Stochastic Chase Decoding of BCH Codes

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***Abstract*—This paper analyzes the complexity-performance trade-off of the Stochastic Chase decoding algorithm for Bose-Chaudhuri-Hocquenghem (BCH) codes over the additive white Gaussian noise (AWGN) channel. It is verified by computer simulations that this algorithm can outperform the traditional Chase algorithm with less complexity decoding.**

Keywords-Block codes; Chase algorithm; decoding complexity; additive white Gaussian noise; frame error rate; BCH code; reliability-based decoding; soft-decision decoding.

I. Introduction

Soft-decision decoding is a decoding process that utilizes the information contained in the unquantized received symbols to improve the error-correcting performance compared to hard-decision decoding. However, the better performance of the soft-decision algorithms comes at the price of higher complexity. Concerning block codes, an important class of soft-decision decoding algorithms is the reliability-based (or probability-based) decoding techniques [1]-[4].

The Chase algorithm [1] is a reliability-based decoding technique that uses a set of test patterns in attempt to find an estimation of the maximum-likelihood codeword. To generate the set of test patterns, the least reliable positions (denoted by p) of the received sequence are considered. For example, considering additive white Gaussian noise (AWGN) channel, the real values of the received sequence correspond to the reliabilities of the Chase algorithm. The higher the value of the reliability, the lower the probability that the corresponding symbol had been strongly affected by the noise. Given the p least reliable positions, the number of generated test patterns is equal to 2^p and it is a way to measure the complexity of the decoding algorithm. With respect to Reed-Solomon (RS) codes and Bose-Chaudhuri-Hocquenghem (BCH) codes, many efforts have been made to find reduced complexity Chase decoding algorithms, including for implementation of VLSI architectures [5][6].

A modification of the Chase decoding algorithm, named Stochastic Chase algorithm, was proposed in [7]. It was assumed that the test patterns are stochastically generated instead of using the least reliable positions of the received sequence. This proposal was investigated for RS codes and it was shown that this modification is a low cost solution for soft-decoding of this class of codes. However, nothing was commented about the use of the Stochastic Chase algorithm for BCH codes. With this in mind, the objective of this paper is to analyze the complexity-performance trade-off of the Stochastic Chase decoding algorithm for BCH codes and how the characteristics of the BCH code influence the performance of the decoding algorithm. For sake of simplicity, hereafter the Chase algorithm and the Stochastic Chase algorithm will be denoted, respectively, by Ch and $S-Ch$ algorithms.

The remainder of this article is structured as follows. In Section II, a modified version of the Stochastic Chase decoding algorithm is described. Section III presents numerical results. At last, conclusions are drawn in Section IV.

II. Stochastic Chase Decoding

Consider a binary linear code $C(n,k,d)$ in which n is the codeword length, k is the dimension of the code and d is the minimum Hamming distance of C. Let $\boldsymbol{v}=(v_1,v_2,\dots,v_n)$ be a codeword in C. For transmission, binary antipodal modulation and an AWGN channel are assumed. At the receiver side, the sequence of real values observed at the output of the matched filters, $\boldsymbol{r}=(r_1,r_2,\dots,r_n)$, and the binary sequence $\boldsymbol{y}=(y_1,y_2,\dots,y_n)$, obtained by hard quantization of $\boldsymbol{r}$, are used as input of the soft-decision decoding algorithm.

In Ch algorithm, the set of test patterns is given by sequences of length n which have any binary combination in the p least reliable positions. After the generation of the 2^p test patterns, they are used as input of the Berlekamp-Massey (BM) hard-decision decoder. If the decoding is successful ($\hat{\boldsymbol{v}}$ is valid), the codeword obtained by the BM decoder is included in the set of candidate codewords $\boldsymbol{\Lambda}$. Maximum likelihood soft-decision decoding is performed for each codeword in this set.

In $S-Ch$ algorithm [7], the test pattern selection is a bit-wise stochastic experiment based on the observation of the sequence $\boldsymbol{r}$. The bit y_i of the m-th test pattern depends on the reliability r_i, which can be either represented in the probability domain as

$$p_i = P(r_i|v_i=1) = \left(1+e^{\frac{2r_i}{\sigma^2}}\right)^{-1} \qquad (1)$$

where σ^2 is the AWGN power. This algorithm has three independent parameters. The variation of the threshold θ changes the number of bits that will be prevented from being inverted. Decreasing θ avoids the flipping of less reliable bits, while increasing θ prevents only the most reliable bits. The parameter β is a positive constant that must be optimized for each BCH code. The parameter τ is the total number of generated test patterns, being each one unique or

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not. We introduce an improvement in the decoding algorithm to reduce the computational complexity compared to the original one. This consists in removing repeated test patterns. Simulation results show that for lower minimum distance codes, the number of repeated test patterns can be very high (see Section III). A summary of the proposed decoding algorithm is shown in Figure 1.

1. **for** $(1 \le i \le n)$ **do**
   if $(p_i \le 0.5 - \theta)$ **then**
   $p_i = 0$, where $0 < \theta < 0.5$
   else if $p_i \ge 0.5 + \theta$ **then**
   $p_i = 1$
   else
   $p_i = \frac{1}{1+e^{-\beta r_i}}$, where $\beta > 0$
2. **for** $(1 \le m \le \tau)$ **do**
   for $(1 \le i \le n)$ **do**
   - Generate an uniformly distributed random value: $s_i \in [0,1]$
   - Generate $y_i^m = 0$ (if $s_i \ge p_i$) or $y_i^m = 1$ (otherwise)

   - Check if $\boldsymbol{y^m}$ is not equal to previous ones. If it is duplicated, remove it.
   - Perform BM hard-decision decoding on $\boldsymbol{y^m}$ to get $\hat{\boldsymbol{v}}$. If $\hat{\boldsymbol{v}}$ is valid, insert it into the set of candidate codewords Λ.
3. Select the codeword $\boldsymbol{d}$ from Λ which has the maximum correlation with $\boldsymbol{r}$.

Figure 1. Description of the modified $S - Ch$ algorithm.

III. SIMULATION RESULTS

Computer simulations of the Ch and the $S - Ch$ algorithms were performed for BCH codes of codeword length $n = 127$ and different error-correcting capabilities ($t = 3$, 6 and 9). The three parameters of the $S - Ch$ algorithm were fixed to $\tau = 1024$, $\theta = 0.45$, and $\beta = 6$.

Table I summarizes the results obtained for the three BCH codes mentioned previously. The performance of the decoding algorithms is given by the frame error rate (FER) and the complexity metric is given by the number of BM hard-decision decoding that are performed as a step of the soft-decision decoding algorithm (Ch or $S - Ch$). For Ch algorithm, the number of BM decodings is 2^p, because every test pattern implies in one BM decoding. For $S - Ch$ algorithm, the amount of hard-decision decodings depends on the average number of distinct test patterns generated to decode each codeword (see the description of $S - Ch$ algorithm in the previous section). We denote the average number of BM decodings by N_{BM}. Both parameters (FER and N_{BM}) were obtained for both algorithms for selected values of the signal-to-noise ratio (SNR) in dB.

We observe from Table I that when $t = 6$ and SNR= 4.0 dB the $S - Ch$ with $N_{\text{BM}} = 772$ outperforms the Ch with 1024 ($p = 10$) test patterns (number of BM decodings). Thus, the $S - Ch$ algorithm operating with the proposed parameters has better performance and less decoding complexity than the Ch algorithm. This trend was also observed for other values of SNR and other values of t (see, for example, the results for $t = 9$ in Table I). We also noticed that the code gain obtained by the $S - Ch$ becomes negligible for small values of t. This is observed for $t = 3$ in the table, where the values of p were selected such that 2^p is close to N_{BM}. In this case, both algorithms have comparable performance with similar complexities.

TABLE I. RESULTS OF PERFORMANCE AND COMPLEXITY OF THE CH AND S-CH ALGORITHMS APPLIED TO DIFFERENT BCH CODES OF CODEWORD LENGTH $n = 127$.

BCH $(127, 106, 7)$ $[t = 3]$				
	$\boldsymbol{Ch}$		$\boldsymbol{S - Ch}$	
SNR	**FER**	$\boldsymbol{N_{\text{BM}}}$	**FER**	$\boldsymbol{N_{\text{BM}}}$
4.0	$\boldsymbol{8.37 \cdot 10^{-3}}$	**512**	$\boldsymbol{6.91 \cdot 10^{-3}}$	427
4.5	$\boldsymbol{1.79 \cdot 10^{-3}}$	**256**	$\boldsymbol{10^{-3}}$	237
5.0	$\boldsymbol{2.7 \cdot 10^{-4}}$	**128**	$\boldsymbol{2.4 \cdot 10^{-4}}$	105
BCH $(127, 85, 13)$ $[t = 6]$				
	$\boldsymbol{Ch}$		$\boldsymbol{S - Ch}$	
SNR	**FER**	$\boldsymbol{N_{\text{BM}}}$	**FER**	$\boldsymbol{N_{\text{BM}}}$
4.0	$\boldsymbol{1.38 \cdot 10^{-3}}$	**1024**	$\boldsymbol{5.25 \cdot 10^{-4}}$	772
4.5	$\boldsymbol{1.1 \cdot 10^{-4}}$	**1024**	$\boldsymbol{3.75 \cdot 10^{-5}}$	607
BCH $(127, 71, 19)$ $[t = 9]$				
	$\boldsymbol{Ch}$		$\boldsymbol{S - Ch}$	
SNR	**FER**	$\boldsymbol{N_{\text{BM}}}$	**FER**	$\boldsymbol{N_{\text{BM}}}$
4.0	$\boldsymbol{8 \cdot 10^{-4}}$	**1024**	$\boldsymbol{2.12 \cdot 10^{-4}}$	992
4.5	$\boldsymbol{5 \cdot 10^{-5}}$	**1024**	$\boldsymbol{8.33 \cdot 10^{-6}}$	840

IV. CONCLUSIONS

In this work, the Stochastic Chase decoding of BCH codes is investigated by a modification of the original algorithm proposed in [7]. Also, the complexity-performance trade-off of the decoding for BCH codes of codeword length $n = 127$ and different error-correcting capabilities is analyzed. Work is in progress to apply this approach to a BCH turbo decoding framework.

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