On the Pseudo-Bayesian Broadcast Control Algorithm for Slotted ALOHA in Multi Packet Reception and under Impaired Channel Conditions

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***Abstract*—The basic concept of slotted ALOHA as a Random Access Protocol (RAP) is commonly implemented for ubiquitous access in many wireless networks. In this paper, we study the generalization of the *network control by Bayesian broadcast* to the environment of M-MRP Multiple Packet Reception (MPR) when channel impairments are considered. In our M-MPR model, up to M data packets transmitted in the same time-slot can be correctly decoded by using capture effect and some advanced signal processing techniques such as Successive Interference Cancellation (SIC) combined with Multiple-Input Multiple-Output (MIMO). We show that the broadcast or permission probability that maximizes the throughput (packets per slot successfully transmitted) is sensitive to channel characteristics. While with ideal channel conditions of maximum capacity a binary feedback – collision versus non-collision – is required, and in the more realistic channel conditions, $M+1$ feedback is needed.**

***Keywords*-Pseudo-Bayesian Control, Multiple Packet Reception.**

I. Introduction

For ubiquitous multi-access in wireless networks, a single channel is shared by a population of devices or users. In order to share this common transmission medium among users, a Medium Access Control (MAC) protocol must be properly designed. When users act in an independent manner, i.e., with minimum coordination between them, we need a suitable Random Access Protocol (RAP). The area of RAPs started with the seminal work by N. Abramson in 1970 [1], where the ALOHA protocol was proposed. Later in 1972 [2], Roberts adds to ALOHA the additional feature of slot synchronization, so the S-ALOHA was proposed as a substantial improvements of its throughput, increasing from the $1/2e \approx 0.1839$ channel utility for pure ALOHA to $1/e \approx 0.3679$ packets/slot for S-ALOHA. Since then, many RAPs based on the ALOHA principles have been proposed for wired Local Area Networks (LANs) and wireless (cellular, Wireless Fidelity (WiFi), etc.) communication systems. The main advantage of ALOHA protocol is its easy and simple implementation. Unlike Carrier Sense Multiple Access (CSMA) protocol, in ALOHA no sensing functioning needs to be performed. Furthermore, the hidden terminal effect that can significantly deteriorate the CSMA performance does not affect the operation of the ALOHA protocol. A basic background on this matter can be found in [3] [5] [7].

ALOHA alike protocols are inherently located at the MAC layer. The improvement of ALOHA protocols can be achieved when combining with other physical layer techniques such as Multi-User Detection (MUD), Multiple-Input Multiple-Output (MIMO) or a combination of both techniques (MU-MIMO). In MUD, a single receiver is able to decode the intended signals from interference and noise. MUD techniques include Maximum-Likelihood (ML), Parallel Interference Cancellation (PIC), Successive Interference Cancellation (SIC), etc. In MIMO technique, more than one antenna at transmitter and at the receiver part are installed to get improvements in parameters such as throughput and channel robustness. For more details, interested readers are referred to [11]. At the physical layer, the use of MIMO, MUD and SIC will benefit the Multiple Packet Reception (MPR) reception technique. So, a cross layer cooperation based on the use of at the physical layer and the S-ALOHA protocol at the MAC layer will bring benefits in the throughput of wireless access system. Thanks to this cooperation, we can enjoy the M-MPR capability.

Additional contributions in the M-MPR area can be found in [6] [10]; where the number of packets that can be received and decoded simultaneously is M, and the stability analysis of MPR is studied in a deep way. In [12], the authors study the M-MPR using the principle of MUD at the Base Station (BS). The authors adopt the adaptive interference canceler employing the Recursive Least Square Maximum Likelihood Sequence Estimation (RLS-MLSE) scheme. Through computer simulation and field trial under a realistic scenario, it is shown that up to three ($M = 3$) simultaneously transmitted packets can be detected, even though they limit their study to $M = 2$. That is, for $M = 2$, very reliable of real time applications, the maximum throughput can exceed 0.7, which is a significant improvement compared to the convention S-ALOHA of $1/e \approx 0.3679$.

In [14], a finite number of devices access to a common wireless channel using S-ALOHA, where the M-MPR scheme with the *all-or-nothing* philosophy is assumed. Devices operate in saturation conditions (there are always packets to be transmitted) and the permission or transmission probability is constant. Their analysis lacks of dynamic adaptation of the transmission probability. In [15], the authors provide an in-depth analysis of the M-MPR protocol for ALOHA and CSMA random access algorithms. However, with regard to ALOHA protocol, the analysis does not take into account the arrival process that could joint backlogged data packets.

In order to avoid total loss of packets to collisions, several strategies supporting power transmission have been proposed for ALOHA packets [17] [18]. Hence, in [18], the authors study the non-orthogonal random access technique for 5th Generation (5G) networks in which due to the different level of the received power at the BS, it enables the BS to decode two packets simultaneously using SIC. The analysis is carried out in terms of access delay, throughput, and energy efficiency.

The capture effect can happen so allowing the decoding of a number of packets lower or equal to the number of packets that simultaneously coincide in the same time slot. The authors of [12] provide an analysis quite parallel to our work but the novelty of our work is the Bayesian estimation of the number of users in contention in a framed-slotted ALOHA environment, as an enhancement to the work by [4]. In [19], the distribution of new plus backlogged packets are assumed to follow a Poisson distribution.

In all these previous studies, the main assumption is that the channel is ideal, i.e., neither fading nor interference happens. All of them consider that the channel capacity is M and, when the number of data packets in one slot is not greater than M all packets can be successfully decoded, otherwise the slot is considered as collision (garbled).

In this work, we assume a general model where $\alpha_{m,k}$ for $0 \leq k \leq m \leq M$ denotes the conditional probability to detect correctly k packets assuming that m packets were transmitted. The aim of this paper is to extend the pseudo-Bayesian broadcast control algorithm of Rivest [4] developed to Single-Packet Reception (SPR) to the case of MPR. Then, first we deal with a finite number of active devices and second we follow with an arbitrary number of active devices. The closest approach to our work or the most related work with our paper is the one presented in [16], but they use the *all-or-nothing* model defined below.

The rest of this paper is organized as follows. In Sec. II, we describe the model of the system under study. In Sec. III, the optimal permission probability for a given number of active devices is derived. Sec. IV deals with the estimation of the number of active devices, so with the updating permission probability based on the Bayesian rules. In Sec. V, we introduce the common assumption of Poisson distribution for the number of active devices and the Pseudo Bayesian procedure is described. In Sec. VI, some particular cases are studied. The paper ends with conclusions in Sec. VII.

II. System model

Consider a time-slotted channel. A finite number of active devices, sufficiently large enough, transmit their packets uplink towards an Access Point (AP) or a 5G BS, i.e., nest generation Node B (gNB). A given device becomes active when it has a packet ready to transmit. Packets are of constant length that fits with the length of the time-slot.

Devices follow the Immediate First Transmission (IFT) principle instead of the Delayed First Transmission (DFT) principle. That is, as soon as a given device becomes active the corresponding packet joints the set of backlogged packets and follows the RAP's rules. In the RAP, all active devices (new or backlogged) transmit with the same broadcast or permission probability provided by the gNB instantly in the beginning of a time-slot. In other words, new and backlogged packets are treated in the same way. The permission probability is updated by the gNB in a slot-by-slot basis according to the observed results in each time slot and according to the expected number of new active devices (the arrival process).

In the M-MPR model, the channel for transmission-reception is represented by a set of conditional probabilities, time invariant, given in the following the MPR stochastic matrix [6],

$$\mathbf{A} = \left[\begin{array}{cccc|c} \alpha_{0,0} & 0 & \cdots & 0 & \alpha_{0,c} \\ \alpha_{1,0} & \alpha_{1,1} & \cdots & 0 & \alpha_{1,c} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \alpha_{M,0} & \alpha_{M,1} & \cdots & \alpha_{M,M} & \alpha_{M,c} \\ \hline \alpha_{M_>,0}=0 & \alpha_{M_>,1}=0 & \cdots & \alpha_{M_>,M}=0 & 1 \end{array}\right] \quad (1)$$

In (1), $M_>$ is identified as *greater than* M and $\alpha_{i,c}$ denotes the probability the receiver interprets as collision when i packets are transmitted. The set $\{\alpha_{i,j}\}$ contains the conditional probabilities that characterize the transmission-reception characteristics of the wireless channel. Each probability $\alpha_{i,j}$ is interpreted as follows. For an arbitrary time slot, first, we assume that no packets are transmitted. Then, with probability $\alpha_{0,0}$, the slot is correctly interpreted by the gNB, i.e., as a hole, and with probability $\alpha_{0,c} = 1 - \alpha_{0,0}$, the empty slot may be seen as a garbled or collision time-slot, for instance, due to the interference and noise of the channel. Second, we assume that a single packet has been transmitted, the second row of the MPR matrix. Then, the gNB interprets, with probability $\alpha_{1,0}$, as an empty slot (the transmitted packet might vanish due to channel fading conditions), with probability $\alpha_{1,1}$, the packet is correctly decoded and with probability $\alpha_{1,c} = 1 - \alpha_{1,1} - \alpha_{1,0}$, the slot is observed as a garbled time slot (collision). Third, we assume that two packets are simultaneously transmitted, the third row of the MPR matrix. Then, with probability $\alpha_{2,0}$, the slot is observed as empty; with probability $\alpha_{2,1}$, one of the two packets is correctly decoded while the other one is lost (the capture effect [8]); with probability $\alpha_{2,2}$, both packets are correctly decoded (using SIC techniques [18]), and with probability $\alpha_{2,c} = 1 - \alpha_{2,0} - \alpha_{2,1} - \alpha_{2,2}$, the observed slot is seen as garbled, as a collision slot. And so on. Finally, when in the same observed time slot more than M packets are transmitted, with probability 1, the gNB interprets as a collision slot, i.e., for $i > M$ we have $\alpha_{i,j} = 0$ and $\alpha_{i,c} = 1$.

In the M-MPR model, the *all-or-nothing* scheme has often been considered. Accordingly, the receiving station is able to successfully decode m simultaneous transmissions with probability one if and only if $m \leq M$ and no decoding can be achieved when $m > M$, which in turns means that $\mathbf{A} = \mathbf{I}$, the identity matrix. This is the typical assumption in many papers such as [15], [16], [19]. Our study generalizes this particular case. For some particular cases, in the same way as in [9], we consider the case where the set of probabilities $\{\alpha_{i,j}\}$ being system feature, are known *a priori* or a good estimation of them is known.

III. Broadcast or Permission Probabilities

We consider a number of active devices N_t, each one with a single packet ready to be transmitted at time-slot t. The idea is to use the optimal broadcast or permission probability that maximizes some relevant function, such as the throughput, defined as the mean number of packets successfully transmitted in time-slot t. Here, we obtain the optimum permission probability, first when the number of active devices is finite and second when this number follows a given distribution.

A. For a Fixed Number of Active Devices

We assume a fixed number of active devices, $N_t = n$, each one with one packet ready to be transmitted at time-slot t. We consider that $n > M$. N_t needs to be estimated, but initially we assume that the gNB has perfect knowledge of it. Each active device will transmit with the probability of permission $b_{M,t}$ and will wait for the next slot with the probability $w_{M,t} = 1 - b_{M,t}$ (the IFT principle). Then, the following events are considered, empty slot (hole), slot with m successes (success=m), with $0 \leq m \leq M$, and slot with collision; i.e., the probability of observing a hole,

$$\begin{aligned} &Pr(hole/(N_t = n, b_{M,t})) = \\ &H_{b_{M,t}}(n) = \textstyle\sum_{k=0}^{M} B_k^n(b_{M,t})\alpha_{k,0}, \end{aligned} \tag{2}$$

where $B_k^n(b_{M,t})$ denotes the binomial distribution,

$$B_k^n(b_{M,t}) = \left(\begin{array}{c} n \\ k \end{array} \right) b_{M,t}^k . w_{M,t}^{n-k}, \quad 0 \leq k \leq n.$$

The probability of observing m successes,

$$\begin{aligned} &Pr(success = m/(N_t = n, b_{M,t})) = \\ &S_{m,b_{M,t}}(n) = \textstyle\sum_{k=m}^{M} B_k^n(b_{M,t})\alpha_{k,m}, \end{aligned} \tag{3}$$

and, the probability to observe a collision,

$$\begin{aligned} &Pr(collision/(N_t = n, b_{M,t})) = \\ &C_{b_{M,t}}(n) = 1 - H_{b_{M,t}}(n) - \textstyle\sum_{m=1}^{M} S_{m,b_{M,t}}(n) = \\ &1 - \textstyle\sum_{m=0}^{M} S_{m,b_{M,t}}(n); \quad (S_{0,b_{M,t}}(n) = H_{b_{M,t}}(n)). \end{aligned} \tag{4}$$

Observe that the event hole can be regarded as the event m = 0 success, i.e., $H_{b_{M,t}}(n) = S_{0,b_{M,t}}(n)$, and this explains the last equality in (4). The mean value of the number of packets successfully transmitted is given, after some simple rearrangement of terms, by

$$\begin{aligned} &E(\#successes/(N_t = n, b_{M,t})) = \\ &\textstyle\sum_{m=1}^{M} mPr(success = m/(N_t = n, b_{M,t})) \\ &= \textstyle\sum_{m=1}^{M} m \sum_{k=m}^{M} B_k^n(b_{M,t})\alpha_{k,m} = \sum_{m=1}^{M} B_m^n(b_{M,t})\overline{\alpha}_m, \end{aligned} \tag{5}$$

where $\overline{\alpha}_m = \sum_{k=1}^{m} k\alpha_{m,k}$ $(0 < m \leq M)$ is the expected number of correctly decoded packets when m packets are transmitted simultaneously in the same time-slot [13]. The maximum of (5) can be computed by differentiating and root finding. Then, from (5), we have found for $\hat{b}_{M,t}$ (also, see (5) in [15] where $\overline{\alpha}_m = m$),

$$\hat{b}_{M,t} = \frac{\sum_{m=1}^{\min(n,M)} m\overline{\alpha}_m B_m^n(\hat{b}_{M,t})}{n\sum_{m=1}^{\min(n,M)} \overline{\alpha}_m B_m^n(\hat{b}_{M,t})} = h_{M,\alpha}(\hat{b}_{M,t}) \tag{6}$$

with $\alpha = [\overline{\alpha}_1, \overline{\alpha}_2, \ldots, \overline{\alpha}_M]$. In (6) we can apply the fixed point iteration method, i.e., $\hat{b}_{M,t}^{(i+1)} = h_{M,\alpha}(\hat{b}_{M,t}^{(i)})$, $i = 1, 2, \ldots$, with $\hat{b}_{M,t}^{(0)} \in [0, 1]$ and the optimum permission probability $\hat{b}_{M,t} = \hat{b}_{M,t}^{(\infty)}$ is obtained, i.e., the iteration always converges to the unique solution. Moreover, explicit expressions can be found for $M = 1, 2$, and 3. When $M = 1$, we have $\alpha_{\geq 2,c} = 1$, equivalent to $\overline{\alpha}_{\geq 2} = 0$ and trivially we obtain $\hat{b}_{1,t} = K_1/n = 1/n$. When $M = 2$, $\alpha_{\geq 3,c} = 1$ (equivalent to $\overline{\alpha}_{\geq 3} = 0$), the optimum value of the permission probability, $\hat{b}_{2,t}$ is,

$$\hat{b}_{2,t} = \tfrac{(n-1)\overline{\alpha}_2 - (n+1)\overline{\alpha}_1 + \sqrt{\Delta}}{n\left[(n-1)\overline{\alpha}_2 - 2\overline{\alpha}_1\right]} > \tfrac{1}{n}; \quad , n = 3, 4, \ldots$$

with $\Delta = (n-1)\left[(n-1)\left(\overline{\alpha}_1^2 + \overline{\alpha}_2^2\right) - 2\overline{\alpha}_1\overline{\alpha}_2\right] = (n-1)\left[n\left(\overline{\alpha}_1^2 + \overline{\alpha}_2^2\right) - \left(\overline{\alpha}_1 + \overline{\alpha}_2\right)^2\right]$. For large values of n we can write,

$$\hat{b}_{2,t} \approx \frac{\overline{\alpha}_2 - \overline{\alpha}_1 + \sqrt{\overline{\alpha}_1^2 + \overline{\alpha}_2^2}}{n\overline{\alpha}_2} = \frac{K_2}{n} > \frac{1}{n}; \quad , n = 3, 4, \ldots$$

with

$$\begin{aligned} &K_2 = 1 + \frac{\sqrt{1 + \left(\overline{\alpha}_2/\overline{\alpha}_1\right)^2} - 1}{\overline{\alpha}_2/\overline{\alpha}_1} = 1 + \tfrac{\sqrt{1+x^2}-1}{x} = \\ &1 + \tfrac{1}{2}x - \tfrac{1}{2^2.2!}x^3 + \tfrac{1.3}{2^3.3!}x^5 - \tfrac{1.3.5}{2^4.4!}x^7 + \tfrac{1.3.5.7}{2^5.5!}x^9 \ldots \\ &\quad \text{and} \quad \mathrm{x} = \tfrac{\alpha_{2,1} + 2\alpha_{2,2}}{\alpha_{1,1}} = \tfrac{\overline{\alpha}_2}{\overline{\alpha}_1}. \end{aligned} \tag{7}$$

Note that when $\overline{\alpha}_2 \to 0$, i.e., $x \to 0$, the evaluation of (7) using the closed form (the expression with a square root) may lead to some imprecise calculation. In this case we could use the approximation given by the Taylor expansion.

Due to the page limit, we omit the exact analytical expression for $M = 3$. In general, for any M, and for large values of n, $\hat{b}_{M,t}$ can be expressed as $\hat{b}_{M,t} \approx K_M/n$. In fact, (5) can be approximated by

$$\begin{aligned} &E(\#successes/(N_t = n, b_{M,t})) = \\ &\approx \textstyle\sum_{m=1}^{M} \frac{(nb_{M,t})^m}{m!}\overline{\alpha}_m e^{-nb_{M,t}}. \end{aligned} \tag{8}$$

B. For a Random Number of Active Devices.

Now, we assume that N_t follows a discrete probability distribution, $p_{n,t}$, with Generator Function (GF), given by, respectively

$$Pr(N_t = n) = p_{n,t}; \quad P_t^*(z) = \sum_{n=0}^{\infty} p_{n,t}z^n. \tag{9}$$

Furthermore, we assume that the gNB has a perfect knowledge of $p_{n,t}$. Therefore, unconditioning (5) with $p_{n,t}$, the throughput, which defined as the expected number of successes at time-slot t, is given by, after some algebra,

$$T_{M,\alpha}(b_{M,t}) = E(Pr(\text{success at slot t})/b_{M,t}) = \sum_{m=1}^{M} \frac{b_{M,t}^m}{m!} \frac{d^m P_t^*(w_{M,t})}{dw_{M,t}^m} \overline{\alpha}_m. \quad (10)$$

The optimum permission probability $\hat{b}_{M,t} = 1 - \hat{w}_{M,t}$ that maximizes (10), a polynomial in the unknown variable $b_{M,t}$, can be computed by differentiating and root finding. In a practical sense, the computation required to obtain $b_{M,t}$, would be time consuming. This can be avoided by using the approximation $\hat{b}_{M,t} \approx K_M/E(N_t)$. However, the pseudo-Bayesian broadcast algorithm described in the next section appears to be an excellent approach [4].

IV. Estimating Number of Active Devices

In an M-MPR channel, the permission probability $\hat{b}_{M,t}$ to be used in time-slot t is evaluated according to the procedure described in previous section. $\hat{b}_{M,t}$ is updated on a slot-by-slot basis. The update procedure is based on the outcomes in time-slot t observed by the gNB and on the arrival process of new packets, i.e., on the number of devices that become active during time-slot t. For the first item, we apply Baye's rule, as suggested in [4]. For the second item we consider a general distribution $\{a_{n,t}\}$ with GF, $A^*(z) = \sum_{n=0}^{\infty} a_{n,t} z^n$. Furthermore, the arrival process is assumed to be independent of the RAP.

A. Bayesian Updating of the Probability Vector

Assume that the procedure to estimate the probability vector N_t, $\overline{p}_t = [p_{0,t}, p_{1,t}, p_{2,t}, \ldots]$, is reasonably good. Now, we describe how the gNB updates this probability vector of N_t, given that slot t was a hole, a success-m, or a collision. Denote E=Evidence (hole, success-m, collision) and H=Hypothesis ($N_t = n$ data packets). The Bayes' rule tells us,

$$Pr(H/E) = \frac{Pr(E/H)Pr(H)}{Pr(E)}. \quad (11)$$

Then, the gNB will use the evidence available up to time-slot t to update $\{p_{n,t}\}$, given the available evidence. This is the so called Bayesian broadcast procedure, since it relies on Bayesian reasoning to estimate $\overline{p}_t = [p_{0,t}, p_{1,t}, p_{2,t}, \ldots]$ according to (11).

Let $p'_{n,t}$ denote the final probability $Pr(N_t = n/E_t)$ where E_t is the slot t evidence (hole, success-m, or collision), i.e., $\overline{p}'_t = [p'_{0,t}, p'_{1,t}, p'_{2,t}, \ldots]$. The probabilities $p'_{n,t}$ ($Pr(H/E)$) are easily obtained using Bayes' rule by multiplying each initial probability $p_{n,t}$ ($Pr(H)$) by the appropriate likelihood $H_{b_{M;t}}(n)$, $S_{m,b_{M;t}}(n)$ or $C_{b_{M;t}}(n)$ ($Pr(E/H)$) (see (2), (3) and (4)), according to whether a hole, success-m, or collision was observed, and then normalizing so that the $p'_{n,t}$ add up to one. Then, the numerator of (11) is evaluated as follows,

If the gNB observes a hole,

$$\overline{p}'_t = \frac{[p_{0,t}H_{b_{M;t}}(0), p_{1,t}H_{b_{M;t}}(1), \ldots]}{Ch_t}. \quad (12)$$

If the gNB observes a success-m event, for $m = 1, 2, \ldots, M$,

$$\overline{p}'_t = \frac{[p_{0,t}S_{m,b_{M;t}}(0), p_{1,t}S_{m,b_{M;t}}(1), \ldots]}{Cs_{m,t}}. \quad (13)$$

Finally, if the gNB observes a collision event,

$$\overline{p}'_t = \frac{[p_{0,t}C_{b_{M;t}}(0), p_{1,t}C_{b_{M;t}}(1), \ldots]}{Cc_t}. \quad (14)$$

where $Ch_t = \sum_{n=0}^{\infty} p_{n,t}H_{b_{M;t}}(n)$, $Cs_{m,t} = \sum_{n=0}^{\infty} p_{n,t}S_{m,b_{M;t}}(n)$ and $Cc_t = \sum_{n=0}^{\infty} p_{n,t}C_{b_{M;t}}(n)$ are the respective normalization constants. Note that the case hole can be regarded as a particular case of success-m when $m = 0$, i.e., $H_{b_{M;t}}(k) = S_{0,b_{M;t}}(k)$ and $Ch_t = Cs_{0,t}$.

B. Modeling Successful Packet Transmission

When the gNB observes the evidence S_m ($m = 1, 2, \ldots, M$), the number of packets pending to be transmitted is m less than the estimated number before the access action. For the evidences H and C the number of packets that are pending to gain the access in the next time slot $t{+}1$ is the same as the one we have at time slot t. Therefore, considering the observations, hole, success-m ($m = 1, 2, \ldots, M$) or collision, we have, including the GF of the probability vector,

If a hole is observed,

$$p''_{n,t} = p'_{n,t} \Rightarrow \quad P''^*_t(z) = P'^*_t(z). \quad (15)$$

If a success-m is observed,

$$p''_{n,t} = p'_{n+m,t} \Rightarrow \quad P''^*_t(z) = P'^*_t(z) z^{-m}. \quad (16)$$

If a collision is observed,

$$p''_{n,t} = p'_{n,t} \Rightarrow \quad P''^*_t(z) = P'^*_t(z). \quad (17)$$

C. Modeling the Arrivals of New Packets

Let us assume that new packets arrive independently of the the contention process. Assuming a memoryless arrival process on a slot basis, we define $a_{n,t}$ the probability that n packets are generated in time slot t with GF $A_t^*(z) = \sum_{n=0}^{\infty} a_{n,t} z^n$. Furthermore, we also assume that $\hat{a}_{n,t}$, ($\hat{A}_t^*(z)$), the estimation of $a_{n,t}$, ($A_t^*(z)$), it can be done with sufficient accuracy.

D. The Probability Vector at Time Slot t+1

Since the arrival process is independent of the RAP, the GF of probability vector at time-slot $t+1$ is the product of the two related generating functions, i.e.,

$$P_{t+1}^*(z) = \sum_{n=0}^{\infty} p_{n,t+1} z^n = P''^*_t(z)\hat{A}_t^*(z) = \begin{cases} P'^*_t(z)\hat{A}_t^*(z), & \text{hole} \\ P'^*_t(z) z^{-m} \hat{A}_t^*(z), & \text{success}-\text{m} \\ P'^*_t(z)\hat{A}_t^*(z), & \text{collision} \end{cases} \quad (18)$$

and the optimum broadcast probability $\hat{b}_{M,t+1}$ for the next slot $t+1$ is derived using the vector probability given by (18) in (10) and the root finding procedure. With this last step, the cycle is completed.

V. The Probability Vector: Poisson Arrivals

The previous procedure can be simplified by assuming that, in the same way as in many other works, [4], [5], [6], [12], [16], [18], [19], the vector of probabilities $p_{n,t}$ at time-slot t, (new arrivals + backlogged packets) can be approximated reasonably, by a Poisson distribution with rate ν_t. In this case, (9) turns as,

$$p_{n,t} = \frac{(\nu_t)^n}{n!} e^{-\nu_t}, \quad P_t^*(z) = e^{\nu_t (z-1)}. \tag{19}$$

Observe that now, the slot-by-slot updating procedure for the probabilities $p_{n,t}$ is simplified to the task of updating the rate $\nu_t = E(N_t)$, i.e., the single parameter that defines the Poisson distribution. We recall that ν_t is the average number of active devices at the beginning of time-slot t and it must be estimated. Then, inserting (19) into (10) and with the notation $x = \nu_t b_{M,t}$, after some algebra,

$$\begin{aligned} &T_{M,\alpha}(x) = E(Pr(\text{success at slott})/b_{M,t}) = \\ &= \textstyle\sum_{k=1}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \overline{\alpha}_k e^{-\nu_t b_{M,t}} = \sum_{k=1}^{M} \frac{x^k}{k!} \overline{\alpha}_k e^{-x}. \end{aligned} \tag{20}$$

Notice that the throughput is a function of the product $x = \nu_t b_{M,t}$. Let $\hat{x}_M = K_M$ be the value that maximizes $T_{M,\alpha}(x)$. Then, setting to zero the first derivative of (20), we have,

$$dT_{M,\alpha}(x)dx = \sum_{k=1}^{M} \left(\frac{x^{k-1}}{(k-1)!} \overline{\alpha}_k - \frac{x^k}{k!} \overline{\alpha}_k \right) e^{-x} = 0 \tag{21}$$

Leaving aside the exponential factor e^{-x}, the condition in (21) can be expressed in the following form,

$$x = \frac{x \sum_{k=1}^{M} \frac{x^{k-1}}{(k-1)!} \overline{\alpha}_k}{\sum_{k=1}^{M} \frac{x^k}{k!} \overline{\alpha}_k} = h_{M,\alpha}(x) \tag{22}$$

where we have defined the function $h_{M,\alpha}(x)$.

In addition, it is trivial to check that $h_{M,\alpha}(x) < h_{M+1,\alpha}(x)$ for $0 < x$, we can assert that, $\ldots \hat{x}_{M-1} < \hat{x}_M < \hat{x}_{M+1} \ldots$. Therefore, additional computing time savings can be achieved by choosing as the initial estimation for $\hat{x}_{M+1}$ the previous value, i.e., $x_{M+1}^{(0)} = \hat{x}_M = K_M$.

For $M = 1, 2, 3$, closed form expressions are obtained for $\hat{x}_M = K_M$; but, in general numerical computation to find K_M is required.

Since N_t is randomly distributed and since $\hat{b}_{M,t}$ is a probability where $\hat{x}_M = \nu_t \hat{b}_{M,t}$ we finally set,

$$\hat{b}_{M,t} = 1 - \hat{w}_{M,t} = min\left(\frac{K_M}{\nu_t}, 1 \right). \tag{23}$$

Clearly, ν_t in (23) is unknown so it needs to be estimated and adapted in a slot-by-slot manner. Let $\hat{\nu}_t$ denote the estimation of ν_t at the beginning of time-slot t (in (23) $\hat{\nu}_t$ will be used instead of ν_t). Then, as we have discussed before, $\hat{\nu}_{t+1}$, the estimation of ν_{t+1}, is supported by two items. First, by the outcomes of slot t observed by the gNB. Second, by the arrival process of new packets that joint the backlogged packets and follow the common RAP. Remember, the algorithm is supported by the IFT principle.

A. Bayesian Updating of the Probability Vector

If the gNB observes a hole, (12) becomes, after the normalization step,

$$p'_{n,t} = \frac{\sum_{k=0}^{M} B_k^n(b_{M,t}) \alpha_{k,0}}{\sum_{k=0}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,0}} \frac{\nu_t^n}{n!} e^{-\nu_t w_t}; \quad n \geq 0. \tag{24}$$

with GF,

$$P_t^{'*}(z) = \frac{\sum_{k=0}^{M} \frac{(\nu_t b_{M,t} z)^k}{k!} \alpha_{k,0}}{\sum_{k=0}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,0}} e^{\nu_t w_{M,t}(z-1)}. \tag{25}$$

We observe that (25) is a weighted sum of $M + 1$ Poisson distributions, where each distribution is obtained by shifting k positions to the right ($k = 0, 1, \ldots, M$) the distribution $e^{\nu_t w_{M,t}(z-1)}$. Consequently, we could reconsider the initial hypothesis of Poisson distribution for $p_{n,t}$ and to inquire about a linear combination of $M + 1$ Poisson distributions as a better distribution for $p_{n,t}$. However, to derive this possibility is beyond the scope of this paper.

The first derivative of $P_t^{'*}(z)$ evaluated at $z = 1$ is,

$$\begin{aligned} &\text{mean value}_{E=H} = \nu_t w_{M,t} + \frac{\sum_{k=0}^{M} k \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,0}}{\sum_{k=0}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,0}} = \\ &\nu_t - x \frac{\sum_{k=0}^{M} \frac{x^k}{k!} (\alpha_{k,0} - \alpha_{k+1,0})}{\sum_{k=0}^{M} \frac{x^k}{k!} \alpha_{k,0}} \end{aligned} \tag{26}$$

with $x = \nu_t b_{M,t}$ and $\alpha_{M+1,0} = 0$, see channel characteristics in (1). Observe that, according to (15), (16), (17), we identify $P_t^{''*}(z) = P_t^{'*}(z)$.

Note that if $\alpha_{k,0} = \delta_{k,0}$ (Kronecker delta) then $meanvalue_{E=H} = \nu_t w_{M,t} = \max(\nu_t - K_M, 0)$. In other words, if $b_{M,t} = 1$, we are certain that the number of data packets ready for transmission was zero. Otherwise, this case cannot be confirmed when $b_{M,t} < 1$.

If the gNB observes the success-m event (13), i.e., m packets are successfully decoded, including the normalization step, we have,

$$p'_{n,t} = \begin{cases} 0; n = 0, 1, \ldots, m-1; \\ \frac{\sum_{k=m}^{M} B_k^n(b_{M,t}) \alpha_{k,m}}{\sum_{k=m}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,m}} \frac{\nu_t^n}{n!} e^{-\nu_t w_{M,t}}; \quad n \geq m. \end{cases} \tag{27}$$

with a generating function,

$$\begin{aligned} &P_t^{'*}(z) = \textstyle\sum_{n=0}^{\infty} p'_{n,t} z^n = \\ &\frac{\sum_{k=m}^{M} \frac{(\nu_t b_{M,t} z)^k}{k!} \alpha_{k,m}}{\sum_{k=m}^{M} \frac{(\nu_t b_{M,t})^k}{k!} \alpha_{k,m}} e^{\nu_t w_{M,t}(z-1)}. \end{aligned} \tag{28}$$

As in (25), we observe that (28) is a weighted sum of $M - m+1$ Poisson distributions, where each distribution is obtained by shifting the same distribution $e^{\nu_t w_{M,t}(z-1)}$ k positions to the right ($k = m, m+1, \ldots, M$). The first derivative of (28) evaluated at $z = 1$, gives us, after some simple algebra,

$$\text{mean value}_{E=S_m} = = \nu_t + m - x + \frac{\sum_{k=m}^{M} (k-m) \frac{x^k}{k!} \alpha_{k,m}}{\sum_{k=m}^{M} \frac{x^k}{k!} \alpha_{k,m}} \quad (29)$$

with $x = \nu_t b_{M,t}$.

As soon as at least one of the parameters $\alpha_{k,m}$ ($k = m, m+1, \ldots, M$) is greater than zero (column m of matrix $\mathbf{A}$, (1)), the first fraction in (29) is greater than or equal to m and it is a non-decreasing function for $x = \nu_t b_{M,t} \geq 0$. First, it is trivial to see that, for $x \to 0$ the fraction approach to m (L'Hopital's rule). To check the non-decreasing property, we proceed in a similar manner to the checking procedure we use for $h_{M,\alpha}(x)$ in (22). Then, the interpretation of (29) is that when the event *success-m* is observed by the gNB at least m packets, those that successfully pursue medium access, were transmitted in the observed time slot. We add further discussions when dealing with two particular cases in Sec. VI.

Then, from (16), the construction of $P_t^{''*}(z)$ implies that, after the observation *success-m*, the distribution of $p'_{n,t}$ must be shifted m positions to the left. We do this action with the term z^{-m}, i.e., $P_t^{''*}(z) = P_t^{'*}(z) z^{-m}$. Also, we remark the fact that the event "hole", (25), (26), can be seen as a particular case of the event *success-m*, (28), (29), for $m = 0$.

When considering the *all-or-nothing* channel model, i.e., when $\alpha_{k,m} = \delta_{k,m}$ for $0 \leq m \leq M$, then mean value$_{E=S_m} = \nu_t w_{M,t} = \max(\nu_t - K_M, 0)$. In other words, if $b_{M,t}$ was one, we are certain that the number of data packets ready for transmission was m. If $b_{M,t} < 1$, some uncertainty exists about such an assumption.

When the gNB interprets as collision, i.e., one garbled slot is observed, (14) becomes, including the normalization step,

$$p'_{n,t} = = \frac{(\nu_t)^n}{n!} e^{-\nu_t w_{M,t}} \frac{1 - \sum_{k=0}^{M} \binom{n}{k} b_{M,t}^k w_t^{n-k} (1-\alpha_{k,c})}{e^{\nu b_{M,t}} - \sum_{k=0}^{M} \binom{n}{k} b_{M,t}^k w_t^{n-k} (1-\alpha_{k,c})} \quad (30)$$

where $\alpha_{k,c} = 1 - \sum_{l=0}^{k} \alpha_{k,l}$ for $k \leq M$ and $\alpha_{k,c} = 1$ for $k > M$. Its generating function is,

$$P_t^{'*}(z) = P_t^{''*}(z) = \frac{e^{\nu_t b_{M,t} z} - \sum_{k=0}^{M} (1-\alpha_{k,c}) \frac{(\nu_t b_{M,t})^k}{k!} z^k}{e^{\nu_t b_{M,t}} - \sum_{k=0}^{M} (1-\alpha_{k,c}) \frac{(\nu_t b_{M,t})^k}{k!}} e^{\nu_t w_{M,t}(z-1)}. \quad (31)$$

where the first equality in (31) comes from (17). Notice that, in opposite way to (25) and to (28), (31) is not represented by a linear combination of Poisson distributions.

The first derivative of (31) in z = 1 gives us, using the notation of $x = \nu_t b_{M,t}$

$$\text{mean value}_{E=C} = \nu_t + x \frac{\sum_{k=0}^{M} (\alpha_{k+1,c} - \alpha_{k,c}) \frac{x^k}{k!}}{e^x - \sum_{k=0}^{M} (1-\alpha_{k,c}) \frac{x^k}{k!}}. \quad (32)$$

Note that it is reasonable to assume that the fraction of (32) is positive for $x > 0$. In fact, obviously the denominator is always positive since $e^x > \sum_{k=0}^{M} (1 - \alpha_{k,c}) x^k / k!$. Also, the numerator is always positive as we admit the common sense assumption that $\alpha_{k+1,c} \geq \alpha_{k,c}$, meaning that the probability of observing a collision with $k+1$ packets is not less than the probability of observing a collision with k packets.

B. Modelling Successful Packet Transmission

For arrival, we also simplify the Poisson process with rate λ_t. Then, inserting (25), (28) and (31) into (18), we observe that, in general the resulting estimated probability vector for time-slot $t+1$ is no longer Poisson, i.e., $P_t^{''*}(z) e^{\hat{\lambda}_t (z-1)} \neq e^{\hat{\nu}_{t+1}(z-1)}$. Nevertheless we can approach the resulting distribution of $P_t^{''*}(z) e^{\hat{\lambda}_t (z-1)}$ by one of Poisson for $p_{n,t+1}$ with mean value $\hat{\nu}_{t+1}$ equal to the mean value of the computed vector probability $P_t^{''*}(z) e^{\hat{\lambda}_t (z-1)}$. In other words, we obtain, by using $x = \hat{\nu}_t b_{M,t}$, that

For a hole,

$$\hat{\nu}_{t+1} = \hat{\lambda}_t + \nu_t - x + \frac{\sum_{k=0}^{M} k \frac{x^k}{k!} \alpha_{k,0}}{\sum_{k=0}^{M} \frac{x^k}{k!} \alpha_{k,0}}; \quad (33)$$

For a success-m,

$$\hat{\nu}_{t+1} = \hat{\lambda}_t + \hat{\nu}_t - x + \frac{\sum_{k=m}^{M} (k-m) \frac{x^k}{k!} \alpha_{k,m}}{\sum_{k=m}^{M} \frac{x^k}{k!} \alpha_{k,m}}; \quad (34)$$

For a collision,

$$\hat{\nu}_{t+1} = \hat{\lambda}_t + \hat{\nu}_t + x \frac{\sum_{k=0}^{M} (\alpha_{k+1,c} - \alpha_{k,c}) \frac{x^k}{k!}}{e^x - \sum_{k=0}^{M} (1-\alpha_{k,c}) \frac{x^k}{k!}}; \quad (35)$$

Then, the deriving cycle is completed.

C. The Pseudo Bayesian Procedure

Here we summarize how the procedure works. At the end of time-slot $t-1$, the gNB estimates the number of devices (new arrivals + backlogged), $\hat{\nu}_t$, that will be active in the next time-slot t. Based on (33), (34) and (35), the gNB needs to,

- inform about the permission probability, $b_{M,t} = \min(K_M / \hat{\nu}_t, 1)$, for time-slot t used by all active devices.
- if the gNB observes a success-m ($m = 0$ is a hole, while $0 < m \leq M$ indicates a success with multiplicity m) decrement the actual estimation $\hat{\nu}_t$ as,

$$\hat{\omega}_t = \hat{\nu}_t - \left(K_M - \frac{\sum_{k=m}^{M} (k-m) \frac{K_M^k}{k!} \alpha_{k,m}}{\sum_{k=m}^{M} \frac{(\hat{\nu}_t b_{M,t})^k}{k!} \alpha_{k,m}} \right); \quad (36)$$

- if the gNB observes a collision increment the actual estimation $\hat{\nu}_t$ as,

$$\hat{\omega}_t = \hat{\nu}_t + K_M \frac{\sum_{k=0}^{M} (\alpha_{k+1,c} - \alpha_{k,c}) \frac{K_M^k}{k!}}{e^{K_M} - \sum_{k=0}^{M} (1-\alpha_{k,c}) \frac{K_M^k}{k!}}; \quad (37)$$

the gNB configures,

TABLE I. M-MPR: OPTIMAL THROUGHPUT $T_{M,\alpha}(\hat{x}_M)$ WITH $\hat{x}_M = K_M$ FOR A CHANNEL WITH MAXIMUM CAPACITY; $\overline{\alpha}_m = m$, $m = 1, 2, \dots M$; I.E. MATRIX $\mathbf{A} = \mathbf{I}$, SEE (1).

$M \rightarrow$	1	2	3	4
$T_{M,\alpha}(\hat{x}_M)$	0.36879	0.83996	1.37110	1.94238
$\hat{x}_M = K_M$	1.00000	1.61803	2.26953	2.94518

$$\hat{\nu}_{t+1} = \hat{\omega}_t + \hat{\lambda}_t \tag{38}$$

where the estimation value $\hat{\lambda}_t$ can be set equal to the number of successful packet transmitted in time-slot t.

VI. SOME PARTICULAR CASES

As illustrative examples, we discuss in this section the obtained results for two cases in M-MPR. First, the *all-or-nothing* model and second the non-perfect capture model.

A. The All-or-Nothing Model

In this case, the channel is characterized by the identity matrix of suitable dimensions, i.e., $\mathbf{A} = \mathbf{I}$, which means that $\alpha_{m,m} = 1$, i.e., $\overline{\alpha}_m = m$ for all $0 \leq m \leq M$ and $\alpha_{m,c} = 1$ for all $m > M$. These are the transmission-reception characteristics used in [16]. Then, the throughput, $T_{M,\alpha}(\nu_t, b_{M,t})$ is given by,

$$T_{M,\alpha}(\nu_t, b_{M,t}) = \sum_{m=1}^{M} \frac{(\nu_t b_{M,t})^m}{(m-1)!} e^{-\nu_t b_{M,t}} = \begin{cases} \sum_{m=1}^{M} \frac{\nu_t^m}{(m-1)!} e^{-\nu_t}; & \nu_t \leq K_M \rightarrow b_{M,t} = 1; \\ \sum_{m=1}^{M} \frac{K_M^m}{(m-1)!} e^{-K_M}; & \nu_t > K_M \rightarrow b_{M,t} < 1. \end{cases} \tag{39}$$

For $M = 1$, the SPR case, $K_1 = 1$ regardless the value of $\overline{\alpha}_1$. The maximum achievable throughput is $e^{-1}\overline{\alpha}_1 \approx 0.3679\overline{\alpha}_1$, then equal to e^{-1} (S-ALOHA) when $\overline{\alpha}_1 = 1$.

For $M = 2$, $K_2 = (1 + \sqrt{5})/2 \approx 1.618034$, see (7), and the maximum throughput is ≈ 0.839962 (coincident with [12]).

For $M = 3$, $K_3 = (S_1 + S_2 + 1)/3 \approx 2.26953084$ where $S_{1,2} = \sqrt[3]{37 \pm 3\sqrt{114}}$, and the maximum achievable throughput is ≈ 1.37110.

For $M > 3$ we do not find a closed form expression, so we resort to numerical calculation as has described above.

Table I shows the maximum throughput $T_{M,\alpha}(x)$ for the *all-or-nothing* model in M-MPR for several values of M, in coincidence with the values obtained in [12].

About the Pseudo Bayesian procedure in this case we notice that in case of hole or success-m, (36) becomes (the same action for all those events),

$$\hat{\omega}_t = \hat{\nu}_t - \hat{\nu}_t b_{M,t} = \hat{\nu}_t - \min(K_M, \hat{\nu}_t) = \max(\hat{\nu}_t - K_M, 0); \tag{40}$$

and in case of collision, we have from (37)

$$\hat{\omega}_t = \hat{\nu}_t + \frac{\frac{K_M^{M+1}}{M!}}{e^{K_M} - \sum_{k=0}^{M} \frac{K_M^k}{k!}}; \tag{41}$$

The final step is achieved when (41) is inserted into (38).

TABLE II. M-MPR: $\Delta\hat{\nu}_t$ FOR A "ALL-OR-NOTHING" CHANNEL; $\overline{\alpha}_m = m$, $m = 1, 2, \dots M$ (MATRIX $\mathbf{A} = \mathbf{I}$, SEE (1)).

$M \rightarrow$	1	2	3	4
hole, $m = 0$	-1.00000	-1.61803	-2.26953	-2.94518
success, $m = 1$	-1.00000	-1.61803	-2.26953	-2.94518
", $m = 2$	-	-1.61803	-2.26953	-2.94518
", $m = 3$	-	-	-2.26953	-2.94518
", $m = 4$	-	-	-	-2.94518
", $m = 5$	-	-	-	-
", $m = 6$	-	-	-	-
collision, $m > M$	1.39221	1.89876	2.34994	2.76516

The fraction in (41) is the bias or error of the *a priori* estimate of $\hat{\nu}_t$ evaluated at the beginning of time-slot t. At the end of this time-slot t, after the observation of the event *collision* has been taken into account, $\hat{\omega}_t$ reflects the *a posterior* estimate of the number of packets involved in that collision. In other words, $\hat{\omega}_t$ is the corrected estimate of $\hat{\nu}_t$. Notice that for $\hat{\nu}_t \rightarrow 0$ the bias approaches to $M + 1$, i.e., $\hat{\omega}_t \rightarrow M + 1$ as expected. That is, since the system is an M-MPR with the *all-or-nothing* capability, $M+1$ is the minimum number of packets involved in one collision, very close to this value for very low traffic. Although, surprisingly, the bias in (41) decreases when $\hat{\nu}_t$ increases from zero up to $\hat{\nu}_t = K_M$ (in this interval the probability $b_{M,t}$ keeps constant equal to one) the net effect is that the *a posterior* estimate $\hat{\omega}_t$ increases when $\hat{\nu}_t$ increases, as common sense dictates. Note that it is straight-forward to check that the first derivative of the bias is negative for any value of $x = \nu_t b_{M,t}$. However, it is also surprising that the bias remains constant, for values of $\hat{\nu}_t > K_M$ (in this case $b_{M,t} < 1$). That is, when $\hat{\nu}_t > K_M$ ($b_{M,t} < 1$) the bias keeps constant, equal to $1.39221, 1.89876, 2.34994, \dots$ respectively for $M = 1, 2, 3, \dots$. Those values are reflected in the row *collision* of Table II and are the positive bias we use for the Bayesian estimation of the number of packets involved in one collision.

Moreover, it is worth mentioning that the maximum achievable throughput per slot, $T_{M,\alpha}(\hat{x}_M)$, increases with M, as expected, i.e., starting with ≈ 0.3679 for SPR, i.e., $M = 1$, then to ≈ 0.839962 for $M = 2$, then to ≈ 1.37110 for $M = 3$, and so on. In fact, it is a linear increasing form.

Then, we conclude that it is trivial to compute the updated broadcast or permission probability $\hat{b}_{M,t}$ as has been summarized in Sec. V-C. We remark that the gNB acts according to a binary feedback, i.e., *non-collision* versus *collision*, as observed in Table II.

B. The Non-Perfect Capture Effect Model

With this model, the gNB has the chance to correctly decode one packet despite the presence of other packets in the same time slot. In general, the probability that one packet is decoded successfully depends on the number of packets involved in the collision [8]. Here we study the simple case of *non-perfect capture*, i.e., according to a noiseless channel based on [3],

$$\alpha_{0,0}=1; \quad \alpha_{m,1}=\begin{cases} 1; & m=1, \\ q^m; & m=2,\dots,M \\ 0; & m>M. \end{cases}; \qquad (42)$$
$$\alpha_{m,c}=\begin{cases} 0; & m=0,1. \\ 1-q^m; & m=2,\dots,M \\ 1; & m>M. \end{cases};$$

so, $\overline{\alpha}_0 = 0$, $\overline{\alpha}_1 = 1$ and $\overline{\alpha}_m = q^m$ for $m = 2,\dots,M$. Equivalently, in matrix form,

$$\mathbf{A}=\left[\begin{array}{cccc|c} 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & q^M & \dots & 0 & 1-q^M \\ \hline \alpha_{>M,0}=0 & \alpha_{>M,1}=0 & \dots & \alpha_{>M,M}=0 & 1 \end{array}\right] \qquad (43)$$

Clearly, we have the perfect capture case when $q = 1$. On the other hand, when $q \to 0$, the model degenerates to the SPR model in which $M = 1$, i.e., no capture effect. In general, a greater capture capability is obtained with large values of q (to deal with how the value of q could be estimated is out of the scope of this paper). Then, the events observed by the gNB are: *hole*, *success*-1, and *collision*, and the corresponding actions associated to (36) and (37) become as,

If a hole is observed

$$\hat{\omega}_t = \max(\hat{\nu}_t - K_M, 0); \qquad (44)$$

If a success-1 is observed ($m = 1$)

$$\hat{\omega}_t = \hat{\nu}_t - \left(\hat{\nu}_t b_{M,t} - \frac{\sum_{k=2}^{M}(k-1)\frac{(q\hat{\nu}_t b_{M,t})^k}{k!}}{\hat{\nu}_t b_{M,t} + \sum k = 2M\frac{(q\hat{\nu}_t b_{M,t})^k}{k!}} \right); \qquad (45)$$

If a collision is observed,

$$\hat{\omega}_t = \hat{\nu}_t + \hat{\nu}_t b_{M,t}.$$
$$\cdot \frac{(1-q)\hat{\nu}_t b_{M,t} + \sum_{k=2}^{M} q^k (1-q)\frac{(\hat{\nu}_t b_{M,t})^k}{k!} + q^M \frac{(\hat{\nu}_t b_{M,t})^M}{M!}}{e^{\nu_t b_{M,t}} - 1 - \hat{\nu}_t b_{M,t} - \sum_{k=2}^{M}\frac{(q\hat{\nu}_t b_{M,t})^k}{k!}}. \qquad (46)$$

From previous expressions, we have ternary feedback in the non-perfect capture effect. The optimal throughput has been evaluated for several values of the parameter q, see (20);

$$T_{M,\alpha}(\hat{x}_M) = \sum_{m=1}^{M} \frac{\hat{x}_M^k}{k!}\overline{\alpha}_k e^{-\hat{x}_M}. \qquad (47)$$

The results are reported in Table III.

VII. CONCLUSIONS

In this paper, we generalize the pseudo-Bayesian broadcast control algorithm when the communication system works in the environment of M-MPR in a time slot-based scheme. Up to M packets that are simultaneously are transmitted in the same time slot can be received and perfectly decoded. To that purpose, the use of capture effect, SIC, and MIMO techniques are essential to increase throughput.

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TABLE III. M-MPR: OPTIMAL THROUGHPUT $T_{M,\alpha}(\hat{x}_M = K_M)$ FOR A CHANNEL WITH NON-PERFECT CAPTURE EFFECT ACCORDING TO (43).

$q \downarrow$	$M \rightarrow$	1	2	3	4
0.0	$T_{M,\alpha}(\hat{x}_M)$	0.36787	-	-	-
	$\hat{x}_M = K_M$	1.00000	-	-	-
0.1	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.36972	0.36978	0.36978
	$\hat{x}_M = K_M$	1.00000	1.00500	1.00533	1.00534
0.3	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.38480	0.38662	0.38676
	$\hat{x}_M = K_M$	1.00000	1.04490	1.05460	1.05578
0.5	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.41659	0.42659	0.42814
	$\hat{x}_M = K_M$	1.00000	1.12310	1.17364	1.18606
0.7	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.46786	0.50236	0.51222
	$\hat{x}_M = K_M$	1.00000	1.23183	1.38575	1.45989
0.9	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.54132	0.63352	0.68095
	$\hat{x}_M = K_M$	1.00000	1.35419	1.67287	1.94236
1.0	$T_{M,\alpha}(\hat{x}_M)$	0.36787	0.58693	0.72603	0.81671
	$\hat{x}_M = K_M$	1.00000	1.41421	1.81712	2.21336

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