A Design of Rich Environment for Teaching Meaningful Mathematics to Low-Achieving Students: Research Implications

Orit Broza
Faculty of Education
Levinsky College
Tel-Aviv, Israel
Email: Oritbroza1@gmail.com

Yifat Ben-David Kolikant
School of Education
Hebrew University of Jerusalem
Jerusalem, Israel
Email: Yifat.kolikant@mail.huji.ac.il

Abstract—**The paper describes implications from a design-based research in which a rich collaborative, computer-supported learning environment was designed to promote meaningful mathematics among low-achieving students. Fifth-grade students interchangeably solved decimal subtraction tasks with peers in the context of a computer game and simulations, and in discussion sessions led by their teachers, in foursomes. We describe the results of the first round of our design-based research, where we traced three such groups, using observations and interviews. We found that the computer context was both constructive and destructive, in terms of students' learning. The group discussions did not yield the rich discussions we had hoped for. Yet, overall, the environment was successful because students gained meaningful mathematical knowledge and practiced active, thoughtful, and collaborative socio-mathematical behavior, which is dramatically different from what they were used to.**

Keywords: low-achieving students; support-rich environment; computer games; scaffolding; computer-supported collaborative learning.

I. INTRODUCTION

The question of how students' construction of meaningful knowledge can be supported represents an important challenge to both researchers and teachers. Teaching the complex topic of mathematics to low-achieving students (LAS) poses a special challenge, owing to LAS's unique cognitive and behavioral characteristics [1]. The teaching and learning processes of LAS have been studied by examining different teaching methods, strategies, and tactics (e.g., [2][3]). However, we found sparse work on the effectiveness of rich environments, let alone environments involving computer-supported collaborative-learning (CSCL), on the learning processes and outcomes of LAS.

In fact, LAS characteristics, which we describe next, might make one doubt the feasibility of teaching LAS basic mathematics, let alone in (Computer Supported) Collaborative Learning (CS)CL settings. Nonetheless, we hypothesized that a rich CSCL environment, involving a computer game, real context mathematics, peer discussions, and teacher mediation may be the key for addressing LAS's unique and diversified needs. Here, we describe the results of the first round of a design-based research we conducted to examine these hypotheses. First, we describe the characteristics of LAS. Then we review the literature and how it influenced our hypotheses and design. Next, we describe a study, the first round of a design-based research in which we examined our hypotheses. We traced the participation of 3 groups of four students each, in the activities we had designed, using various data sources, such as videotapes and audiotapes of the classes, as well as interviews and ad-hoc conversations with students and teachers, along with observations. We will discuss our findings and their practical implications on our design framework and the broader scientific community. Our main conclusion is that CSCL, when carefully designed, can promote LAS learning of meaningful mathematics as well as the development of sociomathematical norms.

In Section II we review the literature on LAS as well as on successful interventions in terms of achieving meaningful learning. Then, we describe our pedagogical design, and the literature that inspired us, such as the decision to involve a computer-game session in which students work in pairs, and small-group discussions led by the teacher (Sections III and IV). We then describe the study (Section V). Next, we examined how a rich environment either hinders or supports students' construction of mathematical meaning. We focused on the mutual interplay between the two contexts in which students worked (on the computer and in group discussions). We present the findings (Section VI) and discuss them (Section VII).

II. LAS AND MEANINGFUL MATHEMATICS

There is no single, definitive profile for LAS [4][5]. In fact, most studies have not focused on the methodological criteria used to identify those students with learning disabilities [5]. LAS are commonly identified based on two factors: teachers' reports and LAS performance on standardized or informal tests (students' score below the 50th percentile on standardized tests; however, they are not diagnosed as having learning disabilities) [2]. In attempting to explain LAS's poor performance, the literature focuses on cognitive deficiencies and on behavioral manifestations of their failures. LAS find it difficult to retrieve basic mathematics knowledge from their memory [6]. Craik [7] terms this difficulty as 'fragile memory', a product of superficial data processing. They also lack meta-cognitive skills [8], and are sensitive to the learning contexts. Thus, they find it much harder than others to solve simple and complex addition and subtraction problems. These difficulties may lead them to use less sophisticated strategies and to make more errors.

Recently, Karagiannakis et al. [9] developed a model that can be used to sketch students' mathematical profiles for four domains (numbers, memory, number line, and reasoning); they empirically examined it to determine whether and how it can differentiate between students with and without difficulties in learning mathematics. According to their analysis, students, both the normal/high achievers and the underachievers, do not all share the same strong or weak mathematical skills. In addition, under achievement in mathematics is not related to weaknesses in a single domain (e.g., numbers, memory, number line, and reasoning). They also suggest that for LAS, just like for other students, cognitive strengths or weaknesses may rely on any of the four domains (mentioned above) of their model. Their findings empirically strengthen the heterogeneity of this population group.

Experiencing repeated failures and difficulties in keeping up with the class might, in turn, decrease LAS's motivation and their sense of internal responsibility and make them more passive learners. It might also lead them to act impulsively, rely on the judgment and feedback of an external authority [3], and avoid collaborative work with peers [10]. Their schooling-purposed interaction in class is largely with the teacher.

These characteristics probably underlie many teachers' beliefs that LAS are unable to deal with tasks involving high-order thinking skills and that the most effective way of promoting mathematical performance in LAS is to 'drill and kill', that is, to focus more on the mathematical algorithms than on the mathematical meaning [11].

However, despite their difficulties, there is empirical evidence that in certain environments LAS are capable of enhancing their mathematical understanding. There is empirical evidence that LAS can exhibit mathematical reasoning orally when placed in intimate and supportive learning environments, such as in small groups where they are tutored [12][11]. Peltenburg et al. [13] show that, in a familiar context with the help of technological tools, LAS can succeed in solving subtraction problems by using an indirect addition strategy spontaneously, rather than the conventional direct subtraction strategy. Karagiannakis and Cooreman [14] suggest that these interventions should be designed for repeated success by building on a student's strengths, while avoiding use of repetitive tasks that cause repetitive failure experiences, thereby maximizing the learning opportunities of all students.

Synthesizing these empirical evidence with the reports on the literature on the diversity among LAS, we assumed that a rich environment that includes technological tools, small groups, and teacher's support building on LAS's strengths might be the key to their success.

III. THE LITERATURE THAT INSPIRED THE DESIGN AND HYPOTHESES

Our design was inspired by the socio-cultural theoretical perspective on learning, especially the notion of distributed scaffolding. Scaffolding is "titrated support that helps learners learn through activity. It helps learners perform tasks that are outside their independent reach and consequently develop the skills necessary for completing such tasks independently" [24, p.306] . Because LAS vary in their behavior, in our design we sought to design distributed scaffoldings [15], i.e., to integrate and sequence multiple forms of support via various means. Different scaffolds interact with each other; sometimes they produce a robust form of support, a synergy [16], and other times, they might sabotage the learning processes and the outcome.

We were also inspired by the Learning in Context approach, namely, the idea of presenting mathematical concepts and procedures in a context relevant to the child's day-to-day life [17], and in particular, the Realistic Mathematics Education (RME) theoretical framework. According to the RME framework, students should advance from contextual problems using significant models that are situation related, to mathematical activity at a higher level (e.g., engaging in more formal mathematical reasoning). As students progress from informal to more formal mathematics, their "*model of*" the situation is transformed into a "*model for*" reasoning. We hypothesized that RME could be the key to promote meaningful learning for LAS, because the subtraction tasks, the mathematics to be mastered, will be associated with real-life experiences, which might mitigate their fragile memory and tendency for superficial processing of new knowledge.

We aimed at transforming students' social and socio-mathematical norms, from passive to active, from isolated to social collaboration, and from impulsive to thoughtful. We were motivated by the premise that digital games, by the nature of their design, have the potential to motivate students to become active rather than passive, by enabling experimentation and exploration without fear of failing in front of the entire class [18][19]. The use of games for teaching may be particularly beneficial for LAS because of their tendency to remain passive and to comply with authoritative voices.

We were aware of the possibility that a hands-on, minds-off strategy might emerge, especially because of the tendency of LAS for impulsivity. This is one of the reasons students were asked to work with peers in front of the computer. We assumed that collaborative settings would trigger twofold interactions: with the system and with the co-learner. Peers would explain their calculations to each other, and question other actions, which would bring about reflection and thoughtfulness [20].

Additionally, every session was designed to include interchangeable students' work in front of the computer with their peers, along with group discussions, led by the teacher. Teachers' interactions with students can create zones of opportunities that can be directed to scaffold students' social and emotional development [21]. The teacher can mediate the use of tools (e.g., computer games, online units), orchestrate the students' activities, and reframe them conceptually [22].

Hence, the students experienced two different collaborative settings. When they worked (in pairs) in front of the computer (computer games or online units), the teachers were asked to observe them and to offer help when necessary (for instance, if students maintain trial and error strategies or are stuck in their calculation process). In the group discussions, the teachers were asked to focus their discussions on various strategies that can be used to solve subtraction tasks, to encourage students to verbalize their thoughts, and encourage them to rely on each other's past experience, thereby facilitating students in learning the meaning of how to participate in the community, i.e., support the transformation of their sociomathematical norms [23]. In these discussions, the teachers also introduced students to new tasks and encouraged them to employ the strategies they previously used in a supposedly new context. As we will explain in the next section, in our design we presented tasks sometimes as stories and sometimes as formal subtraction exercises, and gradually increased the difficulty of calculating the numbers whose decimals are half, to numbers whose decimals include individual units. We assumed that students' sense of security when expressing themselves publicly would increase, since they are in a group of equals, and will experience active (and successful) work with their peers in front of the computer.

IV. The Instructional Design

We developed an extracurricular program for fifth grade LAS. It consisted of ten weekly sessions that focus on subtraction with decimal numbers, a topic that students had not yet learned in their regular classes. Students were categorized into groups of four, according to their regular class, and each group worked with a teacher trained by the second author.

We utilized a *real-life context* simulated by an ice cream shop computer game. Specifically, during the sessions, students played a computer game in which they received orders from random customers, prepared the orders, calculated the price to be paid, and gave change as needed (Fig. 1). Because of the heterogeneity of LAS and their individual needs, we sought to provide a variety of support types. Therefore, students also worked on supplementary online study units concerned with the transition between money and formal representations, as well as change calculations. Students also enacted game-like situations with play money using Israeli bills and coins: New Israeli Shekels (NIS) and agorot (1NIS = 100 agorot; the smallest coin is 10 agorot). In order to support the transition from the concrete to the abstract, real worksheets were designed, which included exercises in concrete, graphic, and abstract forms.

In order to facilitate the delicate transition from the realistic environment (shop simulation) to formal mathematics, subtraction was first presented through monetary simulations and calculations only, and formal representations were interwoven at a later stage. The program progresses in a *spiral-like manner*. With the help of the teacher, students are expected to progress from one level to the next. The tasks at each level maintain an overall forward trend of increasing complexity, and students are able to revisit earlier levels and solve simpler exercises on the computer on their own. The teachers had the flexibility to fine-tune the program, in response to students' emerging needs.

Fig. 1 A screenshot of an online learning unit, where the task at hand is 50-38.6.

In each session, students spent almost half of their time in front of the computer, working in pairs. They were first introduced through online activity to two avatars, a girl, and a boy, each of whom described a strategy he or she uses for calculating the required change. Then they played or worked in pairs on the computer. The other half of the session time was devoted to class discussions, as described above. Specifically, in order to address LAS's tendency to passively rely on external authority and to encourage them to take personal responsibility, the teachers were not supposed to correct students' strategies directly, but rather, to ask questions to encourage them to talk aloud about their thinking processes, thus, making the diagnosis easier and potentially leading them to correct their own mistakes, re-voicing when needed, and referring them to suitable tools in the environment when necessary. The teachers generally followed these instructions closely.

V. The Study

Our goal was to examine our design's hypotheses, i.e., to examine the students' learning processes, focusing on how the rich environment either hinders or supports students' construction of mathematical meaning, especially the mutual interplay between the two contexts in which students worked (on the computer and in group discussions).

A. Participants

We traced 12 LAS (4 male, 8 female) from 3 fifth grade classes in suburban schools within the same city, and who participated in the program. They learned in 3 groups of 4 students, with 3 different teachers (one of them was the first author). All participants were chosen based on the recommendation of their mathematics teachers. They all performed under the 50^{th} percentile on standardized tests, yet were not diagnosed as having learning disabilities.

B. Data Sources

In two groups all sessions were videotaped. In one group they were audiotaped. We observed students in their regular mathematics class two times before they began participating in our CSCL activity. We also observed all the sessions, focusing on the sequence of activities—of both the teacher (e.g., presenting tasks, intervening during the computer sessions, suggesting a tool, getting students' attention, and answering questions) and the students (e.g., how they interact with the computer, with each other, with the teacher, and so forth). We conducted interviews with the CSCL teachers, after the activity as well as ad hoc conversations after every session. We also talked with their previous mathematics teachers and with each student after the CSCL activity.

C. Methods of Analysis

Our report mainly draws on the analysis of the videotapes. A preliminary analysis of the data was presented elsewhere [1]. That analysis was useful to identify patterns of students' interaction with the environment. Here we present in detail the results of a fine-tuned analysis. Specifically, we were inspired by the analysis model of Powell et al. [24] for developing mathematical ideas and reasoning. We fully transcribed one group through videotapes. The transcripts were coded twice by two researchers. We segmented the text into episodes, each beginning with the presentation of a new task and ending with its being accomplished (or the work on it was terminated). For each episode we examined: (1) who participated in it; (2) the knowledge pieces that emerged; (3) the difficulties that arose, including whether they were resolved, and if so, how and by whom, especially (d) the support provided by the teacher; and (5) whether the task was successfully accomplished independently or with help from others. We also coded affective utterances, both positive and negative. We compared the results with the video, audio, and notes taken during the observations in the other groups. Interviews were analyzed thematically. We chronologically traced changes in each student's thinking and behavior, thereby creating data stories. One such data story is presented next.

VI. FINDINGS

A. Students' Interaction with the Rich Environment

1) The computer setting: As we hypothesized, the computerized environment, especially the computer game, encouraged the students to be active as well as engaged in their task. For the most part, they were observed to be very focused on the current task. In fact, in 5 sessions, students continued working (or playing) after the class had ended. The students reported in the interviews and ad hoc conversations that they had enjoyed the activity. The following quotes are but two examples of typical phrases heard throughout the entire program: "it was fun…not a regular class", "playing with the computer gives a sense of fun, [vs.] a blackboard, where you just sit and solve exercises".

On the computer the students (who sat in pairs) usually decided to work in turns. In each turn the one on the keyboard gave ice cream, calculated the price, the change, and returned change. For a few couples, we noticed a different division of labor: the one on the keyboard interacted with the avatar clients and in the meantime, the other did the calculations. In a few cases when one student took over the keyboard the teacher interfered.

During the play, each student solved many subtraction exercises, manifested by the need to give change to customers in the shop.

They did not solve all the exercises successfully right away. However, for the most part failures in this context did not discourage or frustrated them. On the contrary, this is when we observed collaboration, mathematical discussions with their peers and with the teacher. Usually, when they received a response from a "customer" indicating that the change they gave was incorrect, they were observed pausing to think and sometimes they turned to their peers and verbalized their "solution process". Sometimes this verbalization was performed after their peers asked them how they had worked. Often the discussion helped them to correct themselves. This behavior was dramatically different from the observed passivity (or impulsivity) in the regular mathematics classes. Moreover, in this context, the students generally welcomed the teachers' intervention and cooperated with them. Hence, the computer and their peers often generated a synergetic effect on the students.

However, we also observed an appreciable number of situations in which students merely employed trial and error, using the immediate feedback of the computer ("too much" and "too little") to guess the correct answer. Usually the partner became silent in these situations. From the conversations in these situations, we learned that the pressure of time and the wish to gain as many points as possible in the game in a designated time frame encouraged this behavior. In one extreme example, one student (Betty) stopped working because the clients became angry (Fig.2 and Fig. 3), because it took her time to calculate. This episode as well as other important episodes and aspects of Betty's learning process within the environment are presented in Section VI.*B*.

Fig. 2 Speech bubbles turn red as a sign for impatient clients.

Fig. 3 An angry face of an impatient client.

We also noticed that in the initial lessons the teacher had to compete with students' attention to their computer in these situations. We observed the teacher, in such situations, touching the students' hand or shoulder to get their attention.

The next episode demonstrates the teacher's struggle for Etty's attention. Etty stared at the computer screen when the teacher approached her:

282.	Teacher:	Ok, how do you calculate the change?
283.	Etty:	Eh, Eh, Eh…[looking at the screen, trying to concentrate in the game]
284.	Teacher:	Etty, please explain.
285.	Etty:	I am not sure…[keeps playing]
286.	Teacher:	Remember how we did it before?
287.	Etty:	Aha… [Her full body is turned to the computer].
288.	Teacher:	And we saw many ways, the way Dor [avatar] calculated?
289.	Etty	Aha…[she keeps concentrating on the computer]

Obviously, Etty preferred to focus on the computer session. She concentrated on the task, mumbling "Eh…or Aha…" answers to the teachers' requests as if it was difficult for her to split her attention (lines 283, 287).

2) Group discussions: Group discussions revolved around calculations and strategies. Figure 4 illustrates a typical discussion routine.

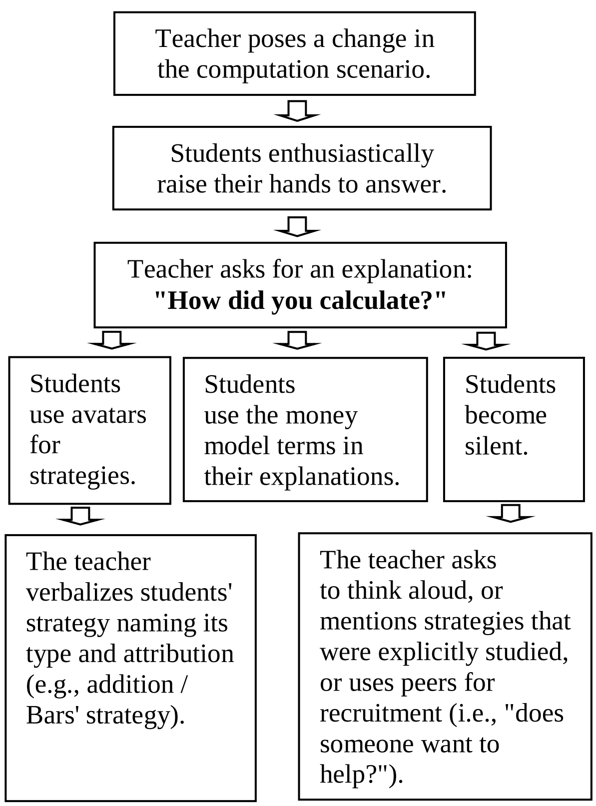

Fig. 4 Group discussion after game play.

The teacher initiated each episode by presenting a subtraction task. After the students solved the task, she then asked them to explain their strategies. We observed many expressions of frustration among the students. The teacher noticed that students tended to take turns when they worked at the computer. She borrowed the idea and asked them to also solve exercises in turns in the group discussions. However, in this setting this idea turned out to be less productive. Generally, the interaction took the form of one student explaining his or her solution process, followed by the teacher's verbalization. Moreover, the teacher sometimes silenced the peers who tried to participate in conversations. In her interview she explained that students' poor discursive habits made her prioritize the individual's learning over building a community and discursive habits. We thus observed almost no rich peer discussions about strategies.

We expected that during the participation the students' ability and wiliness to provide explanations would increase. During the discussion with the teacher (with or without a computer) the students were constantly asked to describe and explain their strategies. The alienation of this request was prominent in their responses. They became silent, gave vague or non-informative answers (e.g., "I just did so"), and sometimes even said, "I don't remember". The following excerpt from lesson 3 illustrates this kind of discourse while a student was struggling in calculating the exercise 10 - 4.1 =

324.	Teacher:	How would you like to solve this problem?
325.	Noya:	I don't know which one is more comfortable to me.
326.	Teacher:	You don't know which one is more comfortable to you. [pause].
327.	Teacher:	Ah! Maybe someone wants to help her explain how should she solve it?
328.	Neomi:	Addition.
329.	Teacher:	Addition. What do you mean? Explain to her.
330.	Neomi:	To add 10 Agorot until we have 10, which means that it is 9 times ten Agorot.
331.	Teacher:	Until we get to 1 Shekel, right? Until we get to a Shekel.
332.	Neomi:	And then we take 5 [Shekels] and 9 like these [10 Agorot coins].
333.	Noya	Aha…

The preference question (line 324) confused Noya. Probably, she was not used to these kinds of questions in her regular math class. When she noticed that Noya became silent, the teacher turned to recruit the group (line 328) and emphasized the meaning of *adding on* strategy (line 331).

The above excerpt also demonstrates that in some of the students' explanations (e.g., for Neomi) there was evidence of a positive change in their discursive manners. In these cases we often found that students relied on the money model (especially the fact that 1 nis = 100 agorot) to explain their subtraction strategies even when the subtraction task was phrased abstractly and not in money terms. Real context mathematics, hence, supported students' leaning.

We expected that the students would develop many strategies for subtraction. Indeed, the teacher posed questions like "in what way would you like to solve this problem?" at least three times in each of the first three sessions. However, we did not observe the emergence of a new strategy. One possible explanation is rooted in our sequencing of students' activities. In the initial lessons, students were introduced by an online unit to two strategies, presented to them by two avatars who dealt with the task of calculating change. Possibly this early exposure, together with students' tendencies to rely on external authoritative voices, brought about a fixation in their thoughts. Moreover, sometimes we were not sure that students understood the meaning underlying these strategies.

Nonetheless, in conversations with the teachers in the regular classes after the program ended, the teachers reported that the behavior of most of the participants in their class improved; specifically, that despite their difficulties they were more motivated and less passive.

B. *Betty's data story*

Betty was diagnosed by her teacher as a low achieving student due to her low academic achievements compared with other students in her class, her impulsivity, and her "short memory", as Betty testified. The teacher reported that once a week Betty used to leave her regular math class to learn in a small group in order to help her keep up with the class.

Table I describes Betty's performance in the subtraction tasks during the activity. The table includes details about each task (whether it was presented in a story form or as a formal exercise), its context (a group discussion led by the teacher, an individual worksheet, or peer interactions while students worked on the computer), whether it was performed orally or in writing, and finally, whether Betty succeeded in solving it, and whether the success was assisted or not. Betty solved more tasks within the computer context, but since we did not record the screen in the first iteration, we have only the tasks in which the teacher was directly involved. Nonetheless, we took notes on her performance within this context as well.

As shown in Table I, Betty's performance was inconsistent. Betty successfully solved oral subtraction tasks within the context of group discussion (tasks A, C). She also experienced some success in written calculation tasks (tasks G, H, J, K) independently. Her failure occurred partly in the computerized context (tasks D, F), probably due to her impulsivity.

1) Starting point: Observations in the regular math class preceding the CSCL activity indicated that Betty was passive, unmotivated, and unengaged, and that she laid her head on the table for most of the lesson as if she was bored. However, her behavior changed dramatically right after the first lesson when we came up with the computer game. Suddenly, she was dominant, controlling the computer, helping her peer. She even took over her peer's role of play (Fig. 5). We could see that Betty was totally engaged.

TABLE I. BETTY'S TASK PERFORMANCE

Task No.	Lesson No.	Task	Performance Of The Task	Context
A	2	Story (50-41.50)	Oral	Group discussion
B	3	Exercise (10-7.20)	Oral	Group discussion
C	4	Story (20-15.60)	Oral	Group discussion
D	4	Story (15-13.20)	Oral	Peers (with computer)
E	5	Exercise (50-14.80)	Oral +Written	Group discussion
F	6	Exercise (15-13.20)	Oral	Peers (with computer)
G	6	Exercise (15-13.20)	Written	Group discussion
H	6	Exercise (20-16.80)	Written	Group discussion
I	7	Story (50-42.60)	Written	Individual worksheet
J	8	Story (100-38.50)	Written	Individual worksheet
K	8	Story (100-57.30)	Written	Individual worksheet

Failure   Assisted success   Independent success

Although her body language expressed her full engagement in the game, we noticed some of Betty's math difficulties. For example, she used her fingers for counting. She also used the game feedback to calculate basic facts (addition and subtraction of whole numbers up to 20) that she did not master. She also used trial and error impulsively. Research reports also strengthen those impressions:

> *"Betty and Neomi cooperate by consulting each other [e.g., 'it said we gave too much; give a bit less'], clicking their answers on the computer, trying to get to the exact answer. When they get positive feedback from the computer, they laugh about their own errors…* [Taken from the second observation]."

Fig. 5 Betty controlling the computer.

In group discussions the teacher tried to confront Betty's impulsivity by encouraging her to think aloud. Sometimes the teachers even used hand touching in order to prevent her from quickly checking on the computer. A representative example is presented in the next episode taken from lesson 3. Betty had to calculate the change from 10 NIS for ice cream, which cost 7.20 NIS (Table I, task B). Betty hesitantly typed a wrong answer and got negative feedback:

356.	Teacher:	[hushing the rest of the group] Girls, please let her concentrate.
357.	Betty:	I thought it was right.
358.	Teacher:	Why?
359.	Betty:	I don't know [she types the same answer. Gets negative feedback]
360.	Teacher:	Betty, how did you try to solve the problem?
361.	Betty:	[disturbed by her peer] Stop it Eve!
362.	Teacher:	Betty, look at the amount to pay.
363.	Betty:	Ehm..7.20.
364.	Teacher:	Seven NIS and twenty Agorot. And what is the amount that was paid?
365.	Betty:	10 NIS.
366.	Teacher:	Ten NIS. According to which strategy is it easier for you to calculate? Ah? [Betty is clicking on the coins on the computer, thereby returning change. Getting negative feedback]
367.	Betty:	Wait, wait it is too much…[reading the negative feedback]
368.	Teacher:	If you give three NIS [change], and he has to pay seven NIS, what number you get?
369.	Betty:	[facing the computer screen] Come on…. but here he [avatar] said that it is too much, so maybe I will add here one [adding one ten Agorot coin and waiting for the computer's feedback. She gets negative feedback].
370.	Teacher:	No Betty! It is impossible! I want you to think for a minute. [Betty counts quietly using her fingers] Betty, answer me…We have Neomi's strategy ...It helped Noya before, did you listen? Did you understand what she said?

The above excerpt demonstrates the leading questions (lines 358, 362, 364, and 366) that the teacher offered in order to help Betty to concentrate on the calculation instead of the trial and error strategy that Betty had used. Betty ignored her and kept the trial and error method, increasing or decreasing the number of coins in order to reach the correct answer (lines 359, 367).

Betty was emotionally involved in the game. Her stress resulting from the response of the avatars is demonstrated in the following episode, taken from lesson 4. Betty and Neomi played the computer game. The teacher noticed that Betty was stuck so she approached them:

279.	Betty:	[to the client avatar] Stop it!!!! [to Neomi] Your turn…[give up and pass the computer mouse to Neomi]
280.	Teacher:	No, No, No, No, No!!!!
281.	Betty:	I can't, they [client avatars] get angry at me!!!I am scared…
282.	Teacher:	Don't look.
283.	Betty:	At the end they will get out of the game and come to beat me.
284.	Teacher:	What is the bill to be paid? Say it loud, what is the bill?
285.	Betty:	[silent]
286.	Teacher:	Thirteen
287.	Betty:	[Mumbling…thinking…] Sh….wait, wait, wait [using her fingers, turning to the computer, tapping an answer and getting positive feedback] I am a genius!!! This is what I did before….

Although Betty was nervous, the teacher did not give up and in a definite statement of "no" (line 280) she decisively did not accept this behavior. Instead, she supported Betty by breaking down the problem into its smaller components (line 284). The use of fingers (line 287) is additional evidence that this time Betty calculated and did not guess, probably the reason for her success. During the lessons, even in the face of the angry avatars, Betty constantly displayed highly enthusiastic behavior while next to the computer. This behavior contrasted with her attitude towards the initial group discussions or the teacher's requests to work on sheets (from lesson 3 onwards) frustrated her. She was observed as impatient, did not take responsibility for her own work, and often relied on the teacher's support. The teacher focused her support on Betty's needs.

2) Turning point: A great change occurred in lesson 5 during a discussion about different ways to solve 50-14.80= (Table I, task E). Betty initiated her participation by asking the teacher to show her own strategy to calculate by writing it on the board:

52.	Betty:	So, I do 50 minus 10 and I do not calculate it now because I do not have the strength. [Writes: 50-10=____] it equals something...
53.	Eve:	Don't you know how much is it?
54.	Betty:	It's…ah…
55.	Teacher:	Sh…….[teacher silence Eve]
56.	Betty:	Thank you, just a moment. [Writes 40 as an answer]. Now 40 minus 4 equals…[writes 40-4=___]
57.	Betty:	Waits…
58.	Betty:	And then I do…
59.	Teacher:	How much do you get here [points at 40-4=___]? How much does it equal?
60.	Betty:	40 minus 4…emmmm….how you call it

		[writes 36]
61.	Teacher:	Great!
62.	Betty:	And then 36 minus…wait not minus…no…
63.	Teacher:	Yes, yes! You are in the correct direction! What is left?
64.	Betty:	Minus 0.8 equals something [writes 36-0.8=__ and turns to the group]
65.	Teacher:	[Puts her hand on Betty's shoulder, turning her back to the board] And how do you solve this?
66.	Betty:	I take from a Shekel…
67.	Teacher:	From a Shekel, great!
68.	Betty:	[Draws circles to represent coins] one, two. I have no power for that…to draw.
69.	Teacher:	OK
70.	Betty:	From a Shekel 8 like this.
71.	Teacher:	Eighty Agorot.
72.	Betty:	So, it comes out 35 plus something [writes 35]
73.	Teacher:	And what is that something? You took a Shekel…
74.	Betty:	[Answers immediately] plus twenty!
75.	Teacher:	Very good, very good! 35 and 20 Agorot because in a Shekel we have 100 Agorot.
76.	Betty:	I am a genius!
77.	Teacher:	Excellent, well done!

Betty's difficulties are manifested by her avoidance of calculating basic facts. Writing on the board helped her to think about intermediate calculations (lines 52, 56, 64, 72). She preferred to focus on the procedure of the strategy using "___" as a place for the calculated result and the teacher accepted that, helping her in critical moments such as encouraging her by saying she is on the correct track (line 63), to focus her attention and avoid distractions (line 55, 65), or giving her hints by using the money terms in critical moments (lines 71, 73).

Most of lessons 6-7 were devoted to practicing next to the computer and solving worksheet tasks. Betty asked to play alone and the teacher let her. Lesson 8 was devoted to the final assessment where Betty succeeded (Table I, tasks J, K).

Overall, Betty's learning process illustrates the fragility of her knowledge, the inconsistency of her performance, her impulsivity in the computerized context, the clear delegitimization of this behavior by the teacher, combined with her support, which led to moments of success by using writing as a tool for self-direction.

VII. Discussion

Here we discuss the hypotheses and factors that influenced the students' learning within the rich environment.

A. *The Computerized Context as a Double-edged Sword*

The computer-peer setting was found to be both supportive and destructive in terms of students' learning. The computer played a major role in making students active and engaged in mathematical discussions about the current subtraction task with their peers and the teacher, despite the students' fragile knowledge. We saw moments of synergy [16] when the presence of peers induced a reflection about a wrong calculation, and a discussion about the strategy applied. Teachers' interventions in this context were welcomed and fruitful. However, as Betty's case study illustrates, there were also situations in which the computer *game* encouraged trial and error because of the time factor and the competitive nature of games. The teacher, in her attempt to disrupt the trial-and-error discourse, often had to compete with students as exemplified above.

B. *Group Discusions: Participation Alongside Silence*

We expected that the group discussions would encourage students to talk about mathematics and therefore, foster their ability and wiliness to provide explanations. The findings indicate that the teachers' requests for explanations, especially the question about which strategy they chose to apply for solving the tasks, were alien. This is probably because in regular math classrooms LAS are rarely asked to explain their answers. Therefore, the group discussions did not yield the rich discussions we had hoped for. Nonetheless, we observed that the ability of most students to provide explanations had developed during their participation. However, these students did not develop new strategies, but rather, used the strategies they had been introduced to at the beginning. This behavior aligns with the LAS's tendency to focus on a given algorithm, given by an external authority. In addition, in this context, as demonstrated in the discussion routine (Fig. 4), students' discursive acts were mostly in response to the teacher and merely addressed her.

C. *Movement within the Rich Environment: Evidence of Diffusion*

In our design we had expected a metaphorical diffusion between the two contexts in which students performed and collaborated—that students' activeness, ability, and wiliness to discuss with their peers when failing to solve a task on the computer would diffuse to the group discussion and that the teacher-led discussions would enrich the mathematical discursive practices, which would then diffuse to the computer context. Apparently, this diffusion is not straightforward and a fine-tuned design is required to support its occurrence.

D. *RME: A Valuable Factor*

In line with other empirical studies [17], RME was found to be a valuable factor in facilitating LAS meaningful learning. Students adapted the real-life money model to resolve the subtraction tasks, even when given in an abstract form. This was evident in their formulation of their solution process in monetary terms as well as in the conceptual scaffold "1 NIS equals 100 agorot", which they often used when they had to reason how they subtracted the decimal.

VIII. Conclusion And Future Work

A. *Implications for the second round*

As we had hypothesized, we found that distributed scaffoldings were beneficial to LAS [15]. The premise was that such a heterogenic population, from the cognitive and behavioral aspects, needs a variety of tools. Indeed, the environment simulated a rich "playground" where students experience diverse tools in order to build and develop new knowledge.

Obviously, more work was required to fine tune the design, in order to better support students' learning. Utilizing the insights gained from our analysis, in the next round we re-designed the group discussions in consultation with the literature on Accountable Talk [25], aiming at better facilitation of establishing the norms of mathematical peer discussions. We minimized the time spent in front of the computer game and instead, added time to the online unit, in which students still simulated the ice cream shop, but without the pressure of time and gaining points. Finally, we aimed at setting the students' mindset right from the beginning by explaining to them that this class is about *their* strategies. We omitted the introduction to the two strategies, and instead, simulated in class an affair where students brought personal items and had to give money and get change and then conducted a discussion on their calculation strategies.

B. *The next round in a nutshell*

Analyzing the rich data of 11 students in the second round, we found vivid mathematical discussions. Discussants were accountable to the group (i.e., engage in talk that builds on the ideas of others), according to the accepted norms of reasoning (i.e., talk that emphasizes logical connections and the drawing of reasonable conclusions using mostly the money model for justifications), and to knowledge (i.e., talk that is based explicitly on facts). While in the first round the teachers' requests for explanations were alien and sometime led to silence among students, in the second round the use of AT practices [25] allow the teacher to explicitly set clear expectations for reasoning and then to proactively support, diagnose, and analyze the development of students' mathematical reasoning.

Consequently, nine of the 11 participating LAS showed evidence of positive change in their mathematical thinking and behavior as a result of their participation in the environment. Students constructed at least one meaningful subtraction strategy using it at a high success rate once it was constructed.

Although the majority of LAS exhibited evidence of meaningful learning of mathematics in constructing and using their own computation strategies, it highlighted the challenges their learning difficulties pose. Their learning processes were inconsistent characterized by progressions and regressions and therefore were difficult to predict by the teacher [26]. In rare cases when the teacher-students discourse reached impasse, the setting of peer work next to the computer on online units was found promoted due to its safe and constructive space for building new knowledge.

Future work is still required. A larger sample of participants is necessary in order to generalize and further explore LAS learning processes and outcomes in this environment and to gain insights as to how to support their learning. Indeed, as shown above, the encounter of LAS with a rich environment might introduce complexity and pose challenges to teachers, who already have to deal with many factors (such as the fragility of students' knowledge, the inconsistency of students' performance, the impulsivity within the computerized context and the silent situations in group discussions). Nonetheless, this study shows that overall, the rich CSCL environment was successful. Not only had students gained meaningful mathematical knowledge, such as strategies to solve subtraction tasks—they also practiced socio-mathematical behavior that differed from what they were used to: they moved from passive reliance on authority, as well as impulsive and individualistic interactions in class, towards active, thoughtful collaboration about mathematical meaning. According to the regular class teachers, to some extent, this behavior has diffused to their regular classes. Thus, we can conclude that meaningful learning of LAS is feasible and furthermore, that LAS can benefit from CSCL settings, which stands in contrast to their characteristics in the literature as passive or even detached individualists [2].

We believe that a rich CSCL environment, involving a computer game, real context mathematics, peer discussions, and teacher mediation may be the key for promoting LAS's learning. In this respect, our work makes a modest step towards achieving equity in mathematics education by extending the teaching of mathematical meaning to academically diverse students.

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