False Alarm Rate Analysis of the FCME Algorithm in Cognitive Radio Applications

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***Abstract*—Cognitive radio is a promising choice to fulfill needs of growing wireless applications in the future. Spectrum sensing is beneficial in several circumstances when secondary user (SU) search empty frequencies for a transmission. One interesting choice for spectrum sensing is the localization algorithm based on the double-thresholding (LAD) method. The LAD method is based on the forward consecutive mean excision (FCME) algorithm that calculates the used thresholds. Threshold setting is based on the usage of the desired false alarm rate, which is sensitive to issues like the length of the integration time. In the real-time applications, integration time is limited. In this paper, the false alarm rate of the FCME algorithm is studied. The false alarm rates of the FCME algorithm in the noise-only case with different integration times (sample vector lengths) are analyzed. The minimum length of the sample vector is defined. The simulation results are verified by the real measurement results in the noise-only case, and a scenario that combines the results is presented. It is also noted that in the noise measurements, the achieved false alarm rates are somewhat lower than the desired ones.**

Keywords–cognitive radio; spectrum sensing; false alarm rate.

I. INTRODUCTION

Cognitive radio technology [1] [2] [3] can be considered as a revolution against the traditional, inflexible frequency allocation. Cognitive radio (CR) enables both dynamic spectrum management and flexible transmission bandwidth [4]. In CR, secondary users (SU) may transmit if there is room aka empty frequencies (white space) in the spectrum and if they are not interfering primary users (PU). Interference-free transmission is a privilege of the PUs. In cognitive radio, SUs may find out empty frequencies using, for example, databases or spectrum sensing [5] [6] [7] [8]. Sensing may be beneficial instead of geolocation and databases, for example, in the wireless local area network (WLAN)-type solutions when transmitters are located close to each other and transmit powers are small. Also, public safety applications when the connection to the outside world is lost may use sensing. Spectrum sensing requires ability to find unused frequencies, which can be done via detecting existent signals.

Many detection methods are based on the use of a threshold. The basic principle is that the threshold separates the samples into two sets: noise and signal sets. Nowadays, most of the methods use adaptive thresholds. Threshold setting is a very demanding task, especially when the threshold is set adaptively. As too high threshold causes missed detections, too low a threshold leads to false detections. Missed detection means that existing signals are not detected, as false detection means that noise samples are falsely detected to be from a signal.

One of these detection methods is the localization algorithm based on the double-thresholding (LAD) [9]. At the core of the LAD method, the forward consecutive mean excision (FCME) algorithm [10] provides the used detection thresholds. The FCME algorithm sets the threshold iteratively based on the mean of sample energies and a pre-selected threshold parameter. This parameter depends on the statistical properties of the noise-only case. Usually, Gaussian assumption is used even though the measured noise is not purely Gaussian [9]. The threshold parameter is defined using the desired false alarm rate $P_{FA,DES}$. It defines how many samples are above the threshold when there is only noise present. The FCME method uses constant false alarm rate (CFAR) principle, so the false alarm probability stays almost constant. However, it is sensitive to the issues like the length of the considered sample vector and noise properties. In the real-time applications, integration time is limited, so the number of considered samples N can not be as large as in the computer simulations.

The performance of the FCME algorithm is highly depending on the false alarm rate. If the achieved false alarm rate differs from the desired one, the performance of the FCME method may degrade. Especially when the signal-to-noise ratio (SNR) is low, the false alarm rate totally defines the performance of the FCME algorithm, and, thus, the LAD method. If the achieved false alarm rate is not close enough to the desired one, the performance of the LAD method may even totally degrade. It is very important to control $P_{FA,DES}$ because it is related to the caused interference as well as the spectrum opportunity loss in cognitive radio applications [9]. Thus, it is very important to study and analyze the false alarm rate of the FCME algorithm.

In this paper, the false alarm rate of the FCME algorithm is studied in the noise-only case. That is, there are no signals present. First, the effect of the length of the considered sample vector (i.e., integration time) to the false alarm rate of the FCME algorithm is analyzed using simulation software generated AWGN noise. Mean, variance as well as minimum and maximum values of achieved false alarm rates are analyzed. Based on those, proper sample vector lengths are recommended. The analysis results are verified by the real measurement results in the noise-only case. The measurements covering a wide range of the spectrum are used to find out the differences in the achieved false alarm rate between the measured and simulation software generated noise. Several measurements up to 39 GHz are used to cover higher frequency areas possible used in future applications as 5G and beyond. The Kruskal-Wallis test is used to provide more statistical information. In addition, a scenario that combines the analysis and measurement results is presented.

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This paper is organized as follows. In Section II, the used FCME algorithm is presented. Section III covers the probability of false alarm analysis of the FCME algorithm, and Section IV describes our scenario. Conclusions are drawn at Section V.

II. THE FCME ALGORITHM

The FCME algorithm [9] [10] [11] was originally proposed for impulsive interference suppression in the time domain. Later on, it was noticed that the method can be used also in other transform domains, e.g., in the frequency domain. Its enhanced version called the LAD method [9], which uses the FCME thresholds was developed to detect narrowband information signals, e.g., for spectrum sensing purposes.

The FCME algorithm is blind and independent of modulation methods, signal types and number of signals. The only requirements are that the signal(s) can not cover the whole bandwidth under consideration, and the signal(s) are above the noise level.

The FCME algorithm is computationally simple but effective. It calculates the threshold iteratively based on the noise properties.

Initial Preparation: When the noise is assumed to be zero mean, independent, i.i.d. Gaussian noise, i.e., samples x_i follow the Gaussian distribution, the FCME algorithm calculates the threshold parameter based on [10]

$$T_{CME} = -ln(P_{FA,DES}), \tag{1}$$

where $P_{FA,DES}$ is the desired clean sample rejection rate (the desired false alarm rate) [10]. For example, if the desired clean sample rejection rate is 1% $(= 0.01)$, $T_{CME} = 4.6$ [9]. After that, energy of samples is calculated. Now, samples $|x_i|^2$ that follow the chi-squared distribution with two degrees of freedom are rearranged in an ascending order according to their sample energy. Then, $m = 10\%$ of smallest samples are selected to form the initial set Q (called also as a "clean set").

Algorithm: The FCME threshold is [10]

$$T_h = T_{CME}\overline{Q}, \tag{2}$$

where $\overline{Q}$ denotes the mean of Q. Samples below the threshold are added to the set Q and new mean and threshold are calculated. This is repeated until there are no new samples below the threshold. Usually, it takes 3-4 iterations to get the final threshold. In the end, samples *above* the threshold are assumed to be signal samples, as samples *below* the threshold are assumed to be noise samples.

The required false alarm rate $P_{FA,DES}$ is related to the threshold. Small $P_{FA,DES}$ value leads to larger threshold. Thus, the amount of false alarms is small. Large $P_{FA,DES}$ value leads to smaller threshold and the amount of false alarms is larger [12]. In cognitive radio applications, it is important to control $P_{FA,DES}$ because it is related to the caused interference as well as the spectrum opportunity loss [9].

It should be noted that (1) is valid when the noise is at least approximately Gaussian. It is also possible to define the used equation to other distributions [9]. Note, that the noise variance has no influence [13].

TABLE I. ACHIEVED P_{FA} WHEN $P_{FA,DES} = 0.01$.

N	mean(P_{FA})	diff	var(P_{FA})	min	max
64	0.025245	0.0152	0.0075937	0	0.9062
128	0.015415	0.0054	0.00062043	0	0.8984
256	0.0141	0.0041	$7.505e-05$	0	0.0585
512	0.013526	0.0035	$3.566e-05$	0	0.0390
1024	0.013313	0.0033	$1.721e-05$	0.00195	0.0341
2048	0.013181	0.0032	$8.356e-06$	0.00341	0.0268
4096	0.013139	0.0031	$4.318e-06$	0.00659	0.0229

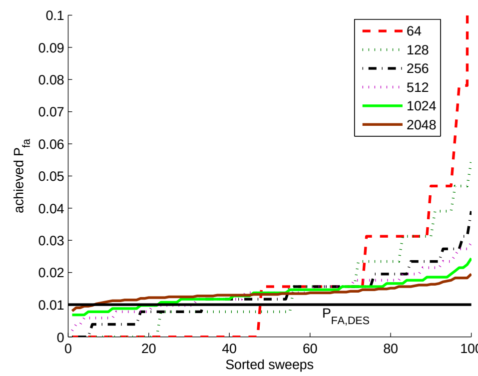

Figure 1. Achieved P_{FA} values for different values of N. MC=100 sweeps. Results are sorted in an ascending order. Matlab-generated noise. $P_{FA,DES} = 0.01$.

III. P_{FA} ANALYSIS OF THE FCME ALGORITHM

Achieved P_{FA} values for different desired $P_{FA,DES}$ values were studied. That is, how close the achieved P_{FA} values are to the desired $P_{FA,DES}$ value. This effects to the performance of the FCME method, especially at low SNR values. Two different commonly used desired $P_{FA,DES}$ values were used, $0.01 = 1\%$ ($T_{CME} = 4.6$) and $0.001 = 0.1\%$ ($T_{CME} = 6.9$) [9]. It means that according to the CFAR principle, when there is only noise present, 1% or 0.1% of the samples should be above the threshold, respectively. In the computer simulations, the effect of the length of the samples N, was considered. The purpose was to find the smallest N when the FCME algorithm is able to operate properly. Measurement results are compared to the Matlab-generated AWGN noise results.

A. Matlab simulations

In the simulations, Matlab software was used. Computer-generated AWGN noise was used as a noise. There were 10 000 Monte Carlo iterations. The length of the samples, N, varied. This is because in the simulations we can use large values of N, but in the real-time implementations, N may be often smaller because of hardware limitations. The purpose was to find smallest N so that the achieved P_{FA} values are in the decent level.

In Table I, achieved P_{FA} values when desired $P_{FA,DES} = 0.01 = 1\%$ and N varies are presented. Diff=$|P_{FA,DES} - P_{FA}|$. As can be seen, means are close to each others when N is large enough, that is, $N >= 256$. Achieved P_{FA} values differ from the desired $P_{FA,DES}$ value 152% ($N = 64$), 54% ($N = 128$), 41% ($N = 256$), 33% ($N = 512$), 33% ($N = 1024$), 31% ($N = 2014$), and 31% ($N = 4096$). It can also

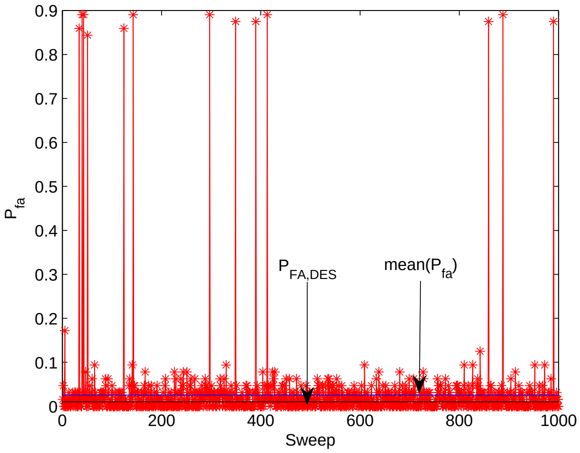

Figure 2. Achieved P_{FA} values when $N = 64$. MC=1000, $P_{FA,DES} = 0.01$.

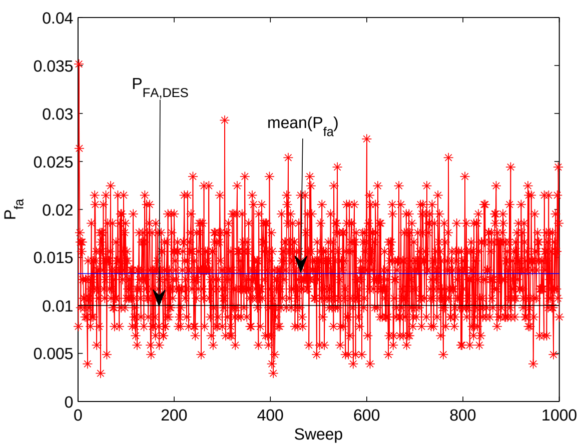

Figure 3. Achieved P_{FA} values when $N = 1024$. MC=1000, $P_{FA,DES} = 0.01$.

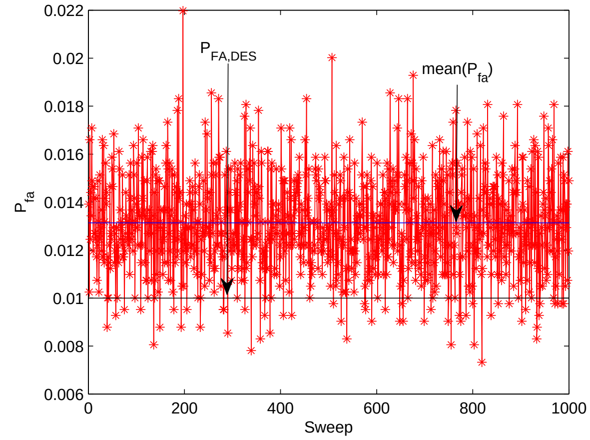

Figure 4. Achieved P_{FA} values when $N = 4096$. MC=1000, $P_{FA,DES} = 0.01$.

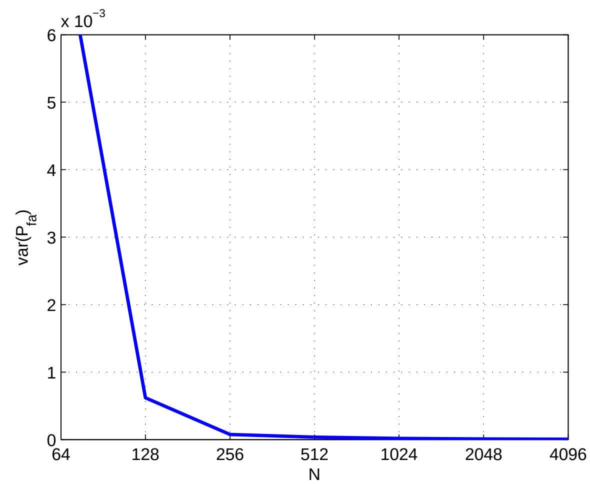

Figure 5. Variance of P_{FA} values for different sample lengths N. Matlab-generated noise. $P_{FA,DES} = 0.01$.

been seen that the smaller N (the shorter data), the higher the variance is.

In Figure 1, achieved P_{FA} values are presented for different values of N. There were 100 iterations (sweeps) and the results were sorted in an ascending order. Horizontal line presents desired $P_{FA,DES}$ value. It can be seen that the more samples, the closer the achieved P_{FA} values stay with the desired $P_{FA,DES}$ value (here, $P_{FA,DES} = 0.01$).

In Figures 2 - 4, achieved P_{FA} values are presented when $N = 64$, 1024 and 4096. It can be noticed that the achieved mean of P_{FA} is slightly higher than the desired $P_{FA,DES}$ value. It can also be seen that when N is small (Figure 2), variance is very high.

In Figure 5, variances of the achieved P_{FA} values are considered as in Figure 6, mean P_{FA}, min P_{FA} and max P_{FA} values are studied. In both figures, N varies. It can be seen that when $N >= 256$, values are on acceptable level.

In Table II, achieved P_{FA} values when desired $P_{FA,DES} = 0.001 = 0.1\%$ and N varies is presented. Diff=$|P_{FA,DES} - P_{FA}|$. $P_{FA,DES} = 0.1\%$ means that when $N = 1000$, 1 sample is above the threshold. Thus, we considered $N >= 1024$ to get realistic results; therefore, smaller values for N were not considered.

TABLE II. ACHIEVED P_{FA} WHEN $P_{FA,DES} = 0.001$.

N	mean(P_{FA})	diff	var(P_{FA})	min	max
1024	0.0010574	$5.74e-05$	$1.1071e-06$	0	0.00683
2048	0.0010685	$6.85e-05$	$5.5935e-07$	0	0.00585
4096	0.0010708	$7.08e-05$	$2.7042e-07$	0	0.00341

B. Measurements at 10 MHz-39.1 GHz

The measurements were performed in wide frequency area to get reliable and wide-ranging results. Here, high-performance spectrum analyzer (Agilent E4446A) [14] was used as in [15]. Note, that the results depend on the used equipment. We used Instrument Control Toolbox to connect Matlab to the spectrum analyzer to enable direct results

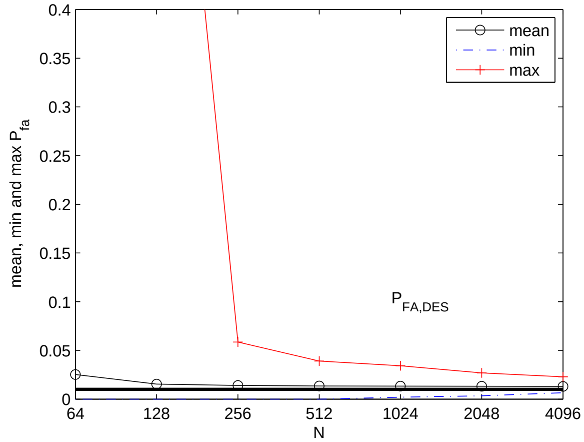

Figure 6. Mean P_{FA}, min P_{FA} and max P_{FA} values for different sample lengths N. Matlab-generated noise. $P_{FA,DES} = 0.01$.

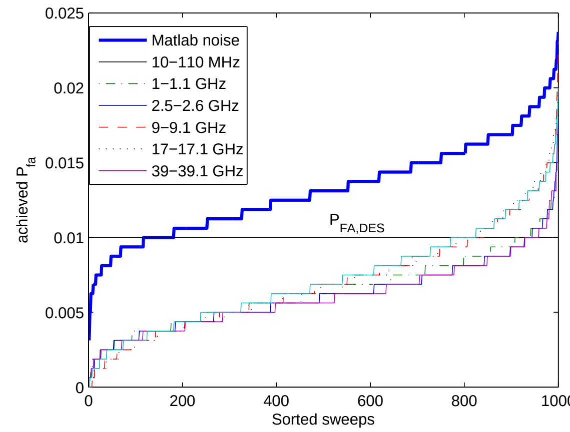

Figure 7. Achieved P_{FA} values for different frequency areas. MC=100 sweeps. Results are sorted in an ascending order. $N = 1601$. Measured noise.

TABLE III. ACHIEVED P_{FA} WHEN $P_{FA,DES} = 0.01$.

freq. range	mean(P_{FA})	var(P_{FA})	min	max
$10 - 110$ MHz	0.006431	$5.8612e-06$	0	0.0149
$1 - 1.1$ GHz	0.0061711	$5.6578e-06$	0.000624	0.0181
$2.5 - 2.6$ GHz	0.0070012	$1.1744e-05$	0	0.0231
$9 - 9.1$ GHz	0.0070244	$1.2441e-05$	0	0.0199
$17 - 17.1$ GHz	0.0060668	$5.5949e-06$	0.000624	0.0149
$39 - 39.1$ GHz	0.0071974	$1.103e-05$	0	0.0199
Matlab-noise	0.013229	$9.7771e-06$	0.000624	0.0237

analysis. At frequency ranges 10-110 MHz, 1-1.1 GHz, 17-17.1 GHz and 39-39.1 GHz, only internal noise level was measured. In frequency ranges 2.5-2.6 GHz and 9-9.1 GHz, broadband antenna was connected, so the noise consists of internal noise and noise from antenna. There were 1 000 time domain sweeps and $N = 1601$ frequency points [15]. Energy of the samples was measured in the frequency domain. Matlab-generated AWGN noise with same N was used for a comparison.

In Table III, achieved P_{FA} values when desired $P_{FA,DES} = 0.01 = 1\%$ and $N = 1601$ are presented. It can be noticed that mean P_{FA} values are very close to each others. Variances are on the same level. It should be noted that now the achieved P_{FA} values are slightly lower than desired $P_{FA,DES}$ value.

In Figure 7, achieved P_{FA} values are presented for different measured frequency bands. There were 1000 iterations (sweeps) and the results were sorted in an ascending order. Horizontal line presents desired $P_{FA,DES}$ value (=0.01). Matlab-generated noise results are presented as a reference. Here, $N = 1601$. It can be seen that the measured results are almost on the same level, and lower than the reference results.

In Figure 8, variances of the achieved P_{FA} values are considered as in Figure 9, mean P_{FA}, min P_{FA} and max P_{FA} values are studied for different measured frequency bands. In both figures, $N = 1601$. It can be seen that there are only small differences.

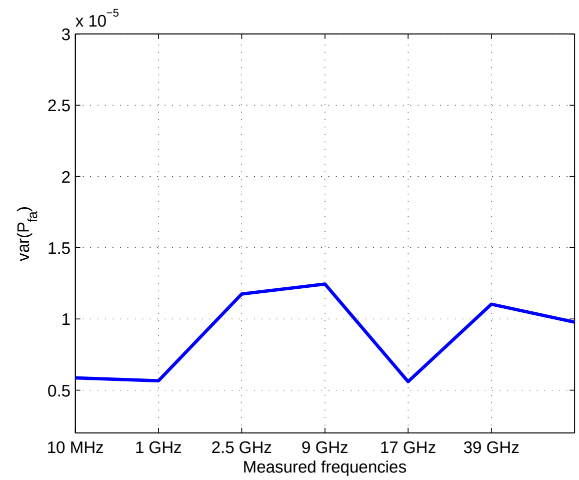

Figure 8. Variance of P_{FA} values for different frequency areas. N=1601. Measured noise.

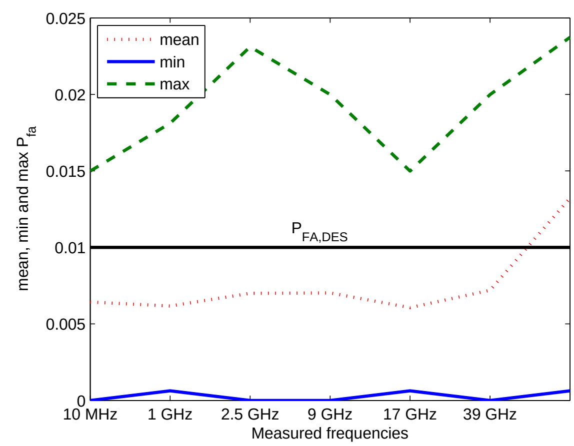

Figure 9. Mean P_{FA}, min P_{FA} and max P_{FA} values for different frequency areas. N=1601. Measured noise.

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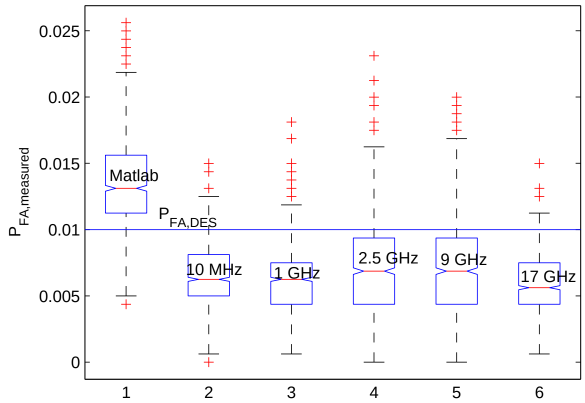

Figure 10. Kruskal-Wallis test to P_{FA} for several measured groups at 10 MHz-17 GHz. $P_{FA,DES} = 0.01$

C. Kruskal-Wallis test

Kruskal-Wallis tests the null hypothesis that samples that are independent and come from two or more groups follow same distribution and their means are equal [16]. There is no normality assumption nor assumptions about the mean and variance. Here, Kruskal-Wallis test is used to produce statistical boxplots.

In Figure 10, Kruskal-Wallis boxplots are presented to achieved P_{FA} for several measured groups at 10 MHz-17 GHz. One boxplot presents five statistics - from bottom to top those are minimum, first quartile, median value (line in the middle of the box), third quartile, and maximum value. This figure confirms the results presented earlier.

IV. SCENARIO

Sensing can be verified using a spectrum analyzer. Here, Agilent E4446A was used, but there are a lot of other equipments, like the wireless open-access research platform (WARP) [17]. The WARP is a platform used to test and prototype wireless networks. The noise level (from internal noise) may vary between the equipments. Therefore, adjusting is needed if it is required that the achieved false alarm rate is controlled. Assume that the LAD method which uses the FCME thresholds is used to perform spectrum sensing. It is desired that the P_{FA} is controlled so spectrum opportunities are not lost. It is possible first to measure the noise in the desired frequency area. As noticed here, the length of the noise vector has to be at least 256 samples when $P_{FA,DES} = 0.01$. It does not matter what is the used sampling rate, however, the same rate should be used later. After measuring the noise level, the FCME threshold can be fixed to correspond the theoretical one. This can be done using a correction coefficient which can be defined when $P_{FA,DES}$ and P_{FA} are known. Note, that this method is valid when the noise is not impulsive.

V. CONCLUSION

The false alarm rate of the FCME algorithm was studied in the noise-only case. A proper length of the sample vector was defined, and analysis results were compared with the results from noise measurements. This result can be used in future simulations and in real-time applications, for example, when implementing the FCME algorithm on the wireless open-access research platform. It was also noted that as in the computer simulations the achieved false alarm rates were slightly higher than the desired ones, in the noise measurements, the achieved false alarm rates were slightly lower than the desired ones. Based on this information, used thresholds can be fixed using a proper correction coefficient in the cases when the achieved false alarm rate need to be as close as the desired false alarm rate as possible. In the computer simulations, the false alarm rate can be reduced as in the measurements and real-time applications, the false alarm rate can be raised.

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