}_m = H_{m,m}x_m + I_{res,m} + W_m, \quad 0 \leq m < M,$$

(7)
where $I_{\text{res},m}$ denotes the residual ICI on the $m$th subchannel. The expression of $I_{\text{res},m}$ depends on the ICI reduction algorithm. In general, the power of each $I_{\text{res},m}$ is proportional to the power of transmitted symbols.

A. Detection Based on the MMSE Principle

We first consider the signal model without ICI, and write (7) as

$$\hat{Y}_m = H_m X_m + W_m, \quad 0 \leq m < M, \quad (8)$$

For the OFDM system, if we have the estimate of $H_m$, we can detect $X_m$ simply by making hard decision on the equalized signal,

$$\hat{Y}_m = \frac{Y_m}{H_m} = X_m + \frac{W_m}{H_m}, \quad 0 \leq m < M. \quad (9)$$

For the SC block transmission, however, since the symbols are in time domain, we need to further take the IFFT of (9). For the scenario of deep fading, or $|H_m| \ll 1$ for some $m$, the detection in (9) incurs the noise enhancement, which results in a large $\frac{W_m}{H_m}$. The noise enhancement may cause large interference on other TD symbols after the IFFT. Therefore, for the SC block transmission, we need to modify (9) as

$$\hat{Y}_m = \frac{H_m}{H_m + 1/\gamma_s} X_m + \frac{W_m}{H_m + 1/\gamma_s} \quad (10)$$

where $\gamma_s = (2\sigma^2_X)/(2\sigma^2_W)$ for the quaternary phase-shift keying (QPSK) modulation. The equalization in (10) is based on the minimum mean-squares error (MMSE) principle. Compared with (9), the item $1/\gamma_s$ can mitigate the effect of noise enhancement when deep fading $|H_m| \ll 1$ occurs, since $\frac{W_m}{H_m + 1/\gamma_s} \leq \frac{W_m}{1/\gamma_s}$. For the OFDM system, on the other hand, the noise enhancement in (9) does not affect the symbols on other subchannels, and we can simply apply (9) for equalization.

B. The Effect of Residual ICI

In (10), we consider the scenario that there is no ICI. When the ICI exists, after the ICI reduction, there exits residual ICI on each subchannel, since the ICI cannot be completely removed in the time-varying multipath fading channel. Substituting (7) for $\hat{Y}_m$ in (10), we have

$$\hat{Y}_m = \frac{H_m}{H_m + 1/\gamma_s} X_m + \frac{W_m + I_{\text{res},m}}{H_m + 1/\gamma_s} \quad (11)$$

for the QPSK modulation and $\gamma_s = \frac{2\sigma^2_X}{2\sigma^2_W}$. When $|H_m| \ll 1$ (deep fading on the $m$th subchannel), we can approximate the second term of (11) as

$$\frac{W_m + I_{\text{res},m}}{H_m + \frac{2\sigma^2_X}{2\sigma^2_W}} \approx \frac{2\sigma^2_X}{2\sigma^2_W} (I_{\text{res},m} + W_m). \quad (12)$$

Since the power of residual ICI, $I_{\text{res},m}$, is proportional to the signal power, or $2\sigma^2_X$. At high SNR, or $|W_m| \ll 1$, we can further approximate (12) as

$$\frac{2\sigma^2_X}{2\sigma^2_W} I_{\text{res},m} \quad (13)$$

which increases with SNR more than linearly. For the OFDM system, this only affects the detection of the transmitted symbols on other subchannels, and we can simply apply (9) for equalization.
symbol $X_m$ on the $n$th subchannel. However, for the SC block transmission, because the transmitted symbols are in time domain, the enhanced ICI in (12) will affect the detection of other TD symbols after the IFFT of (11).

In the above, we show that after the ICI self-reduction, if we apply the principle of MMSE in the FD equalization, the residual ICI may be enhanced at some subchannel which is under deep fading, and this affects all the TD symbols after the IFFT for the SC block transmission. We will show by simulation that the bit-error rate (BER) rises with the increasing SNR at high SNR, and this verifies that the enhanced residual ICI increases with the signal power more than linearly, as (13) indicates.

IV. Maximum-Likelihood Detection

To avoid the effect of “error-rising” caused by the residual ICI when the receiver applies the MMSE FD equalization after the ICI reduction, one can resort to the ML detection or the associated complexity-reduced suboptimal schemes. Referring to Fig. 1, we can write $y_F$ as

$$y_F = H x_F + w_F$$

where $H$ is a diagonal matrix that represents the FD channel responses, and $w_F$ is the vector of noise plus residual ICI. Note that in (14), we assume $H$ is a diagonal matrix after the ICI reduction. Since $x_F = F x$, we can write (14) as

$$y_F = H F x + w_F.$$  \hspace{1cm} (15)

For the SC block transmission, after the ICI self-reduction, as indicated in Fig 4, the ML detection of the transmitted vector of symbols $x$ is given by

$$\hat{x} = \arg \min_{x} \|y_F - H F x\|^2.$$  \hspace{1cm} (16)

Although by the exhaustive search of all possible possible $x$ and calculating the associated norm $\|y_F - H F x\|^2$, we can determine the ML estimate of $x$ by finding the minimum norm in (16), the exhaustive search and calculation leads to much high complexity that grows exponentially with the length of $x$. We can resort to the sphere decoding (SD) algorithm, which can efficiently implement the ML detection, or the corresponding complexity-reduced suboptimal schemes, like the K-Best algorithm. Note that in (16), we do not have explicit equalization process, and this avoids the enhancement of residual ICI and the associated error-rising effect, as we will show in Section V.

V. Simulation Results

In Fig.5 and Fig.6, we consider the detection of an SC-FDE system. The size of a block is 64, and there are 64 virtual subchannels in frequency domain. We apply an improved ICI self-reduction algorithm based on the algorithm in [9]. This algorithm can be applied in the SC-FDE system. We set $P = F$ and this leads to $x = b$ for the transmitter in Fig.1. As [7] indicates, for the ICI reduction in the high-mobility channel, unlike the scenario of carrier-frequency offset, the ICI cannot be completely removed, and there exists residual ICI after the ICI reduction. Our analysis shows that compared with the OFDM system, the residual ICI in the SC-FDE system results in the error-rising effect at high SNR if we apply the MMSE FD equalization. The error-rising effect is more unwanted than the error-floor effect. On the other hand, we can alternatively apply the ML detection that can be efficiently implemented by the SD algorithm, or the corresponding suboptimal complexity-reduced schemes, like the K-Best algorithm. The simulation results verify the error-rising effect, and show that by the ML-based detection, one can effectively remove the error-rising effect.

APPENDIX A

AN ICI SELF-REDUCTION ALGORITHM

Now we give an improved ICI self-reduction algorithm based on the algorithm in [9]. This algorithm can be applied in the SC-FDE system. We set $P = F$ and this leads to $x = b$ for the transmitter in Fig.1. As [7] indicates, for the ICI reduction in the high-mobility channel, unlike the scenario of carrier-frequency offset, the ICI cannot be completely removed, and there exists residual ICI after the ICI reduction. Our analysis shows that compared with the OFDM system, the residual ICI in the SC-FDE system results in the error-rising effect at high SNR if we apply the MMSE FD equalization. The error-rising effect is more unwanted than the error-floor effect. On the other hand, we can alternatively apply the ML detection that can be efficiently implemented by the SD algorithm, or the corresponding suboptimal complexity-reduced schemes, like the K-Best algorithm. The simulation results verify the error-rising effect, and show that by the ML-based detection, one can effectively remove the error-rising effect.

VI. Conclusion

In this paper, we analyze the effect of residual ICI in the detection of SC-FDE system. For the time-varying multipath channel, unlike the case of carrier-frequency offset, the ICI cannot be completely removed, and there exists residual ICI after the ICI reduction. Our analysis shows that compared with the OFDM system, the residual ICI in the SC-FDE system results in the error-rising effect at high SNR if we apply the MMSE FD equalization. The error-rising effect is more unwanted than the error-floor effect. On the other hand, we can alternatively apply the ML detection that can be efficiently implemented by the SD algorithm, or the corresponding suboptimal complexity-reduced schemes, like the K-Best algorithm. The simulation results verify the error-rising effect, and show that by the ML-based detection, one can effectively remove the error-rising effect.
for $0 \leq d \leq N - M$, $0 \leq m < M$. We can write $Y_{m}^{(d)}$ as
\[
Y_{m}^{(d)} = H_{m,m}^{(d)}(X_{m}e^{i2\pi \frac{md}{M}}) + \sum_{m' \neq m} H_{m,m'}^{(d)}(X_{m'}e^{i2\pi \frac{md}{M}}) + W_{m}^{(d)} \tag{A.4}
\]
where
\[
H_{m,m'}^{(d)} = \frac{1}{M} \sum_{\ell=0}^{L_{p}-1} e^{-i2\pi \frac{m'm'}{M}} \sum_{k=0}^{M-1} h_{k+d}^{(\ell)} e^{-i2\pi \frac{(m-m')k}{M}} \tag{A.5}
\]
for $0 \leq m, m' < M$, and
\[
W_{m}^{(d)} = \frac{1}{M} \sum_{k=0}^{M-1} w_{k+d} e^{-i2\pi \frac{md}{M}}. \tag{A.6}
\]
The receiver combines $Y_{m}^{(d)}$'s into
\[
\hat{Y}_{m} = \frac{1}{N-M} \sum_{N-M-1}^{N-1} u_{d} Y_{m}^{(d)} e^{-i2\pi \frac{md}{M}} \tag{A.7}
\]
for $0 \leq m < M$, and the combining weights are
\[
u_{d} = \begin{cases} \left(2M-N+2\right)/2, & \text{if } d = 0 \text{ or } N-M-1, \\ 1, & \text{elsewhere.} \end{cases} \tag{A.8}
\]
When $M = N/2$, note $u_{d} = 1$ for $0 \leq d \leq N - M$.

However, the operations in (A.3) and (A.7) need $(N-M+2)$ FFT's. We can implement an equivalent low-complexity algorithm by substituting (A.3) into (A.7), and with some manipulation, we can further write (A.7) as
\[
\hat{Y}_{m} = \frac{1}{M} \sum_{q=0}^{M-1} \hat{y}_{q} e^{-i2\pi \frac{md}{M}} \tag{A.9}
\]
where
\[
\hat{y}_{q} = \begin{cases} \left( \sum_{k=0}^{q} u_{k} \right) y_{q} + \left( 1 - \sum_{k=0}^{q} u_{k} \right) y_{q+M}, & 0 \leq q < N - M, \\ y_{q}, & N-M \leq q < N. \end{cases} \tag{A.10}
\]
for $0 \leq q < M$. Therefore, we can obtain $\hat{Y}_{m}$'s by taking only one FFT on $\hat{y}_{q}$'s and $\hat{y}_{q}$'s are just the linear combination of $y_{q}$'s by (A.10). Following the operation in Fig.4, the ICI self-reduction transforms $y$ into $\tilde{y}$ based on (A.10), followed by the FFT of $\tilde{y}$ in (A.9).

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