

The Bit Error Rate for Complex SSC/MRC Combiner at Two Time Instants in the Presence of Hoyt Fading

Dragana Krstić

Department of Telecommunications,
Faculty of Electronic Engineering,
University of Niš
Aleksandra Medvedeva 14
Niš, Serbia
dragana.krstic@elfak.ni.ac.rs

Petar Nikolić

Tigartyres, Pirot, Serbia
nikpetar@gmail.com

Goran Stamenović

Tigar, Pirot, Serbia
goran.stamenovic@tigar.com

Mihajlo Stefanović

Faculty of Electronic Engineering
University of Niš
Aleksandra Medvedeva 14
Niš, Serbia
mihajlo.stefanovic@elfak.ni.ac.rs

Abstract —The expressions for probability density function (PDF) of the Switch and Stay Combiner (SSC) output signal to noise ratio (SNR) at one time instant and the joint probability density function of the SSC combiner output signal to noise ratio at two time instants in the presence of Hoyt fading are determined in this paper. Then, these expressions are used for calculation of the bit error rate for complex SSC/MRC (Switch and Stay Combining/Maximal Ratio Combining) combiner versus some parameter values. The results are shown graphically in some figures and the analysis of the parameters influence and different types of combiners is given.

Keywords - Probability Density Function; Joint Probability Density Function; Bit Error Rate, Hoyt Fading; SSC/MRC Combiner

I. INTRODUCTION

The joint probability density function of the SSC combiner output signal at two time instants in the presence of Hoyt fading is done in [1] and based on it the bit error rate for complex SSC/MRC combiner will be calculated in this paper.

The signal propagation through wireless communications channels has received a great deal of research interest [2]-[4]. The random fluctuations of the signal envelope and phase in a radio channel are caused with two propagation phenomena: multipath scattering (fast fading) and shadowing (slow fading).

The multipath fading is modeled by several distributions such as: Rayleigh, Rice, Nakagami- m and Weibull. The Hoyt (Nakagami- q) distribution model has recently received increased attention in modeling fading channels. This fading model provides a very accurate fit to experimental channel measurements in a various communication applications, such as mobile satellite propagation channels [5]. It spans the range of the fading figure from the one-sided Gaussian to the Rayleigh distribution [6]. Similarly, the Hoyt

distribution can be considered as an accurate fading model for satellite links with strong ionospheric scintillation [7]. Recently, in [8], an ergodic capacity analysis is presented, and in [9] the information outage probability of orthogonal space-time block code (OSTBC) over Hoyt fading channels has been studied. Also in [10] this model has been used in outage analysis of cellular mobile radio systems, while in [11] a capacity analysis of Hoyt fading is provided.

In wireless communication systems, various techniques for reducing fading effect and influence of shadow effects are used. Such techniques are diversity reception, dynamic channel allocation and power control. Upgrading transmission reliability and increasing channel capacity without increasing transmission power and bandwidth is the main goal of diversity techniques.

The space diversity combining techniques, based on using multiple antennas at the reception, are very efficient methods used for improving system's quality of service and ensures efficient solution for reduction of signal level fluctuations in channels with fading. Multiple received copies of signal could be combined on different ways among which the most popular are: maximal ratio combining (MRC), equal gain combining (EGC), and generalized selection combining (GSC) [2]-[4]. Their complexity of implementation is relatively high since they require a separate channel for each diversity branch.

Between the simpler diversity combining schemes, the most popular are selection combining (SC) and switch and stay combining (SSC). SC and SSC types of diversity systems process only one of the diversity branches, so they are less complicated. With SSC combining the switching of the receiver between the two receiving antennas is based on a comparison of the instantaneous SNR of the connected antenna with a predetermined threshold what results in a reduction of complexity with regard to SC combining. The simultaneous and continuous monitoring of the SNRs at

both branches is no longer necessary. The price of this simplification is some loss in performances.

Namely, in SSC combiner a particular antenna is selected until its quality drops below a predetermined threshold. When this happens, the receiver switches to another antenna and stays with it for the next time interval, regardless of whether or not the channel quality of that antenna is above or below the predetermined threshold. The consideration of SSC systems in the literature has been restricted to low-complexity mobile units where the number of diversity antennas is typically limited to two [12], [13].

II. RELATED WORK

The use of SSC combiner with great number of branches can minimize the bit error rate (BER) [14]. Dual SSC combiner is considered because the gain is the greatest when dual SSC combiner is used instead of one-channel system. The improvement becomes less with enlarging of the branch numbers [14]. The ratio of price and complexity is the best for dual branch system. Because of that it is more economic using SSC combiner with two inputs.

The probability density function (PDF) of the SSC combiner output signal at one time instant and the joint probability density function of the SSC combiner output signal at two time instants in the presence of Rayleigh, Nakagami-m, Weibull, log-normal and Hoyt fading are determined in [15] - [18] and in [1], respectively.

The authors showed in [19]-[22], based on results obtained in [15]-[18], that the error probability and the outage probability are significantly reduced if the decision is performed in two time instants. The analysis of the complex SSC/SC combiner outage probability at two time instants in the presence of Rayleigh and log-normal fading are done in [19] and [20] and the bit error rate for complex SSC/MRC combiner at two time instants in the presence of log-normal and Rayleigh fading in [21] and [22], respectively.

Based on the expressions for the PDF of the SSC combiner output SNR at one time instant and the joint PDF of the SSC combiner output SNR at two time instants in the presence of Hoyt fading obtained in [1], the bit error rate for complex SSC/MRC combiner at two time instants in the presence of Hoyt fading will be given in this paper.

Because it is shown earlier that the better system performances are obtained by decision in two time instants for other fading influences, the motivation for this work is determination of system performances in the presence of Hoyt fading since Hoyt fading has increasing importance in the study of telecommunications systems now days. This investigation could be useful and important for designers and scientists who deal with the decision based on multiple samples and to those who study the impact of different types of fading on system performances.

The remainder of this work is organized in the following way: Section III introduces the model of dual SSC combiner and determines PDF of the SSC combiner output SNR at one time instant. In Section IV, the joint PDF of the SSC combiner output SNR at two time instants is calculated. Subsequently, in fifth Section the bit error rate calculation for complex Switch and Stay Combining/Maximal Ratio

Combining (SSC/MRC) combiner is calculated and then in the sixth chapter the numerical results are presented graphically. Final part of this paper is conclusion with an analysis of the obtained results.

III. SYSTEM MODEL AND PERFORMANCES AT ONE TIME INSTANT

The system model is presented in Fig. 1. The combiner input signals to noise ratios are γ_1 and γ_2 , with γ being the combiner output signal to noise ratio.

Dual SSC combiner works in the following manner: the probability of the event that combiner first examines the signal from first input is P_1 , and for the second input to be examined first is P_2 . If the combiner tests first the signal from first input and if the value of the signal to noise ratio at this input is greater than the threshold, γ_T , SSC combiner leads this signal to the decision circuit. If the value of the signal to noise ratio from the first input is less than the threshold γ_T , SSC combiner directs the signal from another input to the decision circuit, regardless if it is above or below the predetermined threshold. If the SSC combiner first examines the signal from second input his working algorithm is similar.

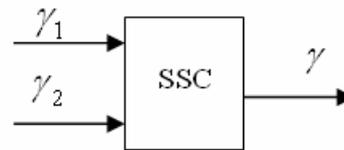


Figure 1. Model of dual SSC combiner with two inputs

The expression for PDF of the combiner output signal to noise ratio will be defined first for the case that input signal to noise ratio is less than the threshold, $\gamma < \gamma_T$. Based on the work algorithm of the SSC combiner in this case, PDF is equal:

$$p_\gamma(\gamma) = P_1 \cdot F_{\gamma_1}(\gamma_T) \cdot p_{\gamma_2}(\gamma) + P_2 \cdot F_{\gamma_2}(\gamma_T) \cdot p_{\gamma_1}(\gamma) \quad (1)$$

In the case $\gamma \geq \gamma_T$ the expression for PDF of the signal to noise ratio at the combiner output is :

$$p_\gamma(\gamma) = P_1 \cdot p_{\gamma_1}(\gamma) + P_1 \cdot F_{\gamma_1}(\gamma_T) \cdot p_{\gamma_2}(\gamma) + P_2 \cdot p_{\gamma_2}(\gamma) + P_2 \cdot F_{\gamma_2}(\gamma_T) \cdot p_{\gamma_1}(\gamma) \quad (2)$$

where γ_T is the decision threshold. The cumulative probability densities (CDFs) are given by [4]:

$$F_{\gamma_i}(\gamma_T) = \int_0^{\gamma_T} p_{\gamma_i}(x) dx, \quad i=1,2 \quad (3)$$

The probabilities P_1 and P_2 are [4]:

$$P_1 = \frac{F_{\gamma_2}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \quad (4)$$

$$P_2 = \frac{F_{\gamma_1}(\gamma_T)}{F_{\gamma_1}(\gamma_T) + F_{\gamma_2}(\gamma_T)} \quad (5)$$

The PDFs of the SNRs at the combiner input, γ_1 and γ_2 , in the presence of Hoyt fading, are [4]:

$$p_{\gamma_1}(\gamma_1) = \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right) \quad \gamma_1 \geq 0 \quad (6)$$

$$p_{\gamma_2}(\gamma_2) = \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right) \quad \gamma_2 \geq 0 \quad (7)$$

where q_i are Nakagami-q fading parameters, which range from 0 to 1 and $\bar{\gamma}_i$ are average SNRs for input channels

The CDFs of the SNRs at the combiner input in the presence of Hoyt fading, after putting of the expressions (6), (7), into (3), are given by:

$$F_{\gamma_1}(\gamma_T) = \int_0^{\gamma_T} \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2 x}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)x}{4q_1^2\bar{\gamma}_1}\right) dx = \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right) \quad (8)$$

$$F_{\gamma_2}(\gamma_T) = \int_0^{\gamma_T} \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2 x}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)x}{4q_2^2\bar{\gamma}_2}\right) dx = \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \quad (9)$$

where $I_e(k, x)$ is Rice's I_e function [23].

After putting of the expressions (8) and (9) into (4) and (5), the probabilities P_1 and P_2 are:

$$P_1 = \frac{\frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)}{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right) + \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)} \quad (10)$$

$$P_2 = \frac{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right)}{\frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right) + \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right)} \quad (11)$$

After putting of the expressions (4)-(7), (8) and (9) into (1), the PDF of the combiner output SNR, γ , for $\gamma < \gamma_T$ is:

$$p_\gamma(\gamma) = P_1 \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma}{4q_1^2\bar{\gamma}_1}\right) \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2\gamma}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)\gamma}{4q_2^2\bar{\gamma}_2}\right) + P_2 \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma}{4q_2^2\bar{\gamma}_2}\right) \cdot \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2\gamma}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)\gamma}{4q_1^2\bar{\gamma}_1}\right) \quad (12)$$

After putting of the expressions (4)-(7), (10) and (11) into (2), PDF of the combiner output SNR γ , for $\gamma \geq \gamma_T$ is:

$$p_\gamma(\gamma) = P_1 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right) + P_1 \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2\gamma_T}{4q_1^2\bar{\gamma}_1}\right) \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right) + P_2 \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2\gamma_2}{4q_2^2\bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2\bar{\gamma}_2}\right) + P_2 \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2\gamma_T}{4q_2^2\bar{\gamma}_2}\right) \cdot \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2\gamma_1}{4q_1^2\bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2\bar{\gamma}_1}\right) \quad (13)$$

The obtained expressions for the PDF of the output signal to noise ratio after diversity combining can be used to study the moments, the amount of fading, the outage probability and the average bit error rate of proposed system.

IV. SYSTEM PERFORMANCES AT TWO TIME INSTANTS

The model of dual SSC combiner at two time instants considered in this Section is shown in Fig. 2.

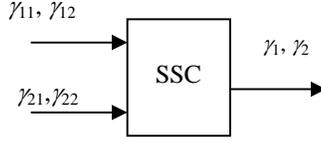


Figure 2. Model of the SSC combiner with two inputs at two time instants

The input SNRs are γ_{11} and γ_{21} at the first time instant and they are γ_{12} and γ_{22} at the the second time moment. The output SNRs are γ_1 and γ_2 .

The indexes for the input SNRs are: first index is the ordinal branch number and the other signs time instant observed. For the output SNRs, the index represents the time instant observed.

The joint PDF of uncorrelated input signals, with Hoyt distribution and the same parameters, is [4]:

$$p_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \cdot \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2 \bar{\gamma}_1}\right) \quad (14)$$

Modified Bessel function of the first kind is defined by [24]:

$$I_m(x) = \left(\frac{x}{2}\right)^m \sum_{k=0}^{\infty} \frac{\left(\frac{x^2}{4}\right)^k}{k! \Gamma(m+k+1)} \quad (15)$$

Four different cases are observed. The first case is: $\gamma_1 < \gamma_T$ and $\gamma_2 < \gamma_T$. In this case all signal to noise ratios at the input are below γ_T , i.e.: $\gamma_{11} < \gamma_T$, $\gamma_{12} < \gamma_T$, $\gamma_{21} < \gamma_T$, and $\gamma_{22} < \gamma_T$.

Let the combiner first treat the signal r_{11} . Because $\gamma_{11} < \gamma_T$, therefore that $\gamma_1 = \gamma_{21}$, and since $\gamma_{22} < \gamma_T$ it is $\gamma_2 = \gamma_{12}$. The probability of this situation is P_1 .

When SSC combiner first treats the signal r_{21} , then it is $\gamma_1 = \gamma_{11}$, because $\gamma_{21} < \gamma_T$. After $\gamma_{12} < \gamma_T$, then it is $\gamma_2 = \gamma_{22}$. The probability of this situation is P_2 . After previous, the joint PDF of the combiner output SNRs at two time instants is obtained using expression (14), for $\gamma_1 < \gamma_T$ and $\gamma_2 < \gamma_T$:

$$p_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) = P_1 \cdot \int_0^{\gamma_T} p_{\gamma_{11}, \gamma_{12}}(\gamma_{11}, \gamma_2) d\gamma_{11} \int_0^{\gamma_T} p_{\gamma_{22}, \gamma_{21}}(\gamma_{22}, \gamma_1) d\gamma_{22} + P_2 \cdot \int_0^{\gamma_T} p_{\gamma_{21}, \gamma_{22}}(\gamma_{21}, \gamma_2) d\gamma_{21} \int_0^{\gamma_T} p_{\gamma_{12}, \gamma_{11}}(\gamma_{12}, \gamma_1) d\gamma_{12} = P_1 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) + P_2 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right)$$

$$\begin{aligned} & \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_1}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) + \\ & + P_2 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\ & \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_2}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) \end{aligned} \quad (16)$$

In the similar manner the other joint PDFs can be derived. The joint PDF is, for $\gamma_1 \geq \gamma_T$ and $\gamma_2 < \gamma_T$:

$$\begin{aligned} p_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) &= P_1 \cdot p_{\gamma_{22}}(\gamma_2) \int_0^{\gamma_T} p_{\gamma_{12}, \gamma_{11}}(\gamma_{12}, \gamma_1) d\gamma_{12} + \\ & + P_1 \cdot \int_0^{\gamma_T} p_{\gamma_{11}, \gamma_{12}}(\gamma_{11}, \gamma_2) d\gamma_{11} \int_0^{\gamma_T} p_{\gamma_{22}, \gamma_{21}}(\gamma_{22}, \gamma_1) d\gamma_{22} + \\ & + P_2 \cdot p_{\gamma_{12}}(\gamma_2) \int_0^{\gamma_T} p_{\gamma_{22}, \gamma_{21}}(\gamma_{22}, \gamma_1) d\gamma_{22} + P_2 \cdot \int_0^{\gamma_T} p_{\gamma_{21}, \gamma_{22}}(\gamma_{21}, \gamma_2) d\gamma_{21} \int_0^{\gamma_T} p_{\gamma_{12}, \gamma_{11}}(\gamma_{12}, \gamma_1) d\gamma_{12} = \\ & = P_1 \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_2}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2 \bar{\gamma}_2}\right) \cdot \\ & \cdot \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) + \\ & + P_2 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\ & \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_1}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) + \\ & + P_1 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_2}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\ & \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_1}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_1}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) + \\ & + P_2 \frac{(1+q_1^2)}{2q_1\bar{\gamma}_1} \exp\left[-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right] I_0\left(\frac{(1-q_1^4)\gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\ & \cdot \frac{(1+q_2^2)}{2q_2\bar{\gamma}_2} \exp\left[-\frac{(1+q_2^2)^2 \gamma_2}{4q_2^2 \bar{\gamma}_2}\right] I_0\left(\frac{(1-q_2^4)\gamma_2}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) \end{aligned} \quad (17)$$

for $\gamma_1 < \gamma_T$ and $\gamma_2 \geq \gamma_T$:

$$\begin{aligned}
 & \cdot \exp\left(-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4) \gamma_2}{4q_1^2 \bar{\gamma}_1}\right) + \\
 & + P_2 \frac{(1+q_1^2)}{2q_1 \bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4) \gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \\
 & \cdot \frac{(1+q_2^2)}{2q_2 \bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2 \gamma_2}{4q_2^2 \bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4) \gamma_2}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right)
 \end{aligned} \quad (19)$$

V. BIT ERROR RATE FOR SSC/MRC COMBINER

The model of the SSC/MRC combiner with two inputs considered in this paper is shown in Fig. 3. We consider the SSC/MRC combiner with two branches at two time instants. The output signals at SSC part are γ_1 and γ_2 and they become the inputs at MRC combiner and the overall output signal is γ . In this case, the signal at output of complex combiner includes both, the time diversity and the space diversity {1}.

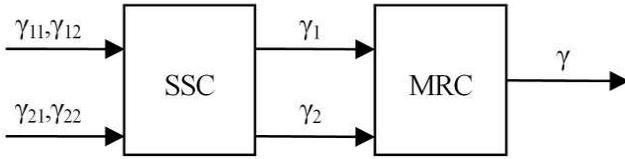


Figure 3. Model of complex dual SSC/MRC combiner at two time instants

The total conditional signal value at the output of the MRC combiner, for equally transmitted symbols of L branch MRC receiver, is given by [3]

$$\gamma = \sum_{l=1}^L \gamma_l \quad (20)$$

For coherent binary signals the conditional BER $P_b(e|\{\gamma_l\}_{l=1}^L)$ is given by [4]

$$P_b(e|\{\gamma_l\}_{l=1}^L) = Q(\sqrt{2g\gamma}) \quad (21)$$

where Q is the one-dimensional Gaussian Q-function [17]

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-t^2/2} dt \quad (22)$$

Gaussian Q-function can be defined using alternative form as [4]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{x^2}{2\sin^2\phi}\right) d\phi \quad (23)$$

Using the alternative representation of the Gaussian-Q function, the conditional BER can be expressed as

$$P_b(e|\{\gamma_l\}_{l=1}^L) = \frac{1}{\pi} \int_0^{\pi/2} \exp\left(-\frac{g\gamma}{\sin^2\phi}\right) d\phi = \frac{1}{\pi} \int_0^{\pi/2} \prod_{l=1}^L \left(-\frac{g\gamma_l}{\sin^2\phi}\right) d\phi \quad (24)$$

The unconditional BER can be obtained by averaging the multichannel conditional BER over the joint PDF of the signals at the input of MRC combiner

$$P_b(e) = \underbrace{\int_0^\infty \dots \int_0^\infty}_{L} \int_0^{\pi/2} \prod_{l=1}^L P_{\gamma_1, \gamma_2, \dots, \gamma_L}(\gamma_1, \gamma_2, \dots, \gamma_L) d\gamma_1 d\gamma_2 \dots d\gamma_L d\phi \quad (25)$$

Substituting (24) in (25), $P_b(e)$ is obtained as

$$P_b(e) = \int_0^\infty \dots \int_0^\infty \int_0^{\pi/2} \prod_{l=1}^L \left(-\frac{g\gamma_l}{\sin^2\phi}\right) d\phi P_{\gamma_1, \gamma_2, \dots, \gamma_L}(\gamma_1, \gamma_2, \dots, \gamma_L) d\gamma_1 d\gamma_2 \dots d\gamma_L \quad (26)$$

For dual branch MRC combiner, $P_b(e)$ is

$$\begin{aligned}
 P_b(e) &= \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) P_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) d\phi d\gamma_1 d\gamma_2 + \\
 &+ \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) P_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) d\phi d\gamma_1 d\gamma_2 + \\
 &+ \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) P_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) d\phi d\gamma_1 d\gamma_2 + \\
 &+ \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) P_{\gamma_1, \gamma_2}(\gamma_1, \gamma_2) d\phi d\gamma_1 d\gamma_2
 \end{aligned} \quad (27)$$

Substituting (1-4) in (27), $P_b(e)$ of SSC/MRC combiner can be obtained as:

$$\begin{aligned}
 P_b(e) &= \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} d\gamma_1 d\gamma_2 d\phi \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) \cdot \\
 & \cdot P_1 \frac{(1+q_1^2)}{2q_1 \bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2 \gamma_2}{4q_1^2 \bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4) \gamma_2}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\
 & \cdot \frac{(1+q_2^2)}{2q_2 \bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2 \gamma_1}{4q_2^2 \bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4) \gamma_1}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) + \\
 & + P_2 \frac{(1+q_1^2)}{2q_1 \bar{\gamma}_1} \exp\left(-\frac{(1+q_1^2)^2 \gamma_1}{4q_1^2 \bar{\gamma}_1}\right) I_0\left(\frac{(1-q_1^4) \gamma_1}{4q_1^2 \bar{\gamma}_1}\right) \frac{2q_1}{1+q_1^2} I_e\left(\frac{1-q_1^2}{1+q_1^2}, \frac{(1+q_1^2)^2 \gamma_T}{4q_1^2 \bar{\gamma}_1}\right) \cdot \\
 & \cdot \frac{(1+q_2^2)}{2q_2 \bar{\gamma}_2} \exp\left(-\frac{(1+q_2^2)^2 \gamma_2}{4q_2^2 \bar{\gamma}_2}\right) I_0\left(\frac{(1-q_2^4) \gamma_2}{4q_2^2 \bar{\gamma}_2}\right) \frac{2q_2}{1+q_2^2} I_e\left(\frac{1-q_2^2}{1+q_2^2}, \frac{(1+q_2^2)^2 \gamma_T}{4q_2^2 \bar{\gamma}_2}\right) + \\
 & + \frac{1}{\pi} \int_0^{\gamma_T} \int_0^{\gamma_T} \int_0^{\pi/2} d\gamma_1 d\gamma_2 d\phi \left(-\frac{g\gamma_1}{\sin^2\phi}\right) \left(-\frac{g\gamma_2}{\sin^2\phi}\right) \cdot
 \end{aligned}$$

VI. NUMERICAL RESULTS

It is simple to present graphically the results, that come from equations we obtained in Sections III-V, using mathematical software, for example “Matlab”, that is used in this paper. For the sake of simplicity, it has been assumed that the variances of both signals at the combiner input are equal without losing in generalization.

The case when one time instant is observed is presented in Fig. 4. The PDF of the combiner output SNR is determined as a function of input SNR γ and the decision threshold γ_T , for three different variance values and for the same distribution parameters at receiver branches.

The Fig. 5. to 7. present the situation when two time instants are observed. The PDF is given versus input SNRs at two time instants, γ_1 and γ_2 , for different values of distribution parameters and decision threshold γ_T .

It is assumed that both input branches have the same channel parameters. It is adopted that γ_T is optimal threshold of the SSC decision [1]:

$$\gamma_T = E(\gamma^1), \tag{29}$$

where [25]:

$$E(\gamma^k) = \Gamma(1+k) {}_1F_1\left(-\frac{k-1}{2}, -\frac{k}{2}; 1; \left(\frac{1-q^2}{1+q^2}\right)^2\right) \bar{\gamma}^k. \tag{30}$$

Substituting (30) in (29), the optimal threshold is:

$$\gamma_T = \bar{\gamma}. \tag{31}$$

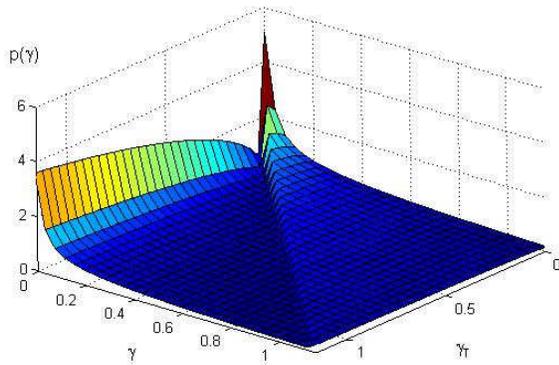


Figure 4. Probability density function of the combiner output signal to noise ratio at one time instant for $\bar{\gamma}_1 = \bar{\gamma}_2 = 1, q_1=q_2=0.5$

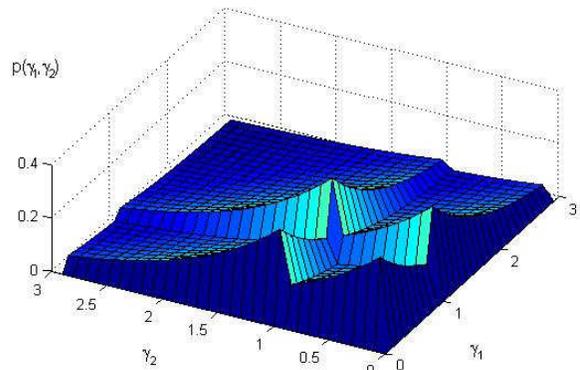


Figure 6. The probability density function of the combiner output signal to noise ratio at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 1, \gamma_T = 1, q_1=q_2=0.9$

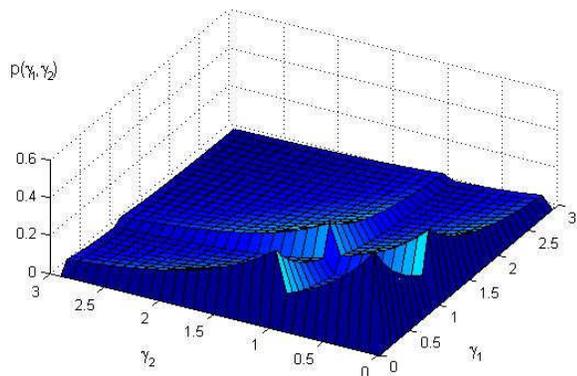


Figure 5. The probability density function of the combiner output signal to noise ratio at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 1, \gamma_T = 1, q_1=q_2=0.5$

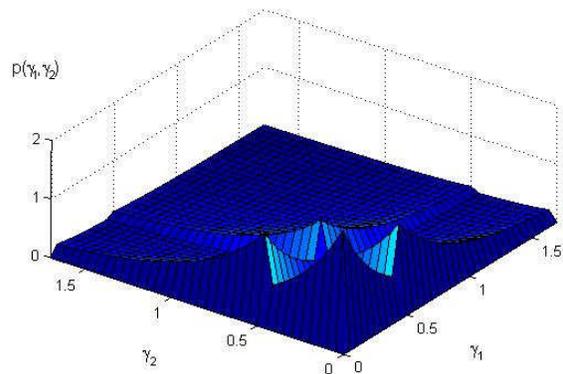


Figure 7. The probability density function of the combiner output signal to noise ratio at two time instants for $\bar{\gamma}_1 = \bar{\gamma}_2 = 0.5, \gamma_T = 0.5, q_1=q_2=0.5$

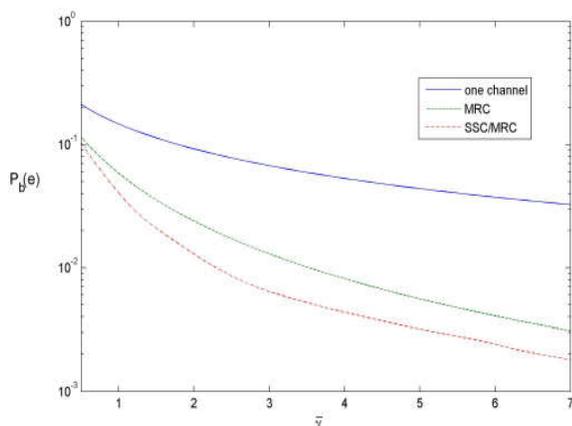


Figure 8. Bit error rate for different types of combiners versus parameter \bar{q} for $q=0.9$

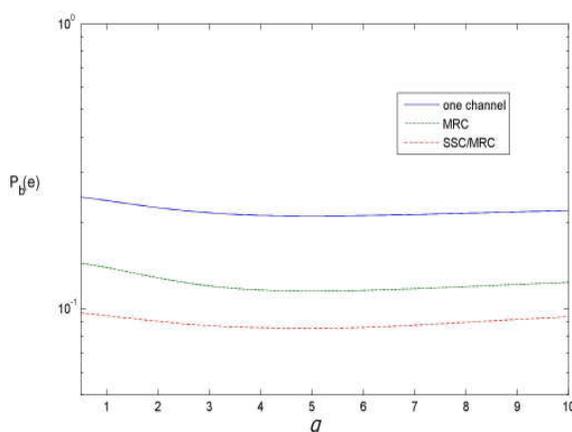


Figure 9. Bit error rate for different types of combiners versus parameter q for $\bar{q} = 0.5$

The families of curves for BER for one channel receiver and for MRC combiner at one time instant and SSC/MRC combiner at two time instants, for uncorrelated case, are shown in Figs. 8. and 9. versus different distribution parameters.

It can be observed that SSC/MRC combiner has significantly better performances for uncorrelated case than MRC combiner at one time instant. It is evident that using this complex SSC/MRC combiner results in better system performances.

VII. CONCLUSION AND FUTURE WORK

The SSC and MRC are simple and frequently used techniques for combining signals in diversity systems. The system performances deciding by two samples can be determined by the joint PDF of dual SSC combiner output signal at two time instants and putting them as inputs of the MRC combiner. In this paper, the PDF of dual SSC/MRC combiner output signal at two time instants in the presence

of Hoyt fading is determined and based on it the bit error probability is calculated.

The obtained results are shown graphically for different parameters. It is apparent that system performances can be significantly improved using the sampling at two time instants and characteristics of complex SSC/MRC combiner comparing with classical SSC and MRC combiners.

In the future works, the other important useful performance measure for diversity systems like outage probability, amount of fading and second order parameters can be determined.

This fact shows that the results obtained in this paper are very significant in the designing and application of diversity receivers.

ACKNOWLEDGMENT

This work has been funded by the Serbian Ministry for Science under the projects TR33035, TR32052 and III 44006.

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