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Analytical and Simulation Modeling of Limited-availability Systems with Multi-service Sources and Bandwidth Reservation

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***Abstract*—The aim of this article is to present a new analytical calculation method for the occupancy distribution and the blocking probability in the so-called limited-availability group with multi-service sources and reservation mechanisms. The limited-availability group consists of links with different capacities. The article considers multi-service limited-availability systems with multi-service sources, in which each single traffic source can generate calls of different traffic classes. The results of analytical modeling of the limited-availability systems with multi-service sources and reservation mechanisms are compared with simulation data, which confirm a high accuracy of the method. Any possible application of the proposed model can be considered in the context of wireless networks with multi-service sources and reservation mechanisms. The proposed model can be also considered in the context of switching networks.**

Keywords-limited-availability systems; multi-service networks; multi-service sources; bandwidth reservation.

I. Introduction

Cellular networks are one of the most rapidly growing areas of telecommunications and one of the most popular systems of mobile communication. They can be used for voice transmission, but are also very efficient for sending data streams from different applications [1] [2] [3] [4]. Data transmission is a type of service that originally was not handled by cellular networks. Over time, data transmission has become more and more popular in expanding mobile networks. Data transmission services offered by operators include video conferencing services, streaming audio services, electronic mail and large file transmission [5] [6] [7] [8] [9].

The increase in intensity of traffic generated by data transmission services is accompanied by an increase in the requirements with respect to the volume of resources offered by networks. This also causes a growing necessity of working out mechanisms that introduce differentiation in Quality of Service (QoS) for particular data classes. The introduction of QoS differentiation mechanisms was conducive in turn to a development of new analytical models for dimensioning of multi-service mobile networks. The initial models of multi-service cellular networks assumed that a single traffic source of a given class could generate only one, strictly defined, type of the traffic stream (traffic sources class unequivocally determined the nature of a traffic stream). In the analytical models of such multi-service systems with single-service sources the class of traffic stream generated is defined by the class of its traffic source. Using the models of the multi-service systems with single-service source both cell groups without QoS mechanisms introduced [10] and cell groups with QoS differentiation mechanisms were considered, including resource reservation mechanisms [11] [12] [13].

With multi-service terminals becoming more and more universal in modern cellular networks, it has become necessary to develop new traffic models. In [14], a model of the multi-service network was presented for the first time, which assumed that a given and defined set of services was related to a single traffic source. The considered system was described as a multi-service system with multi-service sources. The considerations presented in [14] were limited, however, to a model of a full-availability group (single resource) without any QoS differentiation mechanisms introduced. In [1], the model of limited-availability group (a group of separated resources, e.g., a group of cells) with multi-service sources and bandwidth reservation has been presented.

This article proposes a generalized model of a limited-availability group of resources, in which — to facilitate resource usage — a reservation mechanism has been implemented. The model can be used for modeling groups of cells with different capacities in multi-service networks, thus expanding the results presented in [1] in which resources with the same capacities are discussed. In the proposed model, it is taken into consideration that a single terminal can generate various traffic streams corresponding to particular services implemented in the terminal. Additionally, it is assumed that the services cannot be used simultaneously by the terminal. This means that when the terminal is involved in the generation of traffic stream related to one service, it cannot at the same time generate traffic streams related to other services (A case when a single terminal can simultaneously generate a few traffic streams can be also taken into account in the analytical modeling – such a terminal can be treated as a few traffic sources).

The remaining part of the article is organized as follows. In Section II, the generalized model of the limited-availability systems with multi-service sources and resource reservation mechanism is proposed. In Section III, the simulator of multi-service systems with limited-availability is presented. In this section, the results of the blocking probability obtained for limited-availability systems with multi-service sources and reservation mechanisms are compared with the simulation data. Section IV concludes the article and presents further work.

II. Analytical Model of System with Multi-service Sources and Reservation Mechanisms

Let us consider a generalized model of the limited-availability group (GMLAG) with multi-service sources and the capacity V_L, presented in Figure 1. The generalized limited-availability group model is a model of a system that is composed of links with different capacities. The group is composed of z types of links. Each type h is unequivocally defined by the number υ_h of links of a given type and by the capacity f_h of each of the links of a given type (Figure 1). The total capacity V_L of the limited-availability group with different capacities of links is then $\sum_{h=1}^{z} \upsilon_h f_h$. The group can admit a call of a given class for service only when it can be entirely carried by the resources of one of the links.

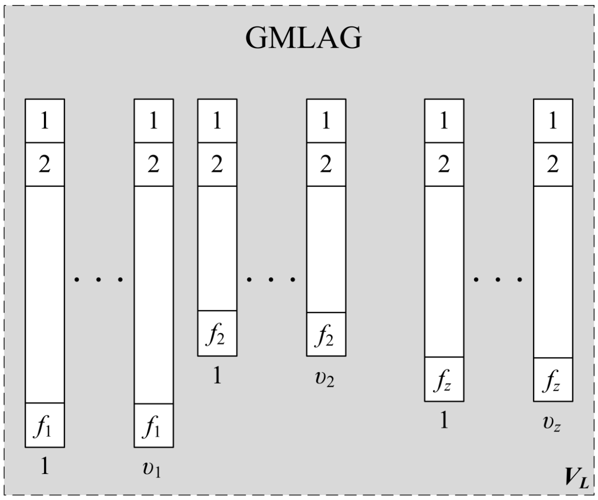

Figure 1. Generalized model of the limited-availability group

In the considered model, m traffic classes that belong to the set $\mathbb{M} = \{1, 2, ..., m\}$ are defined. A given class c requires t_c Basic Bandwidth Units (BBUs) to set up a new connection. The service time for class c calls has an exponential distribution with the parameter μ_c (service rate). In the group, the reservation mechanism has been applied. In accordance with the adopted reservation mechanism for a given class c, the reservation limit Q_c is introduced (Q_c is a certain occupancy state of the system, expressed in the number of BBUs being busy). The reservation mechanism can be applied to selected traffic classes from the set $\mathbb{M}$. The classes, in which the reservation limit has been introduced, are grouped into a new set of classes $\mathbb{R}$, which is a sub-set of the set $\mathbb{M}$. The parameter R_c that determines the reservation area (a certain number of occupancy states of the system) has also been defined. This parameter can be expressed by the following formula:

$$R_c = V_L - Q_c. \tag{1}$$

The system admits a call of class c that belongs to the set $\mathbb{R}$ for service only when this call can be entirely carried by the resources of an arbitrary single link and when the number of free BBUs in the group is higher or equal to the value of the reservation area R_c. A call of class c that does not belong to the set $\mathbb{R}$ can be serviced when this call can be entirely carried by the resources of an arbitrary single link. Thus, this is an example of a system with a state-dependent service process, in which the state dependence is the result of the structure of the group and the introduced reservation mechanism. An example of the limited-availability group with reservation mechanism applied for only class 1 is presented in Figure 2.

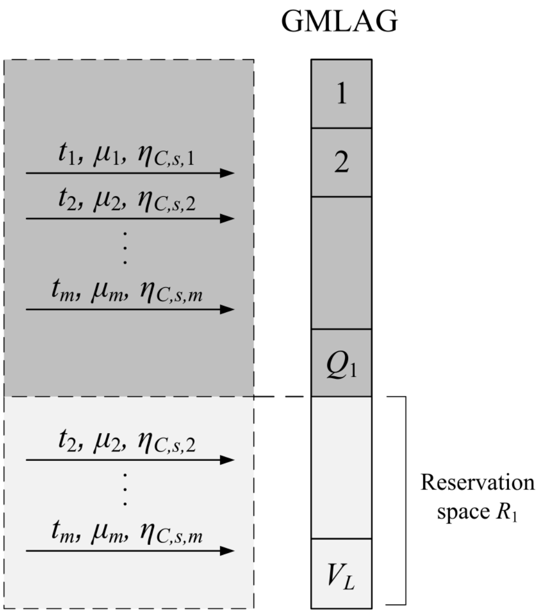

Figure 2. Generalized model of the limited-availability group with reservation mechanisms

The group is offered three types of Erlang (Poisson call streams), Engset (binomial call streams) and Pascal (negative binomial call streams) traffic streams [15]. The selected types of traffic cover three different types of the dependence between the mean arrival rates of calls and the occupancy state of the system: (1) the mean arrival rate of new calls does not depend on the occupancy state of the system (Erlang traffic), (2) the mean arrival rate of new calls decreases with the increase in the occupancy state of the system (Engset traffic), (3) the mean arrival rate of new calls increases with the increase in the occupancy state of the system (Pascal traffic).

Each traffic stream is generated by sources that belong to the corresponding set of traffic sources $\mathbb{Z}_{C,s}$. In set $\mathbb{Z}_{C,s}$, index C denotes the type of traffic stream generated by sources, which belong to this set and takes the value I for Erlang traffic stream, J for Engset traffic stream and K for Pascal traffic stream, respectively, while index s denotes the number of the set, the sources of which generate a given type of traffic stream. In the system, the s_I sets of traffic sources that generate Erlang traffic streams are defined, as well as s_J sets of traffic sources that generate Engset traffic streams and s_K sets of traffic sources that generate Pascal traffic streams. The total number of the sets of traffic sources is $S = s_I + s_J + s_K$. The sources that belong to the set $\mathbb{Z}_{C,s}$ can generate calls from the set $\mathbb{C}_{C,s} = \{1, 2, ..., c_{C,s}\}$ of traffic classes according to the available set of services.

The participation of class c (from the set $\mathbb{M}$) in the traffic structure of traffic generated by sources from the set $\mathbb{Z}_{C,s}$ is determined by the parameter $\eta_{C,s,c}$, which, for particular sets of Erlang, Engset and Pascal traffic sources, satisfies the following dependencies:

$$\sum_{c=1}^{c_{I,i}} \eta_{I,i,c} = 1, \qquad \sum_{c=1}^{c_{J,j}} \eta_{J,j,c} = 1, \qquad \sum_{c=1}^{c_{K,k}} \eta_{K,k,c} = 1. \tag{2}$$

To determine the value of traffic $A_{I,i,c}$ offered by Erlang sources that belong to the set $\mathbb{Z}_{I,i}$ as well as the traffic value $A_{J,j,c}(n)$ offered by Engset sources from the set $\mathbb{Z}_{J,j}$ and traffic $A_{K,k,c}(n)$ offered by Pascal sources from the set $\mathbb{Z}_{K,k}$ that generate calls of class c in the state of n busy BBUs, we use the following formulas [14]:

$$A_{I,i,c} = \eta_{I,i,c}\lambda_{I,i}/\mu_c, \tag{3}$$

$$A_{J,j,c}(n) = [\eta_{J,j,c}N_{J,j} - y_{J,j,c}(n)]\alpha_{J,j}, \tag{4}$$

$$A_{K,k,c}(n) = [\eta_{K,k,c}S_{K,k} + y_{K,k,c}(n)]\beta_{K,k}, \tag{5}$$

where:

- $\lambda_{I,i}$ – the mean arrival rate of new calls generated by a single Poisson source that belongs to the set $\mathbb{Z}_{I,i}$
- $\eta_{J,j,c}$ – the parameter that determines the participation of calls of class c in traffic generated by sources that belong to the set $\mathbb{Z}_{J,j}$,
- $\eta_{K,k,c}$ – the parameter that determines the participation of calls of class c in traffic generated by sources that belong to the set $\mathbb{Z}_{K,k}$,
- $N_{J,j}$ – the number of Engset traffic sources that belong to the set $\mathbb{Z}_{J,j}$,
- $S_{K,k}$ – the number of Pascal traffic sources that belong to the set $\mathbb{Z}_{K,k}$,
- $y_{J,j,c}(n)$ – the average number of calls of class c generated by Engset sources that belong to the set $\mathbb{Z}_{J,j}$ currently serviced in the system in the occupancy state n,
- $y_{K,k,c}(n)$ – the average number of calls of class c generated by Pascal sources that belong to the set $\mathbb{Z}_{K,k}$ currently serviced in the system in the occupancy state n,
- $\alpha_{J,j}$ – the average traffic intensity of traffic generated by a single Engset source that belongs to the set $\mathbb{Z}_{J,j}$, determined by the following formula:

  $$\alpha_{J,j} = \sum_{c=1}^{c_{J,j}} \eta_{J,j,c}\frac{\gamma_{J,j}}{\mu_c}, \tag{6}$$

  where $\gamma_{J,j}$ – the mean arrival rate of new calls generated by a single Engset source that belongs to the set $\mathbb{Z}_{J,j}$,
- $\beta_{K,k}$ – the average traffic intensity of traffic generated by a single Pascal source that belongs to the set $\mathbb{Z}_{K,k}$, defined by the following formula:

  $$\beta_{K,k} = \sum_{c=1}^{c_{K,k}} \eta_{K,k,c}\frac{\gamma_{K,k}}{\mu_c}, \tag{7}$$

  where $\gamma_{K,k}$ – the mean arrival rate of new calls generated by a single Pascal source that belongs to the set $\mathbb{Z}_{K,k}$.

We can notice that – according to Formulas (4) and (5) – with the case of Engset sources, the mean traffic intensity generated by new calls of individual traffic classes decreases with the increase in the occupancy state of the system, whereas in the case of Pascal sources the mean traffic intensity generated by new calls of individual traffic classes increases with the increase in the occupancy state of the system.

The first articles on modeling systems with limited-availability were limited to include multi-service systems with single-service sources only [16] [17] [18]. In [16], an approximate method for a determination of the blocking probability in the generalized limited-availability group model is proposed, while [17] discusses a group model with Engset traffic streams. Limited-availability groups servicing BPP traffic are studied in [18] [19].

The first studies on modeling multi-service systems with limited-availability and multi-service sources are presented in [20], while a method for modeling systems with multi-service sources and the reservation mechanism is described in [1]. The present article, taking advantage of the results presented in [16], proposes an expansion of the method proposed in [1] to provide a possibility of a determination of the occupancy distribution in the generalized limited-availability group model with the reservation mechanism. For this purpose, it is necessary to first determine conditional transition coefficients $\sigma_{c,S}(n)$ for calls of class c. These coefficients define the influence of a particular structure of a group on the process of the determination of the blocking probability. They can be determined on the basis of one of the two following methods: the one presented further on in the text of this section (the combinatorial method), and the convolutional method proposed in [21].

In the combinatorial method, the conditional transition probability $\sigma_{c,S}(n)$ for a traffic stream of class c in the generalized limited-availability group with the parameters: z, υ_h, f_h, V_L, is determined with the assumption, similarly as in the limited-availability basic model [22], that there are n busy BBUs in the considered group and that each distribution of free BBUs is treated as a division of busy BBUs of one class ($t_1 = 1$) between the links.

Let us consider first a limited-availability group composed of links of two types ($z = 2$). The capacity V_L of this group can be represented as the sum of the capacities of the links of the first and the second type, i.e.,

$$V_L = V_1 + V_2, \quad \text{where: } V_1 = \upsilon_1 f_1, V_2 = \upsilon_2 f_2. \tag{8}$$

A determination of the number of all possible distributions x of free BBUs in the group can in this case be successfully performed in two stages:

1) we determine the number of all possible combinations of distribution (division) of x free BBUs in the links of the two types, i.e., all combinations of the division of x BBUs into x_1 BBUs in the links of the first type and $x - x_1$ BBUs in the links of the second type;
2) we determine the number of possible distributions of a given number of BBUs in the links of the same type, i.e., the number of distributions x_1 BBUs in the links of the first type (with the limit of the capacity of a link to f_1 BBU taken into account), and $x-x_1$ BBUs in the links of the second type (with the limit of the capacity of a link to f_2 BBU taken into account) (Figure 3).

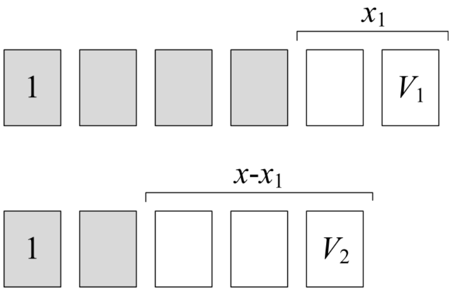

Figure 3. Possible distributions of x free BBUs in links of the two types

The number of distributions determined in the second stage is determined on the basis of the following formula which determines the number of arrangements of x free BBUs in υ links and the capacity of each link is equal to f BBUs and each link has at least t free BBUs:

$$F(x, \upsilon, f, t) = \\ = \sum_{r=0}^{\left\lfloor \frac{x-\upsilon t}{f-t+1} \right\rfloor} (-1)^r \binom{\upsilon}{r} \binom{x - \upsilon(t-1) - 1 - r(f-t+1)}{\upsilon - 1}. \tag{9}$$

Eventually, the number of possible distributions of x free BBUs in the links of the two types – with the assumption that in each link of the first type there are at least t_1 free BBUs and in each link of the second type there are at least t_2 free BBUs – can be expressed by the following formula:

$$F\{x, (\upsilon_1, \upsilon_2), (f_1, f_2), (t_1, t_2)\} = \\ = \sum_{x_1=0}^{x} F(x_1, \upsilon_1, f_1, t_1) F(x - x_1, \upsilon_2, f_2, t_2). \tag{10}$$

The number of possible distributions of x free BBUs in groups composed of links of three types can be determined in a similar way as for the group composed of links of two types:

$$F\{x, (\upsilon_1, \upsilon_2, \upsilon_3), (f_1, f_2, f_3), (t_1, t_2, t_3)\} = \\ = \sum_{x_1=0}^{x} \sum_{x_2=0}^{x-x_1} F(x_1, \upsilon_1, f_1, t_1) F(x_2, \upsilon_2, f_2, t_2) \cdot \\ \cdot F(x - x_1 - x_2, \upsilon_3, f_3, t_3) \tag{11}$$

and then – generalizing the formula (11) – for groups composed of links of l types:

$$F\{(x, (\upsilon_1, \upsilon_2, \ldots, \upsilon_l), (f_1, f_2, \ldots, f_l), (t_1, t_2, \ldots, t_l)\} = \\ = \sum_{x_1=0}^{x} \sum_{x_2=0}^{x-x_1} \ldots \sum\nolimits_{x_{l-1}=0}^{x-\sum_{r=1}^{l-2} x_r} F(x_1, \upsilon_1, f_1, t_1) \cdot \\ \cdot F(x_2, \upsilon_2, f_2, t_2) \ldots F(x_{l-1}, \upsilon_{l-1}, f_{l-1}, t_{l-1}) \cdot \\ \cdot F\left(x - \sum\nolimits_{r=1}^{l-1} x_r, \upsilon_l, f_l, t_l\right). \tag{12}$$

Formula (12) can be eventually rewritten into the following form:

$$F\{(x, (\upsilon_1, \upsilon_2, \ldots, \upsilon_l), (f_1, f_2, \ldots, f_l), (t_1, t_2, \ldots, t_l)\} = \\ = \sum_{x_1=0}^{x} \ldots \sum_{x_{l-1}=0}^{x-\sum\limits_{r=1}^{l-2} x_r} \left\{ \prod_{z=1}^{l-1} F(x_z, \upsilon_z, f_z, t_z) \cdot \right. \\ \left. \cdot F\left(x - \sum_{r=1}^{l-1} x_r, \upsilon_l, f_l, t_l\right) \right\}. \tag{13}$$

Formula (13) makes it possible to determine the transition coefficient for transitions between neighbouring states of the process occurring in the limited-availability group with z types of links in the state of n busy (i.e., $x = V - n$ free)

BBUs:

$$\sigma_{c,S}(n) = 1 - \frac{F\{V-n,(\upsilon_1,\ldots,\upsilon_l),(t_c-1,\ldots,t_c-1),(0,\ldots,0)\}}{F\{V-n,(\upsilon_1,\ldots,\upsilon_l),(f_1,\ldots,f_l),(0,\ldots,0)\}}. \tag{14}$$

The presented combinatorial method for a determination of conditional transition coefficients $\sigma_{c,S}(n)$ that determine a possibility of admittance of a call of class c for service in the occupancy state of n BBUs has been derived with the assumption that each distribution of free BBUs is treated as a result of the occupancy of resources by calls that demand 1 BBU [16]. To determine the parameter $\sigma_{c,S}(n)$, this method uses information on the capacity of individual links, but does not take into consideration, differences in the size of resources demanded by particular classes of calls.

Authors in [21] proposes a convolutional method for a determination of the conditional transition coefficient in the limited-availability group on the basis of independent unavailable distributions of BBUs in individual links. This method takes into consideration the size of required resources demanded by calls of individual traffic classes, but does not take into consideration link capacities. Comparative results provided by the studies for both of the methods, i.e., the combinatorial and the convolutional method, indicate their similar accuracy [21].

Observe that in the case of the considered model of the limited-availability group with multi-service traffic sources and reservation, the operation of the reservation mechanism introduces an additional dependence between the service stream in the system and the current state of the system. To determine this dependence, the parameter $\sigma_{c,R}(n)$ is introduced. The parameter $\sigma_{c,R}(n)$ can be calculated using the following formula:

$$\sigma_{c,R}(n) = \begin{cases} 1 & \text{for } n \leqslant Q_c \wedge c \in \mathbb{R}, \\ 0 & \text{for } n > Q_c \wedge c \in \mathbb{R}, \\ 1 & \text{for } c \notin \mathbb{R}. \end{cases} \tag{15}$$

The reservation mechanism is introduced to the group regardless of its structure, which allows us to carry on with product-form determination of the total coefficient of passing (transition coefficient) $\sigma_{c,\mathrm{Tot}}(n)$ in the generalized model of limited-availability group:

$$\sigma_{c,\mathrm{Tot}}(n) = \sigma_{c,S}(n) \cdot \sigma_{c,R}(n). \tag{16}$$

Having the values of offered traffic $A_{I,i,c}$, $A_{J,j,c}(n)$, $A_{K,k,c}(n)$ and the total coefficient of passing $\sigma_{c,\mathrm{Tot}}(n)$ at our disposal, we are in position to modify the original Kaufman-Roberts formula [23] [24] in order to determine the occupancy distribution in the limited-availability group with multi-service traffic sources and the reservation mechanism:

$$\begin{aligned} n[P_n]_{V_L} &= \sum_{i=1}^{s_I}\sum_{c=1}^{c_{I,i}} A_{I,i,c}\sigma_{c,\mathrm{Tot}}(n-t_c)t_c[P_{n-t_c}]_{V_L} + \\ &+ \sum_{j=1}^{s_J}\sum_{c=1}^{c_{J,j}} A_{J,j,c}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)t_c[P_{n-t_c}]_{V_L} + \\ &+ \sum_{k=1}^{s_K}\sum_{c=1}^{c_{K,k}} A_{K,k,c}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)t_c[P_{n-t_c}]_{V_L}, \end{aligned} \tag{17}$$

where $[P_n]_{V_L}$ is the occupancy distribution (the probability of n busy BBUs) in a system with the capacity V_L, and the parameter $\sigma_{c,\mathrm{Tot}}(n)$ determines the additional dependence between the service stream and the current state of the system resulting from the specific structure of the group and the applied reservation mechanism [25].

Having the values of individual state probabilities $[P_n]_{V_L}$, determined on the basis of Formula (17), we are in position to determine the average number of serviced calls of class c, generated by sources that belong to the sets $\mathbb{Z}_{J,j}$ (Engset sources) and $\mathbb{Z}_{K,k}$ (Pascal sources). For this purpose, we use the following formulas:

$$y_{J,j,c}(n) = \begin{cases} A_{J,j,c}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)[P_{n-t_c}]_{V_L}/[P_n]_{V_L} & \text{for } n \leqslant V_L, \\ 0, & \text{for } n > V_L. \end{cases} \tag{18}$$

$$y_{K,k,c}(n) = \begin{cases} A_{K,k,c}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)[P_{n-t_c}]_{V_L}/[P_n]_{V_L} & \text{for } n \leqslant V_L, \\ 0, & \text{for } n > V_L. \end{cases} \tag{19}$$

The knowledge of the occupancy $[P_n]_{V_L}$ is required to determine the parameters $y_{J,j,c}(n)$ and $y_{K,k,c}(n)$. Whereas, to determine the occupancy $[P_n]_{V_L}$ it is necessary to know the values of the parameters $y_{J,j,c}(n)$ and $y_{K,k,c}(n)$. Equations (18), (19), and (17) form thus a set of confounding equations. To solve a given set of confounding equations it is necessary to employ iterative methods [26] [27].

Assuming that the distribution $[P_n^{(l)}]_{V_L}$ is the occupancy distribution, determined in the l-th iteration, while $y_{J,j,c}^{(l)}(n)$ and $y_{K,k,c}^{(l)}(n)$ define the average number of serviced calls of class c generated by traffic sources that belong respectively to the sets $\mathbb{Z}_{J,j}$ and $\mathbb{Z}_{K,k}$, we can write:

$$y_{J,j,c}^{(l+1)}(n) = \begin{cases} A_{J,j,c}^{(l)}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)[P_{n-t_c}^{(l)}]_{V_L}/[P_n^{(l)}]_{V_L} & \text{for } n \leqslant V_L, \\ 0, & \text{for } n > V_L. \end{cases} \tag{20}$$

$$y_{K,k,c}^{(l+1)}(n) = \begin{cases} A_{K,k,c}^{(l)}(n-t_c)\sigma_{c,\mathrm{Tot}}(n-t_c)[P_{n-t_c}^{(l)}]_{V_L}/[P_n^{(l)}]_{V_L} & \text{for } n \leqslant V_L, \\ 0, & \text{for } n > V_L. \end{cases} \tag{21}$$

The iteration process, involving Formulas (17), (20), and (21), terminates when the assumed accuracy ϵ of the iteration process is reached:

$$\forall_{0\leq n\leq V}\ \left|\frac{y_{J,j,c}^{l-1}(n)-y_{J,j,c}^{(l)}(n)}{y_{J,j,c}^{(l)}(n)}\right|\leqslant\epsilon, \tag{22}$$

$$\forall_{0\leq n\leq V}\ \left|\frac{y_{K,k,c}^{l-1}(n)-y_{K,k,c}^{(l)}(n)}{y_{K,k,c}^{(l)}(n)}\right|\leqslant\epsilon. \tag{23}$$

In case of the proposed model the assumed accuracy ϵ of the iteration process is always reached.

Having determined all probabilities $\sigma_{c,\mathrm{Tot}}(n)$ as well as the occupancy distribution $[P_n]_{V_L}$ in a limited-availability group composed of different types of links, we are in position to proceed with a determination of the blocking probability for individual traffic classes from the set $\mathbb{M}=\{1,2,...,m\}$. The blocking state in the generalized limited-availability group model occurs when none of the links have a sufficient number of free BBUs to service a call of class c. This means that each occupancy state n in a link of the type s, such as: $(f_s-t_c+1\leq n\leq f_s)$, is a blocking state. Possible blocking states for a traffic stream of class c in a limited-availability group composed of links of l types can be thus determined by the following formula:

$$V-\sum\nolimits_{s=1}^{l}\upsilon_s(t_c-1)\leq n\leq V. \tag{24}$$

In addition, in the case of a limited-availability group with the reservation mechanism, the blocking state also occurs – for calls of class c – in all occupancy states of the group that are higher than the reservation limit Q_c, i.e., for $n>Q_c$. On the basis of the determined values of the conditional transition coefficients $\sigma_{c,\mathrm{Tot}}(n)$ and the occupancy distribution $[Q_n]_V$, the blocking probability for calls of class c, that belong to the set $\mathbb{M}=\{1,2,...,m\}$, can be expressed by the formula:

$$E_c=\sum_{n=0}^{V_L}[P_n]_{V_L}\left[1-\sigma_{c,\mathrm{Tot}}(n)\right]. \tag{25}$$

III. Model Verification and Case Studies

A. Description of the simulator

In order to verify the accuracy of the proposed methods for analytical determination of the blocking probability in multi-service systems with the bandwidth reservation mechanism, the authors built a simulator of their own. The simulator was written in C++ with the application of the object-oriented programming rules. The process interaction approach was adopted as method of choice for simulations [28]. The devised simulator makes it possible, among other things, to determine the value of the blocking probability for individual traffic classes in the generalized limited-availability group model with the reservation mechanism.

1) Input data and end task condition: Capacity and the group structure provide input data to the simulator. In addition, for each simulated traffic class we give the number of demanded BBUs, service time and the parameters related to the introduced reservation mechanism (reservation limits). We also define the sets of traffic sources and their type (Erlang, Engset, Pascal). Finally, we give the average value of traffic offered to a single BBU of the system.

Thus, to carry out simulation studies of a system with the capacity V_L, composed of z types of links in which a given type h consists of υ_h links, while the capacity of a single link is f_h, we are to provide values for the following parameters that describe traffic classes:

- the number of defined traffic classes m,
- the number of demanded t_c BBUs necessary to set up a connection of class c and the average service time μ_c^{-1} of a call of class c,
- the number of sets of traffic classes S,
- the set of traffic sources $\mathbb{Z}_{C,s}$ of type C, the number of classes $c_{C,s}$ that belong to a given set, the share of $\eta_{C,s,c}$ calls of class c in traffic generated by traffic sources of a given set, and the number $N_{C,s}$ or $S_{C,s}$ of traffic sources in a given set,
- the set $\mathbb{R}$ of traffic classes to which the reservation mechanism has been introduced,
- the reservation limit Q_c defined for calls of class c from the set $\mathbb{R}$.

In addition, we also give the average traffic a offered to a single BBU of the system. The mean value of offered traffic a can be calculated using following equation:

$$\begin{aligned}a=\Bigg[&\sum_{i=1}^{s_I}\lambda_{I,i}\sum_{c=1}^{c_{I,i}}t_c\eta_c\sum_{c=1}^{c_{I,i}}\eta_c/\mu_c+\\&+\sum_{j=1}^{s_J}\gamma_{J,j}N_{J,j}\sum_{c=1}^{c_{J,j}}t_c\eta_c\sum_{c=1}^{c_{J,j}}\eta_c/\mu_c+\\&+\sum_{k=1}^{s_K}\gamma_{K,k}S_{K,k}\sum_{c=1}^{c_{K,k}}t_c\eta_c\sum_{c=1}^{c_{K,k}}\eta_c/\mu_c\Bigg]/V_L.\end{aligned} \tag{26}$$

On the basis of the above parameters the intensity $\lambda_{I,i}$, $\gamma_{J,j}$ or $\gamma_{K,k}$ of the occurrence of new calls depending on the type of a traffic stream is determined in the simulator. In the case of Engset and Pascal traffic streams, the intensities $\gamma_{J,j}$ and $\gamma_{K,k}$ designate the intensity of the appearance of new calls generated by a single free source. Therefore, the parameters $\lambda_{I,i}$, $\gamma_{J,j}$ and $\gamma_{K,k}$ can be determined on the basis of the following formulas:

$$\lambda_{I,i}=\frac{aV_L}{S\left[\sum_{c=1}^{c_{I,i}}t_c\eta_{I,i,c}\right]\left[\sum_{c=1}^{c_{I,i}}\mu_c\eta_{I,i,c}\right]}, \tag{27}$$

$$\gamma_{J,j}=\frac{aV_L}{S\left[\sum_{c=1}^{c_{J,j}}t_c\eta_{J,j,c}\right]\left[\sum_{c=1}^{c_{J,j}}\mu_c\eta_{J,j,c}\right]N_{J,j}}, \tag{28}$$

$$\gamma_{K,k} = \frac{aV_L}{S\left[\sum_{c=1}^{c_{K,k}} t_c \eta_{K,k,c}\right]\left[\sum_{c=1}^{c_{K,k}} \mu_c \eta_{K,k,c}\right] S_{K,k}}. \quad (29)$$

The next parameters that are determined on the basis of Formulas (27), (28), and (29) are used then as input parameters to call generators that are used by particular traffic sources.

The condition for the simulator to end the simulation is the counted appropriate number of generated calls of the least active class (with the lowest intensity of call generation). This number is selected in such a way as to obtain 95% confidence intervals. The average result of 5 series is then calculated. In practice, to obtain confidence intervals at the level of 95% we need 1,000,000 generated calls of the least active class.

2) Simulation algorithm: The simulation algorithm can be determined using the following steps:

1) Initial configuration of the simulation model.
2) Checking the end task condition of the simulation. If the end task condition is fulfilled, the simulation is terminated and the results are returned.
3) Updating of system time until the first event from the list appears.
4) Execution of the first event from the list.
5) Removal of the first event from the list and return to Step 2.

Two events have been defined in the simulation model: *appearance of a new call* and *termination of the call service*. According to the process interaction approach, these events are serviced by one function. This function has a different form for each type of Erlang, Engset and Pascal traffic streams.

The described approach enables us to define many different traffic classes in the system and to assign them to different types of the sets of traffic sources. All sources that generate calls of different traffic classes are to be created in the initial configuration of the simulation model.

3) Simulation of the system with sets of Erlang sources: Consider a system in which a set of Erlang traffic sources $\mathbb{Z}_{I,i}$ is defined. The set of traffic classes whose calls can be generated by sources from the set $\mathbb{Z}_{I,i}$ looks as follows: $\mathbb{C}_{I,i} = \{1, 2, ..., c_{I,i}\}$. In the initial configuration of the system, it is necessary to plan ahead the appearance of a call of class c from the set $\mathbb{C}_{I,i}$. Thus, the function that executes events that are related to the set of Erlang traffic sources will have the following form:

1) Planning of the appearance of a new call of class c according to the exponential distribution with the intensity $\lambda_{I,i}$ as the parameter. A choice of class c from the set $\mathbb{C}_{I,i}$ on the basis of the parameter $\eta_{I,i,c}$ according to the uniform distribution. Inclusion of the event on the list.
2) Checking network resources for the purpose of admitting a call for service:
   a) Checking if any of the links of the group has at least t_c free BBUs. If not, the call is lost.
   b) In the case of classes that belong to the set of classes $\mathbb{R}$, for which the reservation mechanism has been introduced, checking the occupancy state of the system in relation to the reservation limit Q_c. When the occupancy state is higher than the reservation limit Q_c, a call of class c is lost.

   If any of the conditions is not satisfied, the next steps are omitted.
3) Occupation of resources demanded by a call of class c.
4) Planning (scheduling) of a termination of service according to the exponential distribution with the intensity μ_c as the parameter. Inclusion of the event on the list.
5) Termination of service and release of resources.

4) Simulation of a system with sets of Engset traffic sources: Consider now a system in which a set of Engset traffic sources $\mathbb{Z}_{J,j}$ is defined. The set of traffic classes in which calls can be generated by $N_{J,j}$ sources from the set $\mathbb{Z}_{J,j}$ looks as follows: $\mathbb{C}_{J,j} = \{1, 2, ..., c_{J,j}\}$. In the initial configuration of the system, it is necessary to plan ahead the appearance of calls of class c from the set $\mathbb{C}_{J,j}$, generated by each of $N_{J,j}$ sources. Thus, the function that executes events related to the set of Engset traffic sources will take on the following form:

1) Checking network resources for the purpose of the admittance of a call for service:
   a) Checking if any of the links of the group has at least t_c free BBUs. If not, the call is lost.
   b) In the case of classes that belong to the set $\mathbb{R}$, for which the reservation mechanism has been introduced, checking the occupancy state of the state in relation to the reservation limit Q_c. When the occupancy state is higher than the reservation limit Q_c, a call of class c is lost.

   If any of the conditions is not satisfied, the simulation proceeds to Step 5.
2) Occupation of resources demanded by a call of class c.
3) Planning (Scheduling) of the termination of service according to the exponential distribution where the parameter is the intensity μ_c. Inclusion of the event on the list.
4) Termination of service and network release.
5) Planning of the appearance of a new call of class c according to the exponential distribution where the intensity $\gamma_{J,j}$ is the parameter. Choice of the class c from the set $\mathbb{C}_{J,j}$ on the basis of the parameter $\eta_{J,j,c}$ according to the uniform distribution. Inclusion of the event on the list.

5) Simulation of a system with Pascal traffic sources: Consider also a system in which a set of Pascal traffic sources $\mathbb{Z}_{K,k}$ is defined. The set of traffic classes in which calls can be generated by $S_{K,k}$ sources from the set $\mathbb{Z}_{K,k}$ looks as follows: $\mathbb{C}_{K,k} = \{1, 2, ..., c_{K,k}\}$. In the initial configuration of the system, it is necessary to plan ahead the appearance of calls of class c from the set $\mathbb{C}_{K,k}$, generated by each of $S_{K,k}$ sources. Thus, the function that executes events related to the set of Pascal traffic sources will take on the following form:

1) Planning (scheduling) of the appearance of a new call of class c according to the exponential distribution where the parameter is the intensity $\gamma_{K,k}$. Choice of class c from the set $\mathbb{C}_{K,k}$ on the basis of the parameter $\eta_{K,k,c}$ according to the uniform distribution. Inclusion of the event on the list.
2) Checking network resources for the purpose of the admitting a call for service:
   a) Checking if any of the links of the group has at least t_c free BBUs. If not, the call is lost.
   b) In the case of classes that belong to the set $\mathbb{R}$ classes, for which the reservation mechanism has been introduced, checking the occupancy state of the system in relation to the reservation limit Q_c. When the occupancy state is higher than the reservation limit Q_c, a call of class c is lost.

   If any of the conditions is not satisfied, the next steps are omitted.
3) Occupation of the resources demanded by a call of class c.
4) Planning (scheduling) of the termination of service according to the exponential distribution where the parameter is the intensity μ_c. Inclusion of the event on the list.
5) Addition of a new source related to the admitted call. Planning of the appearance of a new call of class c generated by the new traffic source according to the exponential distribution for which the parameter is the intensity $\gamma_{K,k}$. Choice of class c from the set $\mathbb{C}_{K,k}$ on the basis of the parameter $\eta_{K,k,c}$ according to the uniform distribution. Inclusion of the event on the list.
6) Termination of service and resource release.
   a) Removal of the source related to the call that has just been terminated in service (this source is added at the moment a call is admitted for service).
   b) Removal of the event related to the removed source.

B. Simulation studies of systems with limited-availability and bandwidth reservation

The presented method for a determination of the blocking probability in systems with multi-service traffic sources and reservation mechanisms is an approximate method. In order to confirm the adopted assumptions, the results of the analytical calculations were compared with the simulation data. The research was carried for seven systems described below:

1) Limited-availability system No. 1
   - Capacity: $z = 1$, $\upsilon_1 = 2$, $f_1 = 20$ BBUs, $V_L = 40$ BBUs,
   - Number of traffic classes: 3
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 2$ BBUs, $\mu_2^{-1} = 1$,$t_3 = 6$ BBUs, $\mu_3^{-1} = 1$, $R_1 = R_2 = 33$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1, 2\}$, $\eta_{I,1,1} = 0.6$, $\eta_{I,1,2} = 0.4$, $\mathbb{C}_{J,2} = \{2, 3\}$, $\eta_{J,2,2} = 0.7$, $\eta_{J,2,3} = 0.3$, $N_2 = 60$
2) Limited-availability system No. 2
   - Capacity: $z = 1$, $\upsilon_1 = 2$, $f_1 = 30$ BBUs, $V_L = 60$ BBUs,
   - Number of traffic classes: 3
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 3$ BBUs, $\mu_2^{-1} = 1$,$t_3 = 7$ BBUs, $\mu_3^{-1} = 1$, $R_1 = R_2 = 51$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1\}$, $\eta_{I,1,1} = 1.0$, $\mathbb{C}_{J,2} = \{1, 2\}$, $\eta_{J,2,1} = 0.6$, $\eta_{J,2,2} = 0.4$, $N_2 = 50$, $\mathbb{C}_{K,3} = \{2, 3\}$, $\eta_{K,3,2} = 0.7$, $\eta_{K,3,3} = 0.3$, $S_3 = 50$
3) Limited-availability system No. 3
   - Capacity: $z = 1$, $\upsilon_1 = 4$, $f_1 = 20$ BBUs, $V_L = 80$ BBUs,
   - Number of traffic classes: 4
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 2$ BBUs, $\mu_2^{-1} = 1$,$t_3 = 4$ BBUs, $\mu_3^{-1} = 1$, $t_4 = 9$ BBUs, $\mu_4^{-1} = 1$, $R_1 = R_2 = R_3 = 63$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1, 2\}$, $\eta_{I,1,1} = 0.6$, $\eta_{I,1,2} = 0.4$, $\mathbb{C}_{J,2} = \{2, 3\}$, $\eta_{J,2,2} = 0.7$, $\eta_{J,2,3} = 0.3$, $N_2 = 70$, $\mathbb{C}_{K,3} = \{2, 3, 4\}$, $\eta_{K,3,2} = 0.3$, $\eta_{K,3,3} = 0.2$, $\eta_{K,3,4} = 0.5$, $S_3 = 140$
4) Limited-availability system No. 4
   - Capacity: $z = 2$, $\upsilon_1 = 1$, $f_1 = 20$ BBUs, $\upsilon_2 = 1$, $f_2 = 30$ BBUs, $V_L = 50$ BBUs,
   - Number of traffic classes: 3
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 3$ BBUs, $\mu_2^{-1} = 1$,$t_3 = 5$ BBUs, $\mu_3^{-1} = 1$, $R_1 = R_2 = 35$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1, 2\}$, $\eta_{I,1,1} = 0.6$, $\eta_{I,1,2} = 0.4$, $\mathbb{C}_{J,2} = \{2, 3\}$, $\eta_{J,2,2} = 0.7$, $\eta_{J,2,3} = 0.3$, $N_2 = 60$
5) Limited-availability system No. 5
   - Capacity: $z = 2$, $\upsilon_1 = 1$, $f_1 = 30$ BBUs, $\upsilon_2 = 1$, $f_2 = 40$ BBUs, $V_L = 70$ BBUs,
   - Number of traffic classes: 3
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 4$ BBUs, $\mu_2^{-1} = 1$,$t_3 = 7$ BBUs, $\mu_3^{-1} = 1$, $R_1 = R_2 = 53$ BBUs

- Sets of sources: $\mathbb{C}_{I,1} = \{1\}$, $\eta_{I,1,1} = 1.0$, $\mathbb{C}_{J,2} = \{1, 2\}$, $\eta_{J,2,1} = 0.6$, $\eta_{J,2,2} = 0.4$, $N_2 = 50$, $\mathbb{C}_{K,3} = \{2, 3\}$, $\eta_{K,3,2} = 0.7$, $\eta_{K,3,3} = 0.3$, $S_3 = 50$

6) Limited-availability system No. 6
   - Capacity: $z = 2$, $\upsilon_1 = 2$, $f_1 = 20$ BBUs, $\upsilon_2 = 2$, $f_2 = 30$ BBUs, $V_L = 100$ BBUs,
   - Number of traffic classes: 4
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 3$ BBUs, $\mu_2^{-1} = 1$, $t_3 = 5$ BBUs, $\mu_3^{-1} = 1$, $t_4 = 7$ BBUs, $\mu_4^{-1} = 1$, $R_1 = R_2 = R_3 = 67$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1, 2\}$, $\eta_{I,1,1} = 0.6$, $\eta_{I,1,2} = 0.4$, $\mathbb{C}_{J,2} = \{2, 3\}$, $\eta_{J,2,2} = 0.7$, $\eta_{J,2,3} = 0.3$, $N_2 = 70$, $\mathbb{C}_{K,3} = \{2, 3, 4\}$, $\eta_{K,3,2} = 0.3$, $\eta_{K,3,3} = 0.2$, $\eta_{K,3,4} = 0.5$, $S_3 = 140$

7) Limited-availability system No. 7
   - Capacity: $z = 3$, $\upsilon_1 = 1$, $f_1 = 20$ BBUs, $\upsilon_2 = 1$, $f_2 = 30$ BBUs, $\upsilon_3 = 1$, $f_3 = 40$ BBUs, $V_L = 90$ BBUs,
   - Number of traffic classes: 4
   - Structure of traffic: $t_1 = 1$ BBU, $\mu_1^{-1} = 1$, $t_2 = 4$ BBUs, $\mu_2^{-1} = 1$, $t_3 = 6$ BBUs, $\mu_3^{-1} = 1$, $t_4 = 8$ BBUs, $\mu_4^{-1} = 1$, $t_5 = 10$ BBUs, $\mu_5^{-1} = 1$, $R_1 = R_2 = R_3 = 74$ BBUs
   - Sets of sources: $\mathbb{C}_{I,1} = \{1, 2\}$, $\eta_{I,1,1} = 0.6$, $\eta_{I,1,2} = 0.4$, $\mathbb{C}_{J,2} = \{2, 3\}$, $\eta_{J,2,2} = 0.7$, $\eta_{J,2,3} = 0.3$, $N_2 = 180$, $\mathbb{C}_{K,3} = \{3, 4, 5\}$, $\eta_{K,3,3} = 0.3$, $\eta_{K,3,4} = 0.2$, $\eta_{K,3,5} = 0.5$, $S_3 = 90$

The results of the research study are presented in Figures 4–10, depending on the value of traffic a offered to a single BBU. The results of the simulation are shown in the charts in the form of marks with 95% confidence intervals that have been calculated according to the t-Student distribution for the five series with 1,000,000 calls of each class. For each of the points of the simulation, the value of the confidence interval is at least one order lower than the mean value of the results of the simulation. In many cases, the value of the simulation interval is lower than the height of the sign used to indicate the value of the simulation experiment. We can notice that the results of analytical calculations agree with the simulation data for both lower and higher traffic intensities.

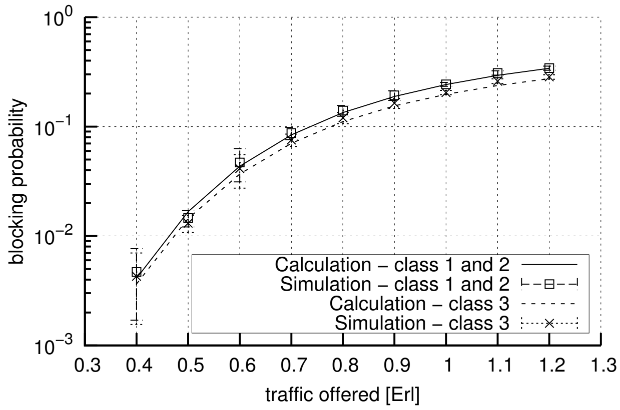

Figure 4. Blocking probability in the limited-availability group No. 1 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1 and 2

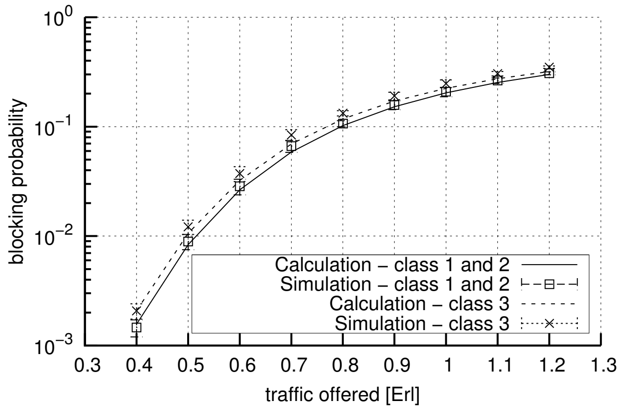

Figure 5. Blocking probability in the limited-availability system No. 2 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1 and 2

IV. Conclusion and Further Work

This article proposes a new method for a calculation of the occupancy distribution and the blocking probability in limited-availability systems with multi-service traffic sources and reservation mechanisms. The method can be used in modeling connection handoff between cells in cellular systems [10], as well as in modeling outgoing directions of switching networks [29]. The proposed method is based on the iterative algorithm for a determination of the average value of traffic sources being serviced in particular states of the system. The results of analytical calculations were compared with the simulation data, which confirmed high accuracy of the proposed method. The proposed method is not complicated and can be easily implemented.

In the further work, we plan to develop analytical models of the multi-service systems with multi-service sources, in which different call admission control mechanisms will be applied, i.e., an analytical model of multi-service networks with threshold mechanisms and multi-service sources, and a model of multi-service systems with hysteresis and multi-service sources.

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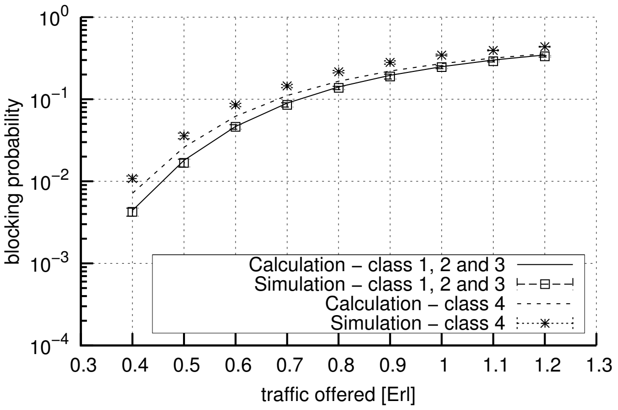

Figure 6. Blocking probability in the limited-availability system No. 3 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1, 2 and 3

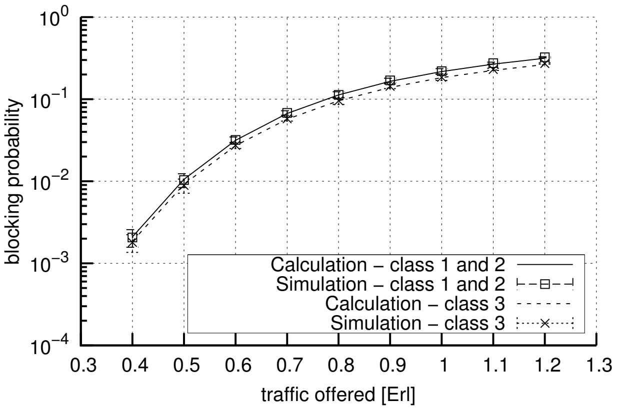

Figure 7. Blocking probability in the limited-availability system No. 4 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1 and 2

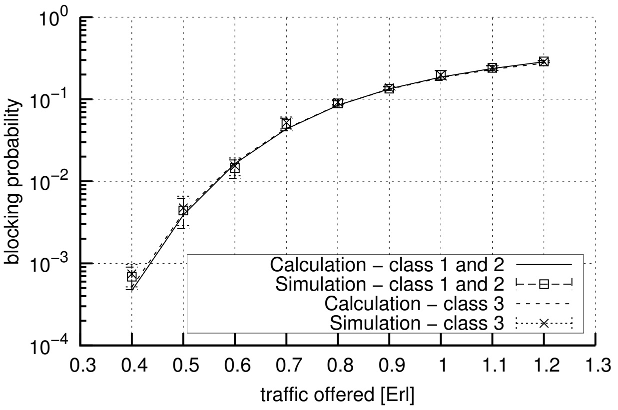

Figure 8. Blocking probability in the limited-availability system No. 5 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1 and 2

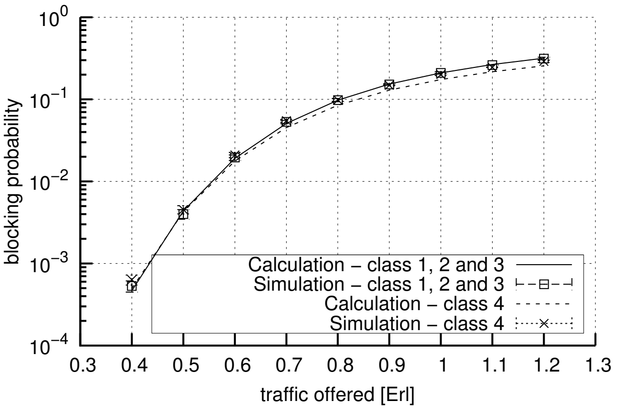

Figure 9. Blocking probability in the limited-availability system No. 6 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1, 2 and 3

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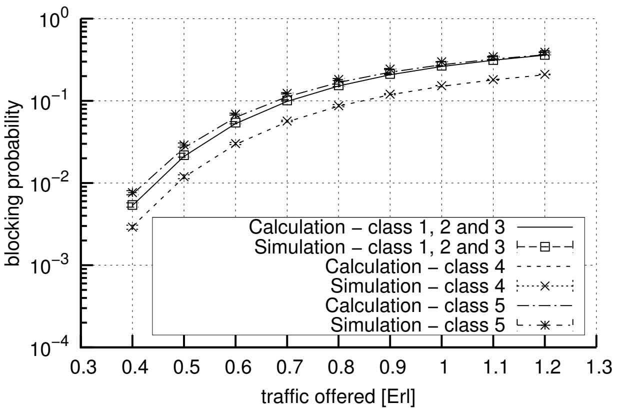

Figure 10. Blocking probability in the limited-availability system No. 7 with reservation mechanism; the reservation mechanism equalizes the blocking probability for calls of class 1, 2 and 3

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