Modeling of Activities as Multivariable Problems in Smart Homes
Case Study – Recognition of Simultaneous Activities

Farzad Amirjavid
Department of mathematics and computer science, UQAC
Chicoutimi, Canada
farzad.amirjavid@uqac.ca

Abdenour Bouzouane
Department of mathematics and computer science, UQAC
Chicoutimi, Canada
abdenour_bouzouane@uqac.ca

Bruno Bouchard
Department of mathematics and computer science, UQAC
Chicoutimi, Canada
bruno_bouchard@uqac.ca

Abstract— **Resident of a smart home, who may be an Alzheimer patient needing permanent assistance behaves non-linearly to achieve the intended goals. He actuates the world by realizing actions, activities which can be observed through the embedded sensors of the smart home. To assist him automatically and live independently at home, it is needed to capture information and knowledge from world to reason if the world state is normal and to evaluate how much the intelligent system succeeds; therefore, for recognizing the activities and their correct realizations, we propose to consider the activities as a sort of fuzzy temporal concepts that can be formalized as a multivariable function. Perceiving the world, an Activity Recognition System makes hypotheses and concepts about the observations. These hypotheses are resumed in a smoothing line and at the recognition time, the activities functions check how much the observations are close to their smoothing line. Finally, the activities are ranked based on the inferred similarities to the observations. All the introduced processes are data-driven and a case study that deals with recognition of simultaneous activities based on the proposed modeling approach is presented.**

Keywords- fuzzy logic; temporal data mining; smoothing; activity recognition

I. INTRODUCTION

Recently proposed works on activity recognition show effective but *unreliable* results. They are still dependent on the expert's knowledge in both learning and recognition steps; on one hand they presume activities are realized in ambient environment, but on the other hand they recognize each activity by consideration of only a few especial attributes. Therefore, they do not propose a totalitarian supervisor system that is capable to reason in all of the possible events that may occur every time and everywhere of the ambient environment. The result is that they cannot verify correct realization of activities.

One more major reason that made these approaches impractical is that their reasoning system is not flexible enough to handle existing uncertainty in input data; especially they are not capable to distinguish for what context, which inputs may play more important roles in activity recognition. In other words, they expect that the activities are performed in standard and rigid structures in order to be recognized.

In order to contribute in activity recognition, in the current work, we propose an approach that not only deals with data-driven activity recognition, but also proposes how to recognize correct realization of activities. Furthermore, we propose an extension of an event-driven approach, which is published in [2] and [9]. In that approach, we formalized an activity as a dynamic entity that can be recognized through recognitions of the fuzzy events caused by the activity realization (see Figure 1).

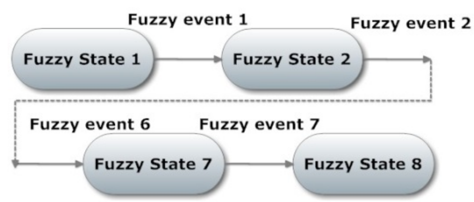

Figure 1. Activity of "coffee making" modeled as chain of fuzzy events

In order to perform a data-driven process to discover the fuzzy-events, we proposed to divide the world into two general parts, one of them representing the static characteristics of the activities (fuzzy context [12]) and the other representing the characteristics that dynamically change while the activities are realized (activities fuzzy states [13]). As a consequence, it is proposed to perform classification process to group the common fuzzy states of all activities in order to provide shortly all the learnt knowledge in a decision tree format. By this modeling approach, we could estimate the intention of the resident and predict the events that may occur in the future when a few elementary actions of a known plan or activity are seen [9].

The mentioned approach includes some limitations that are the subject of the current work: (i) The reasoning in recognition of activities can be done only when an action is performed in the world, so it does not reason in normality of the current momentum observations. Therefore, we desire the ability to do real-time reasoning. (ii) If more than one activity is realized (simultaneous activities), it does not recognize these correct activities as the normal world states. Moreover, interruption of activities cannot be surveyed.

In this paper, we propose to consider the activities as fuzzy temporal concepts that can be formalized as

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multivariable mathematical problems such as the following equation:

$$y = \alpha_1 \tilde{x}_1^{\beta_1} + \alpha_2 \tilde{x}_2^{\beta_2} + \ldots + \alpha_n \tilde{x}_n^{\beta_n} \qquad (1)$$

Here, “y” represents the activity function, “x” represents the variable that activity depends on, and “α” and “β” are the variables’ factors in activity model. In order to achieve this model, we calculate the fuzzy contexts of the activities, then, the hypotheses around the observations taken from the activities realizations are generated and formalized as the fuzzy states of the activity. Each fuzzy state is represented with a fuzzy cluster center. In the next step, by performing smoothing techniques, all the mentioned fuzzy cluster centers are traversed through a line or curve in order to collect all possible activity states so that we calculate the activity’s function (“y”).

The reasoning for activity recognition will be done based on the discovered similarity between the observation and the activity formula. Then, the activities are ranked based on the inferred similarities to the observations. For instance, in Table I, we have illustrated twenty observations from a typical activity through six sensors (activity’s variables), in which one of them is time and the other ones are the distances of the objects to special points in the environments.

TABLE I. OBSERVATION OF THE SIX WORLD ATTRIBUTES IN TWENTY STAGES (SYNTHETIC DATA)

observation number	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5	Variable 6
1	1	1	1	10	12	10
2	2	1	2	9	11	10
3	3	1	1	4	10	10
4	4	4	2	10	9	10
5	5	4	7	19	8	10
6	6	4	8	7	8	10
7	7	7	7	7	9	10
8	8	7	8	5	10	10
9	9	7	12	17	11	10
10	10	9	13	18	12	10
11	11	9	12	20	12	10
12	12	9	14	18	11	10
13	13	20	20	2	10	10
14	14	20	19	5	9	10
15	15	20	18	2	8	10
16	16	16	19	19	11	10
17	17	16	15	14	10	10
18	18	16	14	12	9	10
19	19	12	15	5	9	10
20	20	12	13	1	8	10

In order to break the world observations into two groups of context and activity states, we apply the subtractive clustering method [3]. The cluster estimation process is performed based on the similarities discovered between the data points. In Table II, we illustrated how the world is perceived. We presented several hypotheses in order to explain the observations if different cluster sizes (Influence Range) are desired. For example, one hypothesis is that activity transits four fuzzy states (coded as 3-1, 3-2, 3-3, 3-4 in Table II) and the sixth variable indicates the fuzzy context of this activity (symbolized by © in Table II).

TABLE II. SOME POSSIBLE HYPOTHESES AROUND THE OBSERVATIONS

Fuzzy State Number	Variable 1	Variable 2	Variable 3	Variable 4	Variable 5	Variable 6	Influence Range
1_1	8.0 ©	7.0 ©	8.0 ©	5.0 ©	10.0 ©	10.0 ©	IR=2
2_1	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=1.5
2_2	17.0	16.0	15.0	14.0	10.0	10.0 ©	IR=1.5
3_1	12.0	9.0	14.0	18.0	11.0	10.0 ©	IR=1
3_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=1
3_3	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=1
3_4	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=1
4_1	12.0	9.0	14.0	18.0	11.0	10.0 ©	IR=0.9
4_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=0.9
4_3	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=0.9
4_4	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=0.9
4_5	20.0	12.0	13.0	1.0	8.0	10.0 ©	IR=0.9
5_1	12.0	9.0	14.0	18.0	11.0	10.0 ©	IR=0.8
5_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=0.8
5_3	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=0.8
5_4	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=0.8
5_5	20.0	12.0	13.0	1.0	8.0	10.0 ©	IR=0.8
5_6	5.0	4.0	7.0	19.0	8.0	10.0 ©	IR=0.8
6_1	10.0	9.0	13.0	18.0	12.0	10.0 ©	IR=0.7
6_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=0.7
6_3	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=0.7
6_4	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=0.7
6_5	17.0	16.0	15.0	14.0	10.0	10.0 ©	IR=0.7
6_6	20.0	12.0	13.0	1.0	8.0	10.0 ©	IR=0.7
6_7	5.0	4.0	7.0	19.0	8.0	10.0 ©	IR=0.7
7_1	10.0	9.0	13.0	18.0	12.0	10.0 ©	IR=0.5
7_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=0.5
7_3	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=0.5
7_4	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=0.5
7_5	17.0	16.0	15.0	14.0	10.0	10.0 ©	IR=0.5
7_6	20.0	12.0	13.0	1.0	8.0	10.0 ©	IR=0.5
7_7	5.0	4.0	7.0	19.0	8.0	10.0 ©	IR=0.5
7_8	16.0	16.0	19.0	19.0	11.0	10.0 ©	IR=0.5
7_9	19.0	12.0	15.0	5.0	9.0	10.0 ©	IR=0.5
7_10	3.0	1.0	1.0	4.0	10.0	10.0 ©	IR=0.5
7_11	4.0	4.0	2.0	10.0	9.0	10.0 ©	IR=0.5
8_1	10.0	9.0	13.0	18.0	12.0	10.0 ©	IR=0.4
8_2	7.0	7.0	7.0	7.0	9.0	10.0 ©	IR=0.4
8_3	2.0	1.0	2.0	9.0	11.0	10.0 ©	IR=0.4
8_4	14.0	20.0	19.0	5.0	9.0	10.0 ©	IR=0.4
8_5	17.0	16.0	15.0	14.0	10.0	10.0 ©	IR=0.4
8_6	20.0	12.0	13.0	1.0	8.0	10.0 ©	IR=0.4
8_7	5.0	4.0	7.0	19.0	8.0	10.0 ©	IR=0.4
8_8	12.0	9.0	14.0	18.0	11.0	10.0 ©	IR=0.4
8_9	16.0	16.0	19.0	19.0	11.0	10.0 ©	IR=0.4
8_10	4.0	4.0	2.0	10.0	9.0	10.0 ©	IR=0.4
8_11	19.0	12.0	15.0	5.0	9.0	10.0 ©	IR=0.4
8_12	3.0	1.0	1.0	4.0	10.0	10.0 ©	IR=0.4
8_13	13.0	20.0	20.0	2.0	10.0	10.0 ©	IR=0.4
8_14	15.0	20.0	18.0	2.0	8.0	10.0 ©	IR=0.4
8_15	1.0	1.0	1.0	10.0	12.0	10.0 ©	IR=0.4
8_16	6.0	4.0	8.0	7.0	8.0	10.0 ©	IR=0.4
8_17	18.0	16.0	14.0	12.0	9.0	10.0 ©	IR=0.4
8_18	8.0	7.0	8.0	5.0	10.0	10.0 ©	IR=0.4

Each cluster can be represented by a cluster center. Therefore, instead of direct consideration of the observations in modeling process, we can take the explanative hypotheses (cluster centers). At the next step, we will calculate a function that represents the behavior of the sensors (variables) during the activity’s realization through a smoothing process. Finally, a function representing the activities characteristics is estimated.

The paper is organized as follows. Section II presents some preliminary theories about the proposed approach and justifies the choice of multivariable learning. Section III describes the formalizations and definitions concerning this framework which serves as foundation for modeling the recognition process. Section IV explains the process to estimate the activity function as an important step for recognition. Section V presents our method to survey the simultaneous activities by using the estimation of the activity function model. In Section VI, a case study is presented in order to show experimental results and validation of the proposed approach. Finally, Section VII concludes the paper and outlines the future developments of this work.

II. ACTIVITY AS A FUZZY CONCEPTUAL SYSTEM

In this paper, we regard the "activity" as a type of *fuzzy dynamic conceptual system*. In fact, it is presumed that an intelligent system directs realization of the activity concepts in the virtual world of temporal datasets. In order to explain better this viewpoint we refer to the system theory [14], where, a system is defined as a set of interrelated objects that collaborate together in order to achieve a goal. A system has a boundary with its environment. It takes input from its environment, it processes it, it gives output to its environment, and it directs this output according to the taken feedback from the environment. For a system, we can imagine machine states and a hierarchy of subsystems. Here, the term "*conceptual system*" refers to a system that is composed of non-physical entities, i.e., ideas or concepts and concept is an abstract idea or a mental symbol, typically associated with a corresponding representation in language or symbology [15]. In conclusion, a conceptual system is simply a conceptual model [16].

An activity is a conceptual system because it respects the systems' specifications: (i) it consists of a set of interrelated variables which represent the world attributes, especially, the object's locations; (ii) it is realized to achieve a goal (especially the world state); (iii) it has a boundary with its environment, which is defined through fuzzy state and fuzzy context; (iv) it takes input from its environment by performing observation, it processes it, and it gives output to its environment by accomplishing an action in order to change the world attribute. For an activity we can define fuzzy states [13] and a hierarchy of concepts such as actions [4]. An activity is a dynamic conceptual system because its state depends on time [16] and since fuzzy set operators are applied to model it, an activity is regarded as a sort of *fuzzy dynamic conceptual system*.

A. *Fuzzy contexts of the activities*

Contexts are the surrounding conditions where activities are realized [12]. The fuzzy context refers to a set of variables in which they would keep a stable interrelation while the activities are realized. At the recognition time, any change in the context is interpreted as abnormality of the world state. For example, if a human wakes up at 6 o'clock, then it indicates a normal world state for the activity recognition problem, but if he wakes up at 2 PM, then it can be inferred that he is sick and there is an anomaly.

One other benefit of the consideration of contexts is that it helps with the identification of similar activities. When similar activities are performed in different contexts, they represent different concepts and in this way we can distinguish these different concepts. For example, if a human eats in the morning, it means that he is having his "breakfast", but the same activity (eating) at 12 o'clock means that he is having "lunch". In the next part we will deal with formalization of the fuzzy context.

In our view, context is a fuzzy term and it can be applied to multi-variable problems such as ambient environments, where multiple features of scenarios are observed. In the real world problems, any sensor data may vary (even partially) while the activities are realized. Sometimes these changes should be taken into account because this variation could be significant, but sometimes they should not be taken into account because the variations of sensor data are not significant to recognize the activity. We apply a fuzzy logic based clustering approach in order to survey different levels of details of occurring events in different levels of certainty and survey the activity models in their own contexts. The benefit of fuzzy context estimation is that it reduces the process complexity and focuses the calculations of the modeling on the role-playing variables.

B. *Fuzzy states of the activities*

Fuzzy state represents a general and brief description about the current status of the world. When an activity is realized, the world would transit a chain of fuzzy states (see Table II). However, this transition would be done in a special fuzzy context. Each activity is regarded as a sequence of fuzzy states (see Figure 1). In fact, when an activity is performed, the world would transit a chain of fuzzy states and the system achieves its goal while the activities are realized.

An intelligent system is assigned to direct the realization of an activity. Then, the perception of fuzzy states and fuzzy contexts would indicate how to repeat realization of this activity. For this system, the fuzzy context represents the system environmental and external conditions for realization of the activity, but the fuzzy states represent the procedures or the actions that should be performed by the system in order to realize the activity. In fact, the fuzzy states represent the internal states of this system and the events that would occur inside this system (see Figure 2).

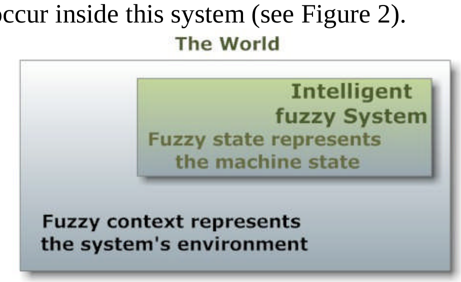

Figure 2. Fuzzy state and fuzzy context for realization of an activity

In Figure 2, we can see that although the types of the fuzzy contexts and fuzzy states are similar, the objective of their consideration is different. In fact, one represents the conditions for a scenario to be realized by an intelligent system, and the other represents the way that the scenario can be realized in the world. Therefore, it is presumed that in order to realize an activity the world is divided into two sections, which are system intern and system extern. The world features that should be unvaried during the activity realization are considered as the fuzzy context members and the attributes that would be varied (played) for activity realization would be taken into account as the fuzzy state elements. The boundary between a system intern and system extern is not a fixed, stable or definitive border. This logical boundary can dynamically change and new attributes can join the system environment (fuzzy context) after a few steps in activity completion.

III. FORMALIZATIONS

Here, we introduce a modeling process to learn the activities. The main function of activity modeling is to recognize them and to reason in the correct realization of the activities. Moreover, we would be able to judge if the world state is normal, or if the smart home resident needs assistance [10].

The "world" of the proposed learning problem is observed through a set of applied sensors "S". "s_i" represents sensor "i" from the set of applied sensors; "n" refers to the number of sensors or variables and "a" refers to a typical activity. Goal "G" is achieved when "a" is realized, so, in rather most of the cases, we can imagine that a goal achievement is equivalent to an activity realization. Presuming that the reality is the state of the world attributes as they actually are, the world is the collection of attributes that are observed from the world accompanying their observed values. In ambient environment the world is observed through a set of embedded sensors "S" where "s_i" observes the i'th attribute of the real world out of the "n" observed features. We refer to "s_i" at time "t" by "$v_{i,t}$"; "T" is the number of times that observation is done.

Definition 3.1 (*world*). The virtual world is formalized as set $World = \{(s_i, t, v_{i,t}) \mid (s_i, t, v_{i,t}) \in S \times \Re \oplus \{0,1\} \times T, \; |S| = n, v_{i,t} = val(s_i, t)\}$

where "s_i" is the i'th sensor of the observing sensor, the *val()* function captures the value of the sensor "s_i" at time $t \in T$. The observations are done through "n" sensors for "T" times. RFID sensors or generally, any kind or sensors that generate any amount of values $v \in \Re$ or the ones that generate 0-1 values are the data types that are accepted.

Because activities are realized and observed in an ambient environment, we expect the observed world is affected by no event occurring out of the ambient environment. Moreover, we presume that all possible world states are observed within "T" observations. Therefore, in this paper, the Activity Recognition Reasoning System (ARRS) supposes the world is closed or in other words it benefits from the Closed World Assumption (CWA), which is a presumption that what is not currently known to be true is false [17]. So, it is presumed that if no explanation for an observation is found, then we infer that the world is abnormal or an erroneous activity is realized.

Definition 3.2 (*momentum observation*). This is the set of digitized numeric values taken from each sensor and registered into a temporal dataset, which indicates the world quality at once. In other words, an observation is a line of data-record inside of a temporal dataset which registers frequently the measured world qualities. It is presumed that per observation, the world attributes are observed simultaneously and synchronously, so that at a record of observation we can find all the measured attributes refering to a unique world state. For example, we will not have the world temperature at time 12:00 with the world light at time 13:00 within a single data record. That is to say every attribute of an observation refers to a single reality or each attribute explains a property of a single object (observation)

A temporal dataset would indicate a set of observations. If the observations concern an activity realization then a temporal dataset consisting from multi-attribute observations is formed. We represent the set of observations in a matrix format, where the consisting elements are the values that indicate the quantified world's qualities. It may be sorted by a world attribute such as time:

$$O_{S,G} = \begin{bmatrix} v_{11} & \cdots & v_{1n} \\ \vdots & \ddots & \vdots \\ v_{T1} & \cdots & v_{Tn} \end{bmatrix}$$

Definition 3.3 (*variable state*). Each variable of the observation matrix represents a value concerning an observed world attribute or an activity feature. This value is given to the variable according to the measurement that a sensor does from the concerning attribute.

For example, in Table I, in each column we can see twenty states per each variable. Here, a set of variable state is defined as the values that a sensor generates. The important point in here is that the definition of a variable or sensor state is dependent on the time, so by elapse of time new data records are created and new variables states can be created. Considering the role of time, we face two groups of variable states. The first group of states refers to the values that in several moments the variable stops and causes a relatively ***stable*** state within that value. For the other group, there are also ***transition*** states that demonstrate transition of a variable from a stable state to other stable states. In order to distinguish these two *transition* and *stable* states in a data-driven manner, we consider the time factor, so when a variable stays at a definitive value for a *relatively long* time, then this value is a sensor (variable) state and if a variable stays *relatively short* on a definitive value, then it is a

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transition state. In this paper, by the world state we refer to the stable state $State_i = \{v_{ti} \in O_a : \exists \mathcal{E} \in [1,T],\ i \in [1,n] \rightarrow v_{t-\mathcal{E}i} = v_{ti} = v_{t+\mathcal{E}i}\}$.

Here, "Ɛ" is the minimum delay for stay of a value in order to be recognized as a stable state. In the following parts of this paper when we talk about variable state, we refer to the stable variable state.

For each action that a smart home resident performs (in order to complete realization of an activity), the state of one or more sensors may be actuated, so their monitoring digital numeric value may be changed. Therefore, the accomplishment of simple actions in smart home is mapped as a *time series* of events in the temporal datasets. During the realization of activities, we can see each sensor stay *temporary* or *permanently* at definitive values. This stability at definitive values causes consideration of a world state.

Definition 3.4 (*world state*). Each record of the observation matrix represents a schema concerning an observed world. We define the variable state as a set $world\ state = \{V_{i,t} \mid V_{i,t} \in O_{S,G}.t\ and\ \vee i \rightarrow V_{i,t} = V_{i,t+1} = V_{i,t+2} = ... = V_{i,t+\varepsilon}\}$ in which "t" refers to the time of observation and indicates all observed attributes at time "t".

World state represents a short estimation from the world quality. The world quality is estimated depending on the observed attributes. If more attributes from the world are observed, then a better estimation from the world state is provided and the difference between similar world states is better distinguishable. For example, in Table II, considering the world is observed frequently, each record of the gathered temporal data represents an instantaneous estimation from the world quality. In other words, each record of the observation matrix represents a momentum world state at the concerning moment.

The important point here is that the definition of world state is dependent on the time, so by the lapse of time new data records are created and new world states can be created. Considering the role of time, we face two main groups of world states. The first sort of states refers to the moments that entire variables stop in their old values and cause a relatively ***stable*** world state. There are also ***transition*** states that demonstrate transition of a world state from a stable state to other stable states. In order to distinguish these two *transition* and *stable* states in a data-driven manner, we consider the time factor, so when the world stays at a definitive sets of values for a *relatively long* time then it causes a world state and if the world stays *relatively short* on a definitive sets of values then it is a transition state.

In the following parts of this paper when we talk about "world state" or "home state", we refer to the stable world state. With the exception of the expression of the world state per *moment of observation*, there are several other ways to express the world states; we can view the world regarding other variables' states in order to estimate the world state. For example, we can view the world represented in Table II by this expression:

"The world state in which the distance of sugar to RFID antenna 2 is high and the distance of glass to the RFID antenna 2 is high."

In this statement, we pointed out to the second and fourth record of the observation matrix. Therefore, world state is a record or a group of records from the observation matrix which are subjected to variable limitations.

Definition 3.5 (*cluster center*). It is a set of observations representing groups of observations that are similar to each other $CC_{i,t} = subtract(O_{i,t})$, where "$CC_{i,t}$" are the cluster centers that represent their own group of similar data points.

In temporal subtractive clustering process, the cluster centers are discovered based on two parameters: (i) the first one is a cluster of similar observations of a single sensor. For each cluster (containing similar data points), a cluster center is discovered in order to represent its concerning cluster members; (ii) the second one is a cluster of similar momentum observations, which are a row in "$O_{i,t}$". For each cluster (containing similar observations), a n-dimensional cluster center is discovered in order to represent its concerning cluster members. The symbol "n" represents the number of observing sensors or the variables: {1<i<n}. For detailed information about the cluster center estimation using subtractive clustering approach, which is not the main focus of this work, please refer to [3]. Through the process of cluster center estimation we can calculate the fuzzy sensors state and fuzzy world state.

Definition 3.6 (*fuzzy sensor state*). It is a set of observations concerning a single sensor, which represent a group of similar observations series $\tilde{S}_i = \{(\tilde{v}_i, \tilde{t}) \in O_{S,G} \times T,\ s_i \in S,\ \tilde{v}_i = CC_{it}.v_i,\ \tilde{t} = CC_{it}.t, 0 \leq IR \leq 1\}$, in which a sensor state is simply indicated as a couple combined from value and time representing the cluster members.

One important point here is that the selection of different influence ranges in cluster center estimation would lead to different interpretations from the sensor observations; so, different data points with different quantity of the cluster centers are proposed as the sensor states. In order to take this parameter into account and point to a special sensor state, we would have:

$$\tilde{S}_{a,i,IR,k} = \{(\tilde{V}_i, \tilde{t}) \mid (\tilde{V}_i, \tilde{t}) \in O_{S,G=a},$$
$$i = 1,2,...,n, s_i \in S, \tilde{V}_i = CC_{i,t}.V_i,$$
$$\tilde{t} = CC_{i,t}.t, 0 \le IR \le 1, k \in \mathbb{N}, 1 \le k \le T\}$$

In here, "a" refers to the activity that is realized to achieve the goal "G", IR is the desired detail/generality from the data and "k" points to k'th cluster center ordered by $\tilde{t}$. "IR" is the influence range or the cluster radius rate, which defines the clusters' sizes; therefore, the cluster centers would represent similar data points, where the similarity criterion is the cluster radius. Influence range factor is a relative factor and it depends on the range of the data points. It should be mentioned that depending on the cluster radius, different cluster centers may be discovered. Therefore, for each temporal dataset, different sets of data records representing the total dataset may be discovered.

In the next part we will discuss that combinations of fuzzy sensors' states would lead to creation of fuzzy world states.

Definition 3.7 (*fuzzy world state*). It is a set of observations concerning all applied sensors, which represents one group of similar observations. It represents an approximate evaluation of a world state and during this state the world is seen as stable. Fuzzy state is formed from groups of similar world states. It is calculated as a result of comparison process between all of the world states and it is indicated to other world states.

The formalization of the fuzzy state is presented in the following:

$$FS_{a,IR,k} = \{(s_i, \tilde{v}_i, \tilde{t}) \mid i = 1,2,..,n,$$
$$s_i \in S, \tilde{V}_i \in O_{S,G=a}, \tilde{V}_i = CC_{i,t}.V_i,$$
$$\tilde{t} = CC_{i,t}.t, 0 \le IR \le 1, 1 \le k \le T\}$$

where "a" refers to the followed activity; "IR" refers to the range of influence or the relative similarity degree, and "k" refers to the k'th (out of "T" possible fuzzy classes) data point that absorbs similar data points around at the influence range of IR; "k" also represents the number of fuzzy states that are transited, so that activity "a" is realized. A fuzzy state may include (subsume) one or more rows of the $O_{S,G}$ matrix. For example, on the data of the Table I we can apply fuzzy clustering process on the data points in order to extract the points (cluster centers) that the data is concentrated around them, so they represent different existing qualities of the data points, which are similar to the majority of the data points. The result of this process is demonstrated in Table II. In there, we have shown that if at cluster radius is selected as IR =0.7, then the world would be divided into seven fuzzy states. If at running time a relatively high similarity between the current observations and the learned fuzzy cluster centers is observed, then it can be inferred that the observations may belong to realization of the surveyed activity.

Definition 3.8 (*fuzzy activity*). It is a set of observations concerning all applied sensors, which represents groups of similar observations. It represents an approximate evaluation from the fuzzy world states that are transited when an activity is realized.

The fuzzy activity is formalized as a set

$$FA_{a,IR} = \{(s_i, \tilde{v}_i, \tilde{t}) : s_i \in S,\ \tilde{v}_i = CC_{it}.v_i \in O_{S,G=a}, \tilde{t} = CC_{it}.t, 0 \le IR \le 1, t.count = m\},$$

where "a" refers to the followed activity; "IR" refers to the considered range of influence or the relative similarity degree, and "m" refers to the number of fuzzy states that the world is transited in realization of "a", so we have: $0 \le k \le m \le T$ and $FS_{a,IR,k} \subseteq FA_{a,IR}$.

In order to calculate the "$FA_{a,IR}$" in matrix format, we perform the subtractive clustering process on the observations matrix:

$$FA_{a,IR} = subtract(O_{S,G}) = subtract(O_{S,G}) = \begin{array}{c} \overbrace{\hphantom{\tilde{t}_1 \quad CC_{1,1} \quad . \quad . \quad . \quad CC_{n,m}}}^{s_i} \\ \begin{bmatrix} \tilde{t}_1 & CC_{1,1} & . & . & . & . \\ . & . & . & . & . & . \\ \tilde{t}_k & . & . & CC_{i,k} & . & . \\ . & . & . & . & . & . \\ \tilde{t}_m & . & . & . & . & CC_{n,m} \end{bmatrix} \end{array} \begin{array}{l} \\ \to FS_{a,IR,1} \\ \\ \to FS_{a,IR,k} \\ \\ \to FS_{a,IR,m} \end{array}$$

"$FA_{a,IR}$" represents the points (cluster centers) that each variable (data of a column) regarding to itself or regarding to other variables (data of other columns) would have a meaningful concentration around. Each row of the "$FA_{a,IR}$" matrix is in fact a fuzzy world state. In this definition, 'n' is the number of columns (variables or sensors), then it can be inferred that activity "a" would 'm' times change the world state to achieve the goal "G".

Definition 3.9 (*Fuzzy context*). Fuzzy context is referred to as "$\tilde{C}$" and it is the set of variables that do not play any significant role in both realization and recognition of the activity "a" such as $\tilde{C}_{a,IR} = \{(s_i, \tilde{v}_i) : s_i \in S, \tilde{v}_i \in O_{S,G=a}, \forall t \in [1,T] \to v_i \approx \tilde{v}_i\}$.

The variable "s_i" during the time of the activity "a" realization does not vary significantly and it is fixed to value "$\tilde{v}_i$"; this value is calculated through the cluster center discovery process. Fuzzy context indicates the surrounding circumstances that scenarios or activities are realized in. The fuzzy contexts of the activities indicate the conditions in which the activities models are valid. A change in the fuzzy context may cause invalidity of the system's perception of the activities; so, it will be taken into account as a new activity model. Therefore, any knowledge extracted from the observations is valid only if the similar context is met.

IV. ACTIVITY FUNCTION ESTIMATION

As it was mentioned earlier, one of our contributions in this paper is to propose a multivariable function in order to recognize it. The goal of this function is to recognize the activities and in order to do that, it transfers the observations to the model space, which is the activity space. This function is represented by "y_a", in which "a" refers to the surveyed activity. For instance, if we consider the positions of the "glass" and "sugar" objects in realization of the "coffee making" activity, then the "$y_{\text{coffee making}}$" transfers the observations of the concerning sensors to the activity space in order to verify how much it is similar to the coffee making activity (see Figure 3).

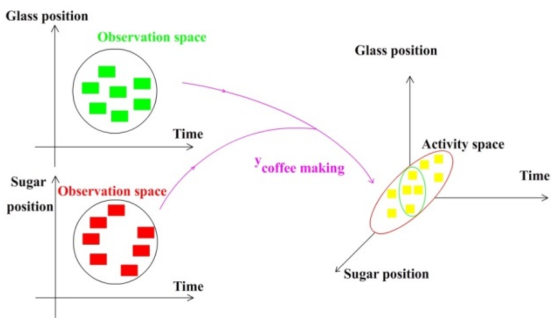

Figure 3. Activity function

The way that this function works is based on the discovered similarities between observations and the activity structure. Therefore, it is expected that this function reasons in both momentum observations and the series of observations to recognize the ongoing activity. In order to discover this function, we calculate equivalent of a curve or a line that traverses the activities' cluster centers. When an observation is recognized as similar to at least one of the activity's fuzzy state, then it can be inferred that it may possibly justify the realization of the activity. The sensors are the variables that the activity function depends on and based on their generated numerical values we can model or recognize the activities. Presuming that we intend to calculate a ***line*** that traverses the cluster centers of the Fuzzy Activity (FA), the resultant line is reported in the following equation:

$$y_a = \alpha_1 s_1 + \alpha_2 s_2 + \ldots + \alpha_n s_n + \alpha_0 \tag{2}$$

Here, "y_a" indicates the similarity degree to the activity model and "a" is the activity.

A. Sensor data smoothing

Dataset smoothing is a creation of an approximating function that attempts to capture important patterns in the data points [18]. The resultant smoothing line, traverses normally the data points otherwise it passes near with relatively closed distance. In the case that the smoothed values can be written as a linear transformation of the observed values, the smoothing operation is known as a linear smoother; the matrix representing the transformation is known as a "smoother" matrix or hat matrix.

There are several ways to smooth the data points and each of them can be customized according to the problem. Here, we suffice to introduce some famous smoothing methods, which are moving average, Local regression using weighted linear least squares with a polynomial model, Savitzky-Golay filter, and Kalman filtering [19]. Generally, these methods are different in the way they treat the existing noise of the data and in the linearity of the smoothing curve. For example, in Figure 4, it is graphically illustrated how a smoothing line resumes the observations of a sensor (location of the glass) using a linear smoother.

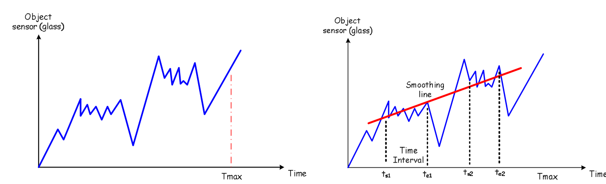

Figure 4. Smoothing of the sensors' observations

In Figure 4, we have presented that the sensors observations can be described by a line calculated by a linear smoothing technique such as linear regression [20].

B. Temporal behavior of the sensors

Behavior of the sensors in realization of the activities can be estimated and model through application of smoothing technique.

Definition 4.1 (*Sensor's linear Temporal Behavior*). Applying linear regression, we can calculate the sensors' data trend while the activity "a" is realized. Performing this process, the observations of the sensor "i" will be calculated in the following equation:

$$\overline{s}_{a,i,t} = \beta_{j,i} t + \tau_{j,i} \tag{3}$$

Here, $\overline{s}_{a,i,t}$ represents the average value of the "s_i" at time "t" in realization of activity "a". "β" and "τ" are the smoothing line factors calculated from the linear smoothing process obtained by the equation:

$$\beta_i = \frac{1}{T}\sum_{t=1}^{T} v_{i,t}.t, \;\; \tau_i = v_{i,t} - \beta_i \tag{4}$$

The calculations are done per each activity in the set of activities "A". The "T" is the activity duration. Because there are several smoothing methods and for each method different factors for data trend are proposed, from here, we refer to the "β" and "τ" by the term "smoothing factors".

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Each sensor during different activities may present different behavior. Therefore, by verifying the sensors' behaviors, we can find which activity is the most possible ongoing activity. Matrix containing all the sensors' smoothing factors in realizations of all activities is estimated by the equation:

$$\gamma = \left[\beta_{j,i}\right].s_i + \left[\tau_{j,i}\right] \tag{5}$$

The symbol "j" refers to the j'th activity out of "m" activities and "γ" represents the matrix of activities. In order to recognize an activity, the current (live) observations are compared to the "γ" and the activities are ranked from small to big deviations to the sensors' temporal behaviors in order to explain the current observations.

C. *Integrating temporal behaviors of the sensors in activities realizations*

Behavior of all sensors in realization of the activities can be integrated and modeled through a curve or line that traverses the fuzzy states of the activities (which are multidimensional data points).

Definition 4.2 (*Activities Linear Temporal Behavior*). It is the collection of data points representing the behavior of the sensors per each activity in smart home. This matrix is represented by "γ" and for the recognition objectives, the most similar behavior would be inferred as the most possible ongoing activity. This concept is formalized as set where "a_j" refers to the j'th activity of the set "A".

$$\gamma = \{x_j \mid x_j \in \overline{s}_{a_j,i,t}, a_j \in A,\ j = 1,2,...,m,$$
$$i = 1,2,...,n,\ t = 1,2,...,T\}$$

In order to apply the "γ" matrix for activity recognition, we would calculate the distance of the current (live) observations. Out of the "m" activities, the one which tells the most similarity in activity trend will be selected as the most possible ongoing activity. The activities may be sorted according to this criterion. In order to calculate the similarity distance between a typical data point ("$v_{i,t}$") and a sensor trend line, we would apply the following equation:

$$d_{a,i,t} = \frac{\left|(v_{i,t}.\overline{s}_{a,i,t}) - (t.\beta_{j,i}t) - \tau_{j,i}\right|}{\sqrt{(\beta_{j,i})^2 + (\tau_{j,i})^2}} \tag{6}$$

In this formula, "$d_{a,i,t}$" is the Euclidian distance of the point ("$v_{i,t}$") to the activity trend line and it represents the similarity measure of the mentioned point to the sensor's behavior in realization of the activity "a". Generally, each sensor can generate numerical values in range from its max to min. Therefore, max ("d_i") and min ("d_i") are the values that may be calculated manually or by the expert's idea. Therefore, we can normalize the similarities using this equation:

$$Nd_{a,i,t} = \frac{d_{a,i,t}}{\max(d_i)}, \tag{7}$$
$$\min(d_i) \le d_{a,i,t} \le \max(d_i)$$

In order to take similarity between the observation and the total activity trend, we may apply several methods in order to calculate the similarity degree. In here, because our objective is to demonstrate only a general schema from this process, we would suffice to the average similarity method given by the following equation:

$$\pi(v_{i,t} \in a) = \frac{\sum_{i=1}^{n} \frac{1}{Nd_{a,i,t}}}{n} \tag{8}$$

Here, the "y_a" refers to the possibility that "$v_{i,t}$" belongs to realization of the goal "a". In order to recognize the activities at a glance and to calculate the *linear activity multivariable function*, we can perform a multiple regression on the output and input of the activity's formula given by the following equation:

$$y_a = \mathit{regression}\ \left(\frac{\sum_{i=1}^{n} \frac{1}{Nd_{a,i,t}}}{n}, Nd_{a,i,t}.v_{i,t}\right) \tag{9}$$

The symbol "y_a" represents the activity function and it is resulted as the result of some linear operations on the observations.

In this section, we discussed that an activity may be recognized using linear statistical analysis methods. A big problem of the application of this method is that it does not help with the recognition of correct realization of an activity. If two or more activities are different in just a few actions (one or a few more sensors), they will not be recognizable because the calculation of their portion in similarity degree would not be noticeable,. Moreover, if an activity is realized fast or slowly, then it will cause noticeable differences in $\pi(v_{it} \in a)$. Experimentally, two different activities may cause very similar trends, so we cannot rely well on the results. The weight of every data point is equal; hence a noise may cause an undesirable high similarity degree. Finally, it can be mentioned that generally, calculation of the similarity degree would depend on three factors: quantity of the training data records or the time of activity realization, domain of the sensors' generated numerical values, and quantity of the observing sensors. In order to improve this method, we would perform the smoothing process on the fuzzy cluster centers of the activities. We calculate fuzzy states of the sensors and perform the smoothing on the cluster centers (see Figure 5). Some advantages of this process are that we would be able to eliminate the transition states and also eliminate the data points that point continuously to a repetitive point.

Moreover, the amount of the smoothing data points would be significantly decreased.

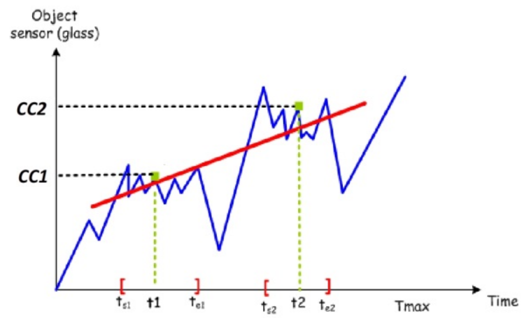

Figure 5. Smoothing fuzzy sensor states

In Figure 5, it is illustrated that the process of smoothing can be performed on the points that are the most similar to their neighbors. This way we can redirect the similarity degrees to the best representing data points rather than to probable noise or transition states; therefore, we can distinguish better the activities. In the next section we will discuss this issue.

V. SIMULTANEOUS ACTIVITIES

In order to realize an activity, several actions get accomplished simultaneously, which are directed by smart home resident. For each action, two more sensors may observe the consequences of the action; therefore, even a simple action can be interpreted as a set of simultaneous operations. For example, when an object is moved two sensors may observe this movement and two operations (in direction to each sensor) can result.

Using the set theory, we can combine or analyze the constituting elements of the concepts and as the result we can recognize high or low level concepts rather than individual activities. The application of this contribution is data-driven recognition of simultaneous activities and also lower-level concepts such as actions; therefore, we can reason for the activities interferences.

The recognition of simultaneous activities requires to analyze the fuzzy-world states in order to discover realization of which activities may possibly cause the world state. By using traditional set theory, we would apply the union (U) operation to find the constituting elements. When two singular activities (“a_k” and “a_u”) are realized together then we would have $FA_s=FA_{au}\cup FA_{ak}$ in which “FA_s” refers to the simultaneous activity. In here, we would treat the fuzzy entities (fuzzy clusters) with the traditional set theory. The result is analyze/combine of the learnt concepts. In order to analyze the “FA_s” based on the known activities we apply the set theory algebra, which indicates $FA_s=FA_{au}+FA_{ak}-(FA_{au}\cdot FA_{ak})$.

Therefore, “a_k” refers to the known activity and “a_u” refers to the unknown activity. A key point in here is that the fuzzy context of both activities should be respected in order for both activity models to be validated. Therefore, the resultant world state should be subjected to $\tilde{C}_{as}=\tilde{C}_{au}\cap\tilde{C}_{ak}$. .

Definition 5.1 (*simultaneous activities*). Simultaneous activities are the sequence of fuzzy events that follow achievement of two or more goals. Each fuzzy state concerning the realization of a social activity should be valid for both of the activity patterns. In other words, combination of individual activities is called simultaneous activities. The concept of “simultaneous activity” refers to the perception of the activities and the learned models from them. Regardless of the quantity of activity performers, when combination of two or more concepts is inferred, then a simultaneous activity is recognized. The simultaneous activity can be represented as $FA_s=FA_{au}\cup FA_{ak}$ subject to $\tilde{C}_{as}=\tilde{C}_{au}\cap\tilde{C}_{ak}$. Constraints of each of the running simultaneous activities should be realized in a space that satisfies all of the known goals. This space is made using fuzzy logic in a way that each activity saves its general structure and has partial flexibility if it faces the deviations from its previously learned structure.

VI. CASE STUDY

In order to validate the proposed approach, we implemented a series of activities in both singular and simultaneous manners in the LIARA laboratory [11]. More than 500 activity features were observed during realization of these activities. Singular realization of the coffee making and hand washing activity was modeled. The simultaneous realization of them was also modeled and we calculated the simultaneous realization of these singular realizations. We verified the similarity of these activities to their models and finally, in Figure 6 we have represented the inferred similarity in the range of [0-10].

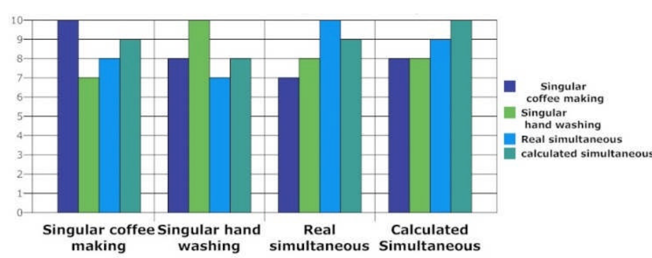

Figure 6. Similarity degrees in recognition of activities

In our experiments, we saw that the calculated simultaneous activity could approximately replace the real simultaneous model. Other experiments proposed in [10] confirmed reliable results, too. The important point here is that we could achieve this result through application of the activities multivariable function that could explain the activities’ dynamicity at a glance.

VII. CONCLUSION

In this paper, we discussed how to model the activities as multivariable problems and proposed to apply fuzzy logic, especially the fuzzy time concept in order to model the dynamicity of an activity in a mathematical function. This function draws a fuzzy space for realization of the activities. One benefit of this method is the modeling the simultaneous activities. On one hand, the uncertainty in human behavior is considered, and on the other hand, the imprecision of the sensors is handled. However, the current solution needs improvements. For example, the proposed approach learns the activity models at first. Then, it uses the learnt knowledge at the runtime. We did not propose an online learning technique because we cannot distinguish between anomaly and a new manner of correct activity realization. A possible solution for this limitation is to make a border and definition for tolerable anomalies, then any unfamiliar patterns, which do not cross that border could be taken into account as a new correct activity. This task would need new definition on normality.

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