Transient Analysis of a Single-stage Vapor Compression Refrigeration System Using Lumped Parameter Approaches

Analysis and simulation validation based on a reduced order differential equation with few degrees of freedom

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***Abstract*—Refrigeration and air conditioning systems need to have enough capacity to maintain the desired temperature at a worst-case, design load operating condition. In this paper, a dynamic analysis of a single-stage vapor-compression refrigeration system is presented. The model is constructed by applying the lumped parameter approach to each component of the refrigeration system; the first low of thermodynamic is applied to individual components to determine the mass and energy balances; then, a linear dynamical system is obtained. The model is implemented by MATLAB and simulation results are given for comparison with real values. The results of the simulation match with the manufacturer's specifications.**

Keywords-Heat exchangers; Refrigerants; Dynamic Model; Household refrigeration; Transient conditions; Control volume.

I. INTRODUCTION

Refrigeration and air conditioning are an active and fleet developing technologies. These devices are closely related to the living standard of people and to the outdoor environment, due to ozone depletion and global warming.

Mathematical modeling is the most practical way of studying the basic behavior of cooling cycle performance, the relative losses in various components and their interactions. Standard science and engineering formulations are applied to describe mathematically the basics processes occurring in the Vapor Compression Refrigeration (VCR) systems. Mathematical modeling is a step towards simulation optimization [1], [2].

Dynamic models are often classified using such terms as white box, gray box, or black box. The term white-box models refer to physics based models that are described using physical laws, such as conservation equations. These models also appear in the literature as mechanistic models or first principles models [1], [3].

At the other extreme, black-box models refer to empirical or data-driven models, where transient experimental data is used to identify a dynamic model. This process is also known as system identification or time-series analysis, and it can be used to construct models in the time or frequency domain. In black-box model one tries to estimate the functional form of relations between variables and the numerical parameters with no need of detailed information about the components of the system [1], [3]. Examples of empirical models include regression analysis, polynomial curve fits and artificial neural networks.

The bulk of modeling efforts for VCR systems are most appropriately termed as gray-box, due to they are largely based on the governing physics but including semi-empirical terms, such as efficiency maps, heat transfer correlations, etcetera, that come out from experimental test. Physics-based modeling paradigms include;

- lumped parameter approaches that capture the gross pressure and cooling transients qualitatively,
- moving boundary approaches, which model the dynamic variations in phase transition points, and

- finite control volume approaches, which use discretized models including temperature and parameter gradients, in an effort to achieve greater accuracy [1], [3].

TABLE I. NOMENCLATURE

Symbol	*Description*	*Units SI*
$c_{p,m}$	*Specific heat of material*	*J/kg-K*
c_{pi}	*Specific heat of refrigerant*	*J/kg-K*
C_{Ti}	*Thermal capacity*	*J/K*
g	*Gravity*	m/s^2
h	*Refrigerant enthalpy*	*J/kg*
m_r	*Mass flow rate*	*Kg/s*
n	*Politropic coefficient*	
Q_1	*Evaporator heat rate*	*W*
Q_3	*Codenser heat rate*	*W*
Q_{P1}	*Heat transfer rate to the surrounding in the evaporator*	*W*
Q_{P2}	*Heat transfer rate to the surroundings in the compressor*	*W*
Q_{P3}	*Heat transfer rate to the surroundings in the condenser*	*W*
Q_{P4}	*Heat transfer rate to the surroundings in the expansion device*	*W*
P	*Pressure*	*Bar*
T	*Temperature*	*K*
T_a	*Ambient temperature*	*K*
T_1	*Evaporator temperature*	*K*
T_2	*Compressor temperature*	*K*
T_3	*Condenser temperature*	*K*
T_4	*Expansion valve temperature*	*K*
R_{Ti}	*Thermal resistance*	*K/W*
T_R, W_C	*Compressor power input*	*W*
ω	*Angular velocity*	*rad/s*
V_c	*Volumetric displacement of compressor*	m^3
η_p	*Volumetric efficiency of compressor*	
υ	*Specific volume*	m^3/kg
u	*Specific internal energy*	*J/kg*
z	*Height*	*m*

The dynamic modeling of VCR systems has been subject on interest since the late 1970s, where first principle models were used to describe the heat exchangers. Lumped parameter and moving boundary models are shown in [4]-[6]. In [7], MacArthur initiated a series of works focusing on a distributed parameter formulation. Nonlinear models have shown good approximation [8], [9], [10], but the complexity level increases. Later on, in 1990s, traditional feedback control has been investigated [11], as well as the multivariable control strategy [9], [10], [12], [13]. Other strategies have been developed to select among the degrees of freedom of the control variables, so that an optimal operation is closely obtained [14], [15].

Due to thermal dynamics of VCR systems are typically slower than the mechanical dynamics, the model complexity generally resides in the heat exchangers. Previous literature reviews [16], [17] indicate that most of research efforts are focused on capturing two-phase flow dynamics in the heat exchangers, seeking a balance between simplicity and fidelity. For the purpose of this paper, the four elements of the thermal system will be classified into lumped parameter models. Lumped parameter models refer to models that apply lumped parameter assumptions to the entire heat exchanger or to fluid phases within the heat exchanger (i.e., Individual lumped models for superheated vapor, two-phase fluid, and subcooled liquid), and the result is a set of algebraic and first-order ordinary differential equations to render a simpler model computationally. In this paper, the term “lumped parameter model” means that each heat exchanger is modeled as a single-control volume (or multiple control volumes for each fluid phase). Most of the literature in the lumped parameter classification are early efforts [3], [17], carrying out a computational simplicity to ensure feasible calculation times. These modeling efforts use few dynamic equations and few (lumped) parameters.

Most of analytical models are used to simulate steady-state performance, but leaving out the transient evolution. In this paper, a method for predicting the cooling performance of a VCR system during transient and steady-state is presented. The dynamic model proposed in this paper is similar as in [18], an advantage in this analysis is because the refrigeration system is simplified only four control volumes.

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The paper is structured as follows. In Section II, a review of the refrigeration cycle principles is shown. In Section III, an explanation of the linearized model of the VCR system is presented. In Section IV, the balance of mass and energy of each element of the system, as well as the four ordinary differentials equations modelled in Matlab Simulink, are presented. In Section V, the validation of the model is shown. In Section VI, conclusions are described.

II. REFRIGERATION SYSTEMS

Vapor-compression refrigerating systems used in modern refrigerators vary considerably in capacity and complexity, depending on the application. They are hermetically sealed and normally require no replenishment of refrigerant or oil during its useful life. System components must provide optimum overall performance and reliability at minimum cost. In addition, all safety requirements of the appropriate safety standard (i.e., IEC Standard 60335-2-24 [19]; UL Standard 250 [20]) must be accomplished. The fully halogenated refrigerant R-12 was used in household refrigerators for many years. However, because of its strong ozone depletion property, appliance manufacturers have replaced R-12 with environmentally acceptable R134a or isobutene [21].

Design of refrigerating systems for refrigerators and freezers has been improved through new refrigerants and oils, wider use of aluminum, and smaller and more efficient motors, fans, and compressors. These refinements have kept the vapor-compression system in the best competitive position for household application.

A. Refrigerating circuit

A VCR system, in its simplest form, consists of two heat exchangers, an expansion valve, and a compressor, Fig. 1. The ideal VCR system consists of four processes:

- Isentropic compression,
- Isobaric heat rejection and condensation,
- Isenthalpic expansion, and
- Isobaric heat absorption and evaporation.

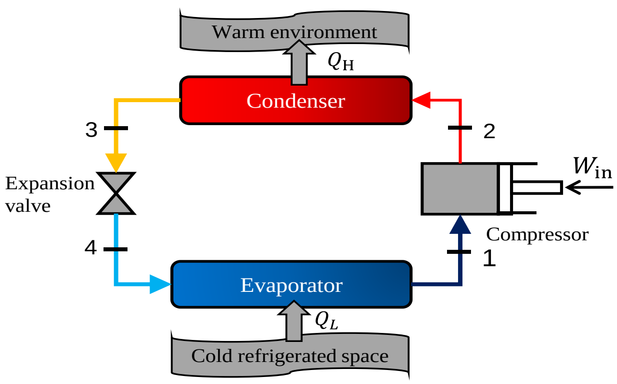

Figure 1. Single stage ideal vapor compression cycle: schematic diagram [21].

Fig. 2 shows the refrigeration cycle on p-h diagram. The refrigerant evaporates entirely in the evaporator and produces the refrigerating effect. Then, it is extracted by the compressor at state point 1, compressor suction, and is compressed isentropically from state point 1 to 2. Next, it is condensed to liquid in the condenser, and the latent heat of condensation is rejected to the heat sink.

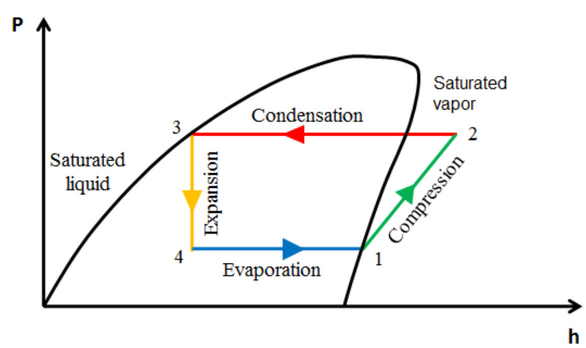

Figure 2. Diagram pressure (p) vs enthalpy (h) [22].

The liquid refrigerant, at state point 3, flows through the expansion valve, which reduces it to the evaporating pressure. In the ideal vapor compressor cycle, the throttling process at the expansion valve is the only irreversible process, usually indicated by a dotted line. Some of the liquid flashes into vapor and enters the evaporator at state point 4. The remaining liquid portion evaporates at the evaporating temperature, thus completing the cycle [21].

Note that energy enters the system through the evaporator (heat load) and through the compressor (electrical input). Thermal energy is rejected to the environment by the condenser and compressor shell. A portion of the capillarity tube is usually soldered to the suction line to form a heat exchanger. Cooling the refrigerant in the capillary tube with the suction gas increases the capacity and efficiency.

A strainer-drier is usually placed ahead of the capillary tube to remove foreign material and moisture. Refrigerant charges of 150 g or less are common. A thermostat (or cold control) cycles the compressor to provide the desired temperatures in the refrigerator. During the off cycle, the capillary tube allows pressures to equalize throughout the system [21], [22].

III. ANALYSIS OF THE LINEARIZED MODEL

Fig. 3 summarizes the thermodynamic model of commercial VCR system. The internal temperature of the cooler is a function of the angular speed of the compressor motor and the high temperature of the coolant, which can be obtained from the condenser model. It can be seen the system has several degrees of freedom represented by: the speed of the compressor, the difference of the expansion valve, and the temperature differences of the compressor and the evaporator. The steam compression cooling system can be studied by a system of multiple input and single output states (MISO).

Due to the number of degrees of freedom at the entrance and the singular existence of the exit, the MISO control systems are highly effective; however, the technological limitations of the devices implemented in the VCR systems are responsible for the algorithms developed for these systems to be of the single input and single output (SISO).

This is because commercial devices only apply the control action in the form of activating and disabling a constant speed compressor, since both the expansion valve area differential, the temperature differentials in the condenser and in the evaporator are all almost constant.

In Fig. 3 the input signal can be a change in the angular speed of the compressor motor, and with regard to the control of these devices, several control algorithms have been tested for the internal temperature [2], [23]. The best temperature regulation results are obtained from controls where the thermodynamic model has been simplified but retaining its non-linear nature [2], [23]. If the compressor motor speeds are pre-calculated to keep the internal temperature constant in a permanent state, basic control algorithms such as the PID have been applied to maintain the speed of the compressor, whose dynamics are modeled as a second order system [24].

The technology developed and applied in most vapor compression systems have constant speed compressors, and therefore, the control of the temperature inside the cooler is carried out through on-off control actions [25]. Also, there are some applications of on-off controls adjusted according to the thermodynamic model defined in Fig. 3 [26], [27].

These controls have been developed to be able to propose algorithms for the optimization of energy consumption, because is the main objective to cover for most research groups working in the area of cooling systems.

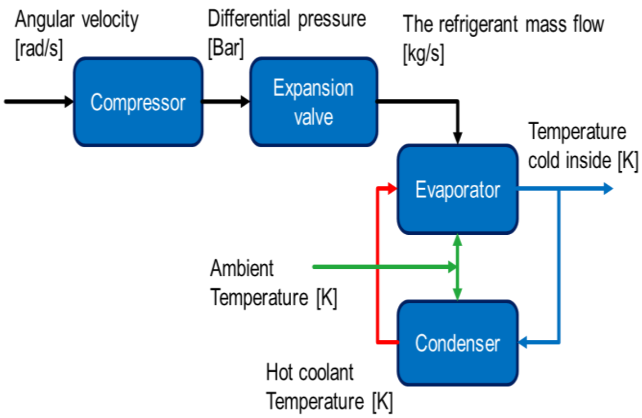

Figure 3. Thermodynamic model of commercial coolers based on vapor compression [2].

Considering the dynamic response of commercial coolers based on vapor compression, various specialists have decided to simplify the dynamic model based on the property of retain heat and the thermal resistance of its barriers [2], [28], [29]. These models consider the evaporator element as a source of heat, which can extract or inject energy from the thermodynamic system. Thus, *C* is considered as the thermal capacity of the volume inside the cooler and, *R* is heat resistance of its barriers. The linearized mathematical model is established in (1).

$$Q(t) = C\frac{dT_i}{dt} + [T_i - T_e]/R \tag{1}$$

Q(t) is the heat injected or absorbed by the source, in this case the evaporator. T_i and T_e are considered internal and external temperatures, respectively. The model is easily recognizable as a first-order dynamic system and the internal temperature depends on the motor compressor angular speed, as shown in Fig. 4.

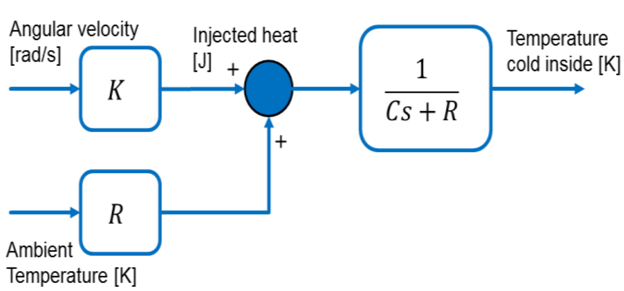

Figure 4. Linearized model dependig on the speed of the compressor [1].

In the same approach of the open-loop model, there are several control strategies implemented to regulate the internal temperature, by applying a control law to manipulate the compressor speed.

Such strategies are tuned considering the parameters of the linearized model [30], [31]. Similar analysis is employed to define the conditions of the hysteresis [32], [33].

The typical tools to model dynamics of the vapor compressor systems are Neural Networks [34], [35], fuzzy logic [36], and genetic algorithms [37], which are heuristic algorithms.

Artificial intelligence techniques, such as AAN (artificial neural networks), fuzzy theory and expert system, belong to non-model method. They do not need mathematical models but have high adaptability. The artificial intelligence technique was used to predict the performance of refrigeration and air conditioning appliances [38], [39], [40], [41]. But, the unsolvable problem in using such a method is because of the imperfection of the artificial intelligence technique itself and the limitation of the user's understanding.

The conventional mathematical model method has been theoretically studied and practically applied for many years. With the mathematical model is more likely to ensure the qualitative precision of simulation than the intelligent method. It is a good way to combine the conventional mathematical method with the intelligent method together in order to take the advantages and to avoid the shortages of both methods. When the modern artificial intelligence techniques are combined with mathematical models of refrigeration systems, called as model based-intelligent simulation [42], the simulation software has certain "intelligence" for simulating the actual complex objectives and becomes more practical.

With the model-based intelligent simulation method, the predicted result of the model can well fit the experimental data as its empirical coefficients can be adapted by an artificial intelligence module. The training task of the artificial intelligence module will be reduced, and the training speed can be accelerated if the calculated results by the theoretical model are used as the initial or prior assumed values for the artificial intelligence module. The adjustment process of the empirical coefficients in the mathematical model can be converted into the training process of the artificial intelligence module, and can be executed by the computer itself.

In this way, less or even no artificial adjustment is needed in the simulation, and self-learning, self-adjusting and self-adapting function can be realized. On the other hand, the number of input parameters and the dimension of the artificial intelligence module will be decreased since many important parameters including configuration parameters are already included in the mathematical model.

Those complicated, empirical and even uncertain factors can be incorporated into the artificial intelligence module and so the mathematical model can be simplified [42].

Fig. 5 shows the simulation process of volumetric efficiency with compound fuzzy model.

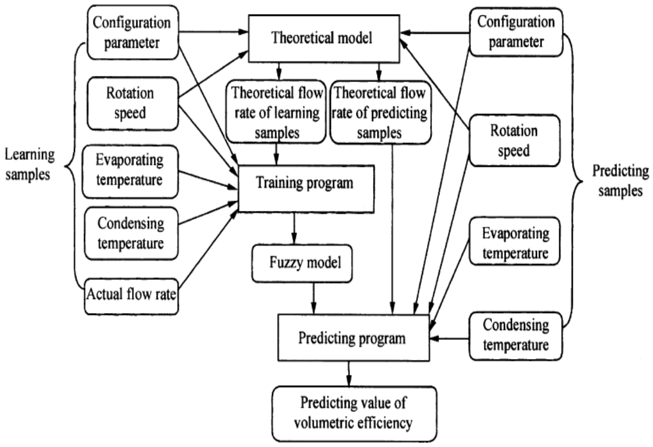

Figure 5. Fuzzy simulation for volumetric effiency of refrigeration compressor [43].

The disadvantage of these heuristic models is that internal variables do not have any physical interpretation, although they have an effective estimation of internal temperature. The non-linearity influences the performance of the linearized model described above.

IV. Dynamic Model of the VCR System

Henceforth, the refrigeration cycle of a reciprocating refrigerating system is considered a closed cycle, and the system is operated in steady state (i.e., in an equilibrium state).

A. Energy Conservation Law

Therefore, according to the principle of continuity of mass and energy balance, the mass flow rates of refrigerant flowing through the evaporator, compressor, condenser and expansion (float) valve must all be equal. In addition, the total amount of energy supplied to the refrigeration system must be approximately equal to the total energy rejected from the system.

The flow is continuous, and the properties of the refrigerant at any point in the system do not vary over time. Therefore, during the design of a refrigeration system, the system components selected should have approximately equal mass flow rates of refrigerant at steady conditions [22], [44].

For the general case of multiple mass flows with uniform properties in and out of the system, the energy balance can be written (2):

$$\dot{Q} + \dot{W} = \dot{m}\left[\left(h_o - h_i\right) + \frac{v_o^2 - v_i^2}{2} + g\left(z_o - z_i\right)\right] \tag{2}$$

In (2), $\dot{Q}$ is transferred to the system by the surrounding heat flow, $\dot{W}$ is the work performed by the electric motor of the compressor, h is enthalpy function that is associated with the sum of the internal energy and the work flow, $u + Pv$, the linear kinetic energy $v^2/2$ and potential energy is caused by attractive forces existing between molecules, or the elevation of the system gz. The subscripts i and o refer to the initial and final states, respectively. In the absence of appreciable variations of kinetic and potential energy, the equation above reduces to:

$$\dot{Q} + \dot{W} + \dot{Q}_{Pi} = \dot{m}_r c_p\left(T_o - T_i\right) \tag{3}$$

This relationship is based on the consideration that all variables along the finite volume (control volume) are homogeneous. Equation (3) applies to each volume of control of the four stages of the refrigeration cycle. In (3), $\dot{m}_r$ is the mass of the refrigerant in circulation, c_{pi} is the specific heat of the refrigerant at room temperature (considered constant), see Table II. Q_{Pi} represents the heat loss or heat generated by the system, and corresponds to the Newton's law of cooling, dT/dt is the cooling speed. For this analysis, the refrigerant temperature change is as shown in [45].

$$Q_{Pi} = C_{Ti}\frac{dT_i}{dt} + \frac{\left(T_i - T_o\right)}{R_{Ti}} \tag{4}$$

In the volume control 1, C_{Ti} is the thermal capacitance to the interior of the space confined to the evaporator element and for other elements is the capacitance of the material, expressed as $C_{Ti} = mc_{p,m}$. Thus, m is the mass of the elements and $c_{p,m}$ is the heat capacity of the material, T_i is the temperature inside the system, and T_a is the room temperature. The enthalpy of the model is considered for an ideal gas in (5).

$$\Delta h = c_p(T_o - T_i) \tag{5}$$

Fig. 6 shows the physical quantities involving the system analyzed.

The following simplifications are considered:

- The physical properties related to the refrigerant are considered uniform in the heat exchanger transversal section.
- The refrigerant liquid and vapor phases are in thermodynamic equilibrium.
- The heat exchangers have a perfect thermal insulation.
- The axial heat conduction in the pipes is ignored.

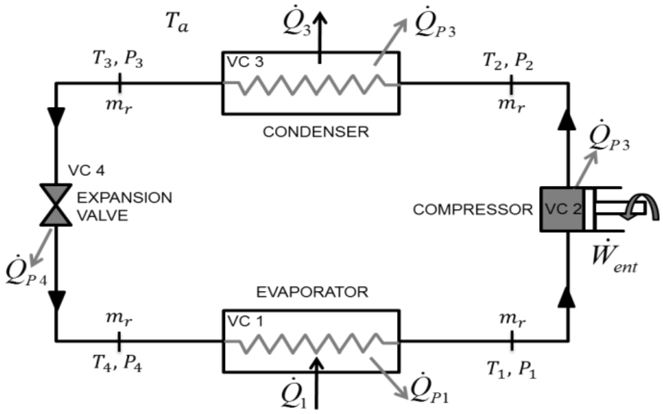

Figure 6. The Dynamic Model Analyzed Scheme [1].

B. Evaporator Mathematical Model

The term $\dot{Q}$ is considered as the heat removed by the refrigerant from the confined space, which is also known as thermal load. Considering the work $\dot{W} = 0$ in equation (3) arises:

$$\dot{Q}_1 + \dot{Q}_{Pi} = \dot{m}_r c_p (T_1 - T_4) \tag{6}$$

Substituting (4) in (6), provides the following equation:

$$\frac{dT_1}{dt} = T_1\left(\frac{c_p \dot{m}_r}{C_{T1}} - \frac{1}{C_{T1}R_{T1}}\right) - T_4 \frac{c_p \dot{m}_r}{C_{T1}} + \frac{T_a}{C_{T1}R_{T1}} - \frac{\dot{Q}_1}{C_{T1}} \tag{7}$$

C. Mathematical Model of Compressor

Motor shaft dynamics are modeled from an angular momentum balance between the driving and braking torques. The torque-speed characteristics of the motor are obtained from manufacturer's specifications. The driving torque and speed of the compressor are known.

The following hypothesis is established; there is no friction in the compressor and is an adiabatic process $\dot{Q}_1 = 0$. The compressor equation is as follows [18].

$$\dot{W}_C = \frac{n}{(n-1)} \eta_p V_c \omega_c p_1 \left[1 - \left(\frac{p_2}{p_3}\right)^{\frac{n-1}{n}}\right] \tag{8}$$

The torque provided by the motor of the compressor $\dot{W}_C = \dot{T}_R$ and ω is the angular velocity of the shaft. Using equation (3),

$$\dot{W}_R + \dot{Q}_{P2} = \dot{m}_r c_p (T_2 - T_1) \tag{9}$$

Substituting (4) in (9), the resulting balance of energy is:

$$\frac{dT_2}{dt} = -T_1 \frac{c_p \dot{m}_r}{C_{T2}} + T_2\left(\frac{c_p \dot{m}_r}{C_{T2}} - \frac{1}{C_{T2}R_{T2}}\right) + \frac{T_a}{C_{T2}R_{T2}} - \frac{\dot{W}_1}{C_{T2}} \tag{10}$$

D. Mathematical Model of Condenser

Only the superheat vapor is considered. The vapor phase is considered in the thermal equilibrium and moving in the same velocity. Work is not performed in this element $\dot{W} = 0$. $\dot{Q} = 0$ it is the heat released into the environment. From equation (3), it is obtained the following,

$$\dot{Q}_3 + \dot{Q}_{P3} = \dot{m}_r c_p (T_3 - T_2) \tag{11}$$

Substituting (4) into (11), you get the following:

$$\frac{dT_3}{dt} = -T_2 \frac{c_p \dot{m}_r}{C_{T3}} + T_3\left(\frac{c_p \dot{m}_r}{C_{T3}} - \frac{1}{C_{T3}R_{T3}}\right) + \frac{T_a}{C_{T3}R_{T3}} - \frac{\dot{Q}_3}{C_{tT3}} \tag{12}$$

E. Mathematical Model of the Expansion Valve

It is considered no interaction of work and heat $\dot{W} = 0$ and $\dot{Q} = 0$. The expansion is isenthalpic. Thus, from equation (3), it is obtained the following,

$$\dot{Q}_{P4} = \dot{m}_r c_p (T_3 - T_4) \tag{13}$$

Substituting (4) into (13), the balance of energy gives:

$$\frac{dT_4}{dt} = -T_3 \frac{c_p \dot{m}_r}{C_{T4}} + T_4\left(\frac{c_p \dot{m}_r}{C_{T4}} - \frac{1}{C_{T4}R_{T4}}\right) + \frac{T_a}{C_{T4}R_{T4}} \tag{14}$$

F. State-space Representation of VCR System

Previously, equations (7), (10), (12) and (14) have been presented as first-order linear ordinary differential equations, with the temperature of the refrigerant as the system output. The following expressions are expressed in matrix form [2], [46]:

$$\begin{aligned} \dot{T}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) + Du(t) \end{aligned} \tag{15}$$

Equations (7), (10), (12) and (14) must be replaced in (15):

$$\begin{bmatrix} \dot{T}_1 \\ \dot{T}_2 \\ \dot{T}_3 \\ \dot{T}_4 \end{bmatrix} = \begin{bmatrix} \left(\frac{c_p \dot{m}_r}{C_{T1}} - \frac{1}{C_{T1}R_{T1}}\right) & 0 & 0 & -\frac{c_p \dot{m}_r}{C_{T1}} \\ -\left(\frac{c_p \dot{m}_r}{C_{T2}}\right) & \left(\frac{c_p \dot{m}_r}{C_{T2}} - \frac{1}{C_{T2}R_{T2}}\right) & 0 & 0 \\ 0 & -\left(\frac{c_p \dot{m}_r}{C_{T3}}\right) & \left(\frac{c_p \dot{m}_r}{C_{T3}} - \frac{1}{C_{T3}R_{T3}}\right) & 0 \\ 0 & 0 & -\frac{c_p \dot{m}_r}{C_{T4}} & \left(\frac{c_p \dot{m}_r}{C_{T4}} - \frac{1}{C_{T4}R_{T4}}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix}$$

$$+ \begin{bmatrix} \frac{1}{C_{T1}R_{T1}} & -\frac{1}{C_{T1}} & 0 & 0 \\ \frac{1}{C_{T2}R_{T2}} & 0 & -\frac{1}{C_{T2}} & 0 \\ \frac{1}{C_{T3}R_{T3}} & 0 & 0 & -\frac{1}{C_{T3}} \\ \frac{1}{C_{T4}R_{T4}} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_a \\ \dot{Q}_1 \\ \dot{T}_R \\ \dot{Q}_3 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \end{bmatrix} + 0$$

G. Numerical prediction

The mathematical model obtained in the previous section for the prediction of the dynamic behavior, was programmed in Matlab Simulink. The typical values of input parameters are presented in Table II. The input parameters include refrigerant type, environmental temperature and the initial temperature of the compartment, etcetera.

The values of c_{pi} of the refrigerant were defined according the average of the phase in relation to p vs h diagram.

H. Modeling

Considering the thermodynamic cycle starts-up when the system is powered by the angular speed of the electric motor of the compressor, the flow of the refrigerant quickly tends to a steady state. It can be noted that the refrigerant tends to decrease its temperature at the evaporator element, trying to keep the relationship of equilibrium of pressure and temperature. Fig. 7 shows the behavior of the refrigerant in the evaporator [47].

TABLE II. Input Values of MATLAB Algorithm

Value	**Units**	**Value**	**Units**
C_{p1} = 1330	J/kg-K	R_{T1} = 0.090	K/W
C_{p2} = 1400	J/kg-K	R_{T2} = 0.025	K/W
C_{p3} = 1138	J/kg-K	R_{T3} = 0.048	K/W
C_{p4} = 1318	J/kg-K	R_{T4} = 3.20	K/W
C_{T1} = 4500	J/K	Q_1 = 195	W
C_{T2} = 2500	J/K	Q_3 = -200	W
C_{T3} = 1250	J/K	T_a = 298	K
C_{T4} = 500	J/K	T_R = 5	W
m_r = 0.000035	kg/s		

The behavior of the refrigerant in the compressor where the temperature and pressure increase, the curve tends to rise starting from room temperature as shown in Fig. 8.

The refrigerant follows its course towards the condenser, where the heat extracted from the confined spaces towards the environment that surrounds it is released, we can see that the coolant temperature is approximately the same, receiving compressor discharge line as it is located in the area of high p and t, Fig. 9 [2], [47].

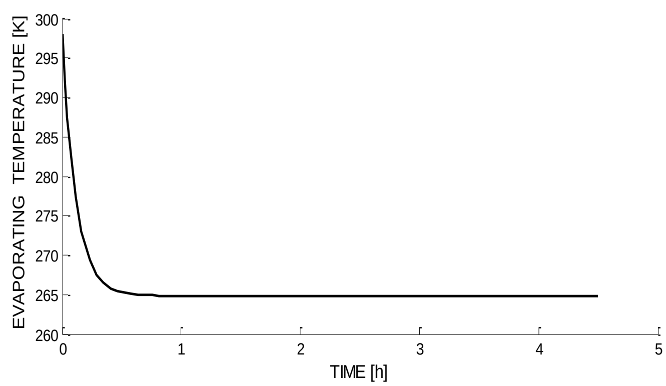

Figure 7. Evaporation temperature of the refrigerant.

In the capillary tube or expansion valve, temperature and refrigerant pressure decreases due to strangulation, to a lower temperature so that heat transfer can be done appropriately. Fig. 10 shows the temperature of the refrigerant which tends to decrease from ambient temperature.

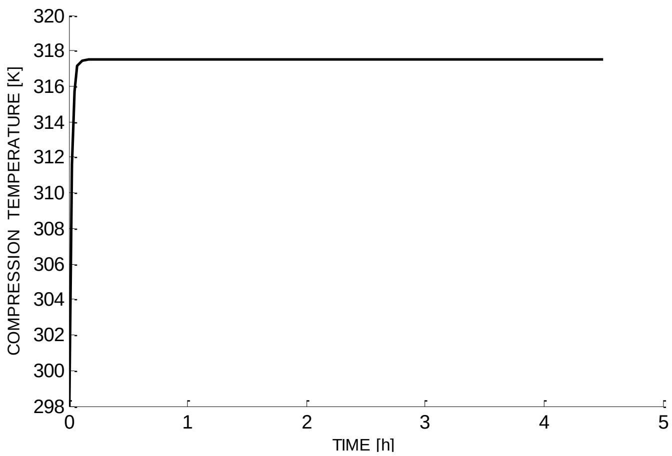

Figure 8. Behavior of the refrigerant in the compression process.

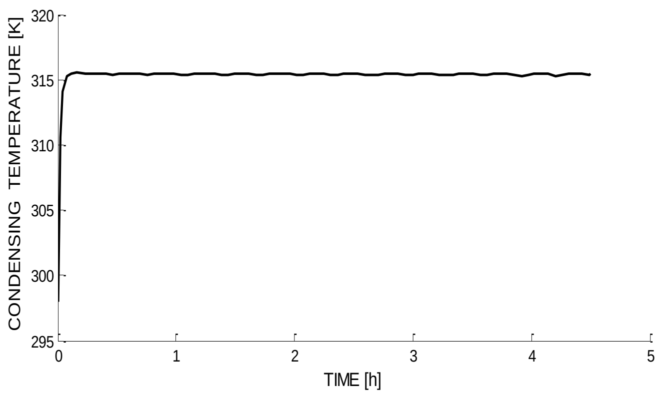

Figure 9. Behavior of the temperature in the condensing process.

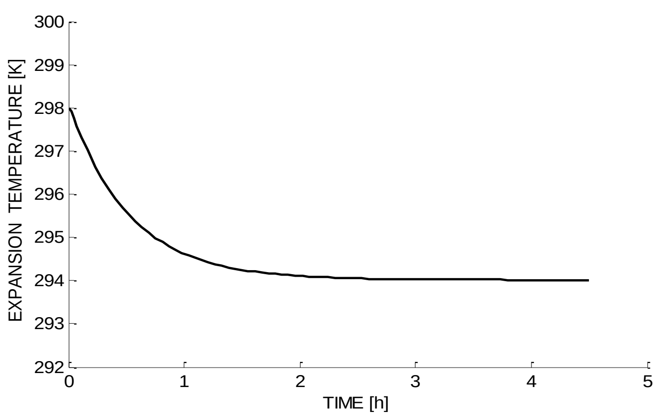

Figure 10. Behavior of the temperature in the strangulation process.

In order to have a frame of reference with which to validate the results obtained from the predicted model are compared with the manufacturer's specification of refrigerant R134a.

V. Validation of the Modeling

In this Section, the estimated model is validated with specifications of the refrigerant R134a, since it is the fluid that is most often used in domestic refrigerators.

The following figures show the dynamic evolution of the refrigeration cycle of two critical variables during 4.5 hours in the heat exchangers: temperature and pressure starting from environmental temperature; then, it approaches the steady state.

Fig. 11 shows the temperature evolution in the compartment of the refrigerator. The model is compared with Embraco Data [47], using R-143a. It can be observed that takes about 10 minutes in the transient state, then the room temperature reaches the steady state condition.

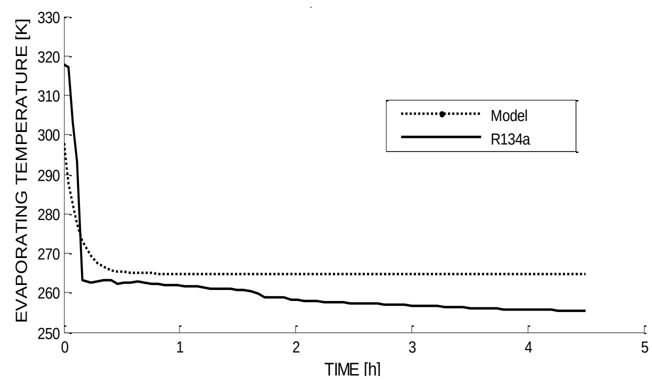

Figure 11. Behavior of the temperature of the evaporation system.

Fig. 12 shows the pressure evolution in the compartment of the refrigerator. It can be observed the same approach when it starts from environmental temperature related to the manufacturer's refrigerant. Fig. 12 shows that the pressure reaches the steady state condition, which replicate the process 4 to 1 in the *p* vs *h* diagram, in Section II.

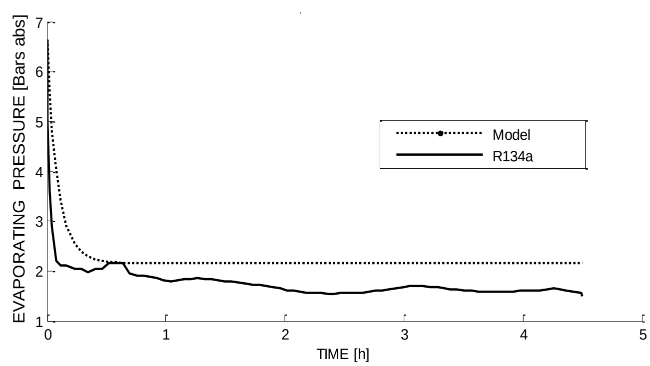

Figure 12. Behavior of the pressure of the evaporation system.

Fig. 13 shows the behavior of the refrigerant temperature in the condenser element. The temperature of R134 reaches 335 K in the first 5 minutes and the estimated model only reaches 315 K. This value is acceptable since it is above the environment and there is a difference greater than 7 K that guarantees the transfer of energy to the surroundings.

The temperature of the estimated model is enough to guarantee the operation of vapor compression equipment that works with natural convection. If a refrigerator is considered to work with forced convection, this reached value guarantees the release of enough energy for the refrigerant to change of phase from gaseous state to liquid state.

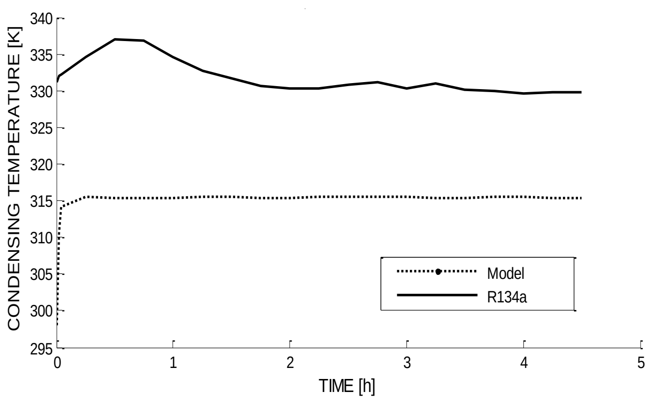

Figure 13. Behavior of the condenser temperature.

Fig. 14 shows the behavior of the refrigerant pressure. The fluid maintains its relationship T and p to reach condensation temperature above the environment and must also maintain a high pressure, 17 bars of the R134a.

For the estimated model, the pressure of 10.5 bars is reached. In the case of a domestic refrigerator we observe that the value reached in the pressure compliance for optimal performance.

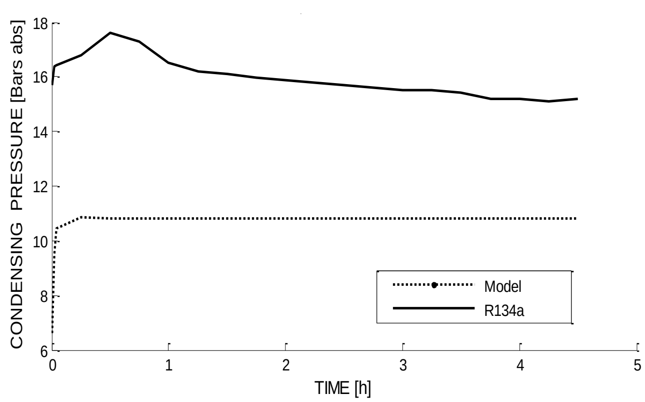

Figure 14. Behavior of the pressure in the condenser.

A good agreement between the estimated model and measure values from [22], [47] was observed for the whole period. In the evaporator, during the first 10 minutes, the pressure decreases from 6.5 bars to 2.2 bars, then tends to decrease slowly until reach the steady state operation.

The behavior of the evolution of the cooling curve are similar, although there is a difference in terms of temperatures, however, are considered within the working area in the diagram pressure-enthalpy [22].

VI. Conclusion and Future Work

In this work, a methodology to model the dynamic behavior of the refrigerator has been developed. This would act as basis for improvements on modeling domestic refrigerators, using lumped transient model. The lumped parameter modeling will reduce the overall cycle time used to predict the compartment temperature, decreasing the experimental effort.

The results agree qualitatively with temperature time evolution shown in the literature. The transient analysis of domestic refrigerators, using the computer program developed in this paper, will be further validated with experimental testing. An advantage of the simplified model here described is the possibility of using personal computer due to the relatively low computational effort required for the calculations.

A valid starting point for the study of cooling systems applying state variables has been introduced in this paper. The first improvement strategy is a state feedback, since the model considers a linear approximation; however, there are nonlinearities, so a parameters identification algorithm and adaptive control strategies are required.

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