Simulation hypotheses

A proposed taxonomy for the hypotheses used in a simulation model

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***Abstract*— Defining a simulation model implies the use of different knowledge of the system we are going to model. Often, this knowledge is not complete or lacks in the needed detail in order to fully explain the behavior and the structure of the system. In that case, different hypotheses must be used in order to constrain the reality, or allow the needed complete and unambiguous definition of the model. However, all the hypotheses used do not lie in the same category. In this paper, we propose taxonomy for the hypotheses used in a simulation model in order to detect, previously to any implementation or model definition the possible lacks in the model construction.**

Keywords-simulation hypotheses; validation; verification; formal languages.

I. DEFINING A SIMULATION MODEL

Building a simulation model is an iterative process where usually different personnel are involved. This process starts always describing the system we want to represent, and what are the key elements that must be taken in consideration in order to define the model. Once we have the definition of what is "my system" we can go further to describe what is the problem we try to solve. As is stated by Professor George Box, "*all models are wrong, but some models are useful.*" It is interesting to keep this present in order to assure that we are performing a good validation process of the model as we can see next. Our model, although maybe is not correct, must be useful to our purposes, a good cite of this can be found on [1]: "*A simulation model should always be developed for a particular set of objectives. In fact, a model that is valid for one objective may not be for another.*"

Regarding to the process of Validation, Verification and Accreditation (VV&A) of a simulation model, the phases are based on the definition of "my system", the conceptual model, and the implementation of the model. Sargent [2] proposes the Figure 1 as the cycle that a simulation project follows until its completion. As we can see in Figure 1, data validation is assumed to be a central point in the whole simulation process. In the next sections we explore how the different model hypotheses work in each one of the different stages of the model validation that accomplishes the "Conceptual Model Validation", the "Operational Validation" and the "Data Validity".

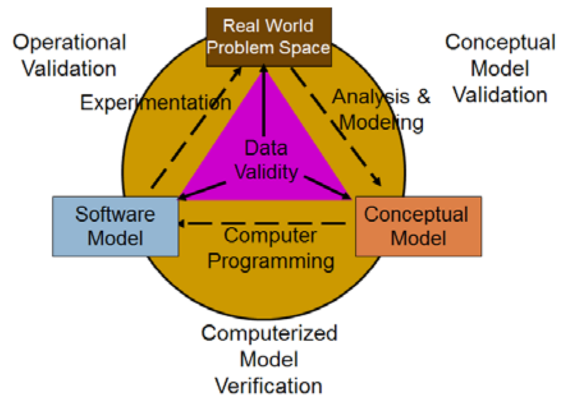

Figure 1. Validation and Verification process in a simulation model [2].

II. WORKING WITH THE HYPOTHESIS

Often the system is a complex reality. Even though we can work with simple systems, we need to use hypotheses in order to constrain what is "my system". From this definition of "my system", the scope of my experimental framework, we need to go further defining a model that represents the hypotheses, the structure and the behavior of the elements that compose "my system".

This model must be defined using a formal representation, independent of the tool selected to perform any implementation. The need of define a formal representation of the system is widely exposed in several books and papers, but maybe three key aspects must convince us of the need of use a formal representation of the model.

First, the formal representation of a model, as is stated on [3], can be considered a product by itself. This is quite interesting since sometimes the representation of the knowledge that rules the different processes in a system (for example an industry) can be more interesting than the simulation by itself. The formal representation of a model helps to understand how the model behaves, and consequently how the system is constructed. Second, the formal representation of the model simplifies its implementation and enhances its maintainability. And third, a formal representation of a model simplifies the understanding of the model by all the different actors that are involved in the simulation project, improves the communication.

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The formal representation of any simulation model starts by the definition of a hypotheses document that represents how I understand the behavior and the structure of “my system”. The formal representation of a simulation model is the formal representation of the simulation model hypotheses, hence the correspondence between the model and the hypotheses must be cleared understood, and this is often unusual. The main concern here is that in simulation literature, usually do not exist any classification between all the different hypotheses that can take part in the definition of the formal model. Not all the hypotheses have the same effect on the simulation model, and not all the hypotheses are needed on all the stages of the simulation model development. For that reason we propose to classify the hypotheses in three different categories as is explained next. This helps the model validation, because the modelers can focus its attention on the hypotheses related to each stage of validation. Also this taxonomy helps to understand the implications on the modifications on the knowledge we have of the system, or in the technology we use to implement the simulator.

III. HYPOTHESES TAXONOMY

First are those hypotheses that allow defining how the system behaves. Those hypotheses usually describe the system. In some papers related to VV&A are known as structural hypotheses. Mainly we want to add as many as we can of these hypotheses, since helps us in the description of the model. If we have a deeper knowledge of the system we can use a lot of these hypotheses describing its behavior. We know (or believe) that these hypotheses are true. We propose to name these hypotheses **Systemic**, since they describe the behavior of the system. These hypotheses can also be divided in two categories, those who are related with the data of the model, representing the flow of the elements of the system, and those related with the structure of the model, representing the underlying elements of the system.

Systemic Data hypotheses are those related with the data assumptions, which define the different probability distributions that rule the behavior of the model elements. **Systemic Structural hypotheses** can be those that represent the relations between the different elements that compose the model, the model behavior.

The other category is composed by the hypotheses that simplify the model we are going to build, named **Simplification** hypotheses. These hypotheses are useful in order to reduce the complexity of the model. Always, because the resources, the time we have to implement the model, and the knowledge we have of the system are limited, we must use this kind of hypotheses.

These three categories of hypotheses encompass all the hypotheses that can be used in a simulation project, with the main objective of simplify and help in the VV&A process and to help in the understanding of the nature the decisions taken during the modeling process.

IV. VALIDATING A SIMULATION MODEL

Validated means that the hypotheses are assumed as true by all the parts involved in the simulation project. However, we cannot assure that a model obtained from a set of hypotheses is true; we can only assure that a model is false. Also modeling can be useful for many other reasons but predict [4], we can assume that a model is valid, and for that the applied hypotheses are valid, for a specific purpose. In order to conduct the Validation process in a simulation project, Naylor and Finger [5] proposed combining the three historical methods of rationalism, empiricism, and positive economics into a multistage process of validation. This validation method consists of:

1. Developing the model’s assumptions on theory, observations, and general knowledge.
2. Validating the model’s assumptions where possible by empirically testing them.
3. Comparing (testing) the input-output relationships of the model to the real system.

Figure 1 mainly shows this process, and as we can see on it we must validate three aspects:

1. Data validation.
2. Conceptual model validation: logical structure and hypothesis. Conceptual model validity is determining that (i) the theories and assumptions underlying the conceptual model are correct and (ii) the model’s representation of the problem entity and the model’s structure, logic, and mathematical and causal relationships are “reasonable” for the intended purpose of the model.
3. Operational validity: In this step, see if the outputs of the model have the accuracy required in accordance with the problem.

Also we can validate according [6].

1. Experimental validation: analyze if the experimental procedures used to obtaining the results are sufficient accurate.
2. Solution validation: in this validation the focus is on the accuracy of the results obtained from the model of the proposed solution. This validation is useful for the modellers in order to learn.

In this paper we are focused on the three first aspects and we avoid analyzing the experimental validation and the solution validation.

Since the validation process is the process of comparing the behavior of the model and the behavior of the real system in order to assure that we build the correct model, we focus in the formal representation of the model, although some techniques to test the model validity (as we can see next) uses a specific implementation of the model. This implies that we accept that the implementation of the model is correct (verification is done correctly). We avoid in this paper of talking about the verification of the tool that implements the simulation model.

Also, since the formal representation of the model lies on the hypotheses, the process of validate a model is the process of validate the different hypotheses used to build the formal representation of the simulation model.

In the next table, we define what kind of hypotheses must be validated for each one of the different aspects that must be validated.

TABLE I. HYPOTHESES VALIDATED IN EACH ASPECT.

	Systemic / Structural	Systemic / Data	Simplification
Validation of data		X	
Validation of the conceptual model	X		X
Operational validity	X	X	X

In order to validate the data, we must focus our efforts in the Systemic Data hypotheses. This implies that the nature of the tests that can be performed must focus on analyze the data we are going to use in the model. Here, we propose to distinguish between two aspects related to the data, the **structure** and the **nature**. The structure represents the shape that the data follows, while the nature means the source of this data, where the data lives. In that case, we can perform Chi-square tests in order to assure that the structure of the data is correct [7], [8]. To analyze the nature of the data, we must detect if this data will be always up to date, often assuring that an institution or research center take care of this work. This is usually a key aspect in order to perform environment simulation models. As an example a wildfire model needs information regarding of the vegetation, the digital terrain model (DTM), the winds, etc. All the validations tests proposed, related with the Systemic Data hypotheses, focuses on the data structure. The validity of the nature of the data is based on assure that the institutions (structures, enterprises, etc.) needed in order to obtain the data and keep this data up to date during the life of the simulator, exists. The validation of data structure is the validation of the Systemic Data hypotheses.

The conceptual model represents the structural relations of the different model elements and the behavior of the different element that compose the model. To validate the conceptual model we must focus on the formal representation of the model or in the implementation of this formal representation in order to validate mainly, the Structural Systemic and the Simplification hypotheses.

In brief, to validate the conceptual model we must validate Structural Systemic hypotheses and Simplification hypotheses. This implies that we must define techniques in order to assure that (i) the Structural Systemic hypotheses are correct for our purposes, and (ii) the Simplifications hypotheses do not transform the model in a caricaturing of the reality.

The Operational validity is focused in the results that we can obtain from an implementation of the model. Some of the methodologies presented here, like Black Box validation among many others, imply the use of all the model hypotheses, since the modeler try to validate the whole behavior of the computer program that implements the model. In that case we often cannot distinguish if the results (in case that the results are wrong) are due to an incorrect (Data or Structural) Systemic hypotheses, due to a wrong Simplifying hypotheses, or due to an incorrect implementation of the model introducing the verification phase into account.

In order to detect the sort of hypotheses that is validated depending on the validation methodology used, we propose a classification of some of well-known validation methods, and what are the hypotheses that they try to validate. In that case we are focused on the validation of a specific class of hypotheses, instead to try to validate the whole model.

V. VALIDATING THE HYPOTHESES

Different papers and books describe several tests related to the model validation [1] [2] [7] [8] [9]. Taking some of the tests described on [2] we are going a little further trying to define what are the hypotheses that are tested on each test. This will help us to define what are the batteries of tests we must use in order to build a good model, since as we can see next, not all the tests are focused on the same kind of hypotheses. The tests we analyze are (i) Validation face (ii) Black Box validation, (iii) Turing tests, (iv) Comparing with other models, (v) Degenerate tests, (vi) Extreme Condition tests, (vii) Event Validity and (viii) Variability of the Parameters and Sensitivity Analysis. For each one of these test we propose a classification depending on the hypotheses that we argue that mainly check.

In **Face** validation, the experts analyze the results obtained from the simulation model. From this analysis they can recognize the correctness of a model. One example could be to test if a simulation model of a specific machine behaves similar to the system machine. In this case, like in **Black Box** validation, the model is seen as a whole, implying that the validation is done over the complete set of hypotheses. Other similar case is **Turing** tests; in that case, the simulator generates fake documentation that is merged with real documentation. Again, the experts determine, examining the documentation that contains real and fake documents, what are the fake documents generated by the simulator. Finally, in this category, the comparison of the model outputs with **Historical Data**, allows to understand if the model is behaving as expected, at least for a scenario that is reproducing the behavior of an existing system. Looking these tests we can argue that the model is tested as a whole, for that the hypotheses tested are all, the Structural and the Simplification hypotheses. This family of validation tests is related with the Operation validity.

On the tests based on the **comparison with other models**, the underline idea is that if other models work fine, its outputs must be similar. As an example, if we have an analytical model, we can compare the outputs of this model with a new simulation model. On this kind of test we can chance the input data used and the model parameters in order to validate if both models follows the same patterns for the results. This allows determining if the structural relations of the models are correct. This test focuses on the structure of the model, we modify the data and we assume that the data is correct. For that in this kind of tests often only Systemic Structural hypotheses are tested, although the Simplification hypotheses can be tested too, since some of the decisions on the structure of the model rely on them.

On the **Degenerate Tests** is analyzed the model's behavior modifying the values of input and some selected internal values. The objective is to test if the modification of these parameters is coherent with the expected result. As an example if we increase the service time of a server we expect that the number of elements in the queue increases. Similar to this on **Extreme Condition Tests** is supposed that the model structure and outputs should be credible although using any extreme and unlikely combination of values for the variables. On **Fixed Values** tests we analyze the outputs for a well know values for the parameters of the model. In this test we look the outputs in order to compare them with the expected results. On these tests we are focused on understand if the relation between the elements are correctly described, for that these tests focuses on the Systemic Structural hypotheses. Note that in these tests we assume that the model is valid and we look the model to understand if the relations between the model elements are correct, no Simplification hypotheses are tested here.

Comparison with other models, Degenerate, Extreme Condition and Fixed Values tests are related with the validation of the conceptual model.

Variability of the parameters and **sensitivity analysis** allows analyzing the factors that have greatest impact on the performance measures. This allows determining what elements must be modeled carefully and detecting possible errors on the definition of the relations of the model elements. In this sort of tests we are focused on the Systemic Structural and Data hypotheses. We can detect if the probability distributions are correctly represented and if the relations between the different elements are correctly implemented. As an example, if we add between two model elements a causal relation when in the system only a correlative relation exists, we are introducing an error that can be detected with this test. This test is related with the validation of the conceptual model and the validation of the data.

On the **Event Validity** we compare the occurrences of some events with the real occurrence of those events in the system. As an example, the number of "broken" event occurrences in a specific machine of the model. This kind of test can be useful to test the Systemic Data hypotheses since usually the events that rule the behavior of a simulation model are defined using known probability distribution or an empirical distribution obtained from a database. This test is related with the data validation.

Other methods exists to validate the model, like Internal Validation, Predictive Validation, the use of Traces or the use of the Animation to understand if the model behaves as expected [2]. In Table II the description of the hypotheses tested on each one of the tests is shown. Subsequent to this table, if we want to validate our simulation model, at least is needed to test once all the hypotheses. This implies that we must select the tests that allow doing this, for instance selecting Compare with other models and Events tests, or Degenerative, Variability of the parameters and Black Box tests.

TABLE II. HYPOTHESES VALIDATED ON SOME TYPICAL TESTS

	Systemic / Structural	Systemic / Data	Simplify
Validation "Face"	+	+	+
Turing tests	+	+	+
Black box	+	+	+
Historical data	+	+	+
Compare with other models	+		+
Degenerative	+		
Extreme conditions	+		
Fixed values	+		
Variability of the parameters, sensitivity analysis	+	+	
Events		+	

Following the approach proposed by Naylor and Finger, and understanding that we need to validate all the hypotheses, we can start with the validation related to the Data, then continue with the conceptual model and finally perform an operational validity once a preliminary version of the model is constructed. We can use Table I to understand the hypotheses that must be validated on each validation, and Table II to select the appropriate test. We can start performing the goodness tests for the distributions we are going to use in our simulation model, and selecting some test that allows validating the Structural Data hypotheses, like the Events test. These tests are focused on the **structure** of the data. It is also needed to assure the validity of the **nature** of the data, or at least that is enough for our project purpose. As we said previously that means that an institution or enterprise assures that we have the data up to date in order to use it in our simulation model. Once we have the validation of the data we can perform the validation of the conceptual model. We can select some of the test that verifies this. Since we have the Systemic Data hypotheses validated, we can focus our efforts in the validation of the Systemic Structural hypotheses, using as an example the Degenerative test. Also, we can use the compare with other models test to test again the Systemic Data hypotheses (note that comparing with other models can be time demanding due to we need to have other models to perform this comparison).

Finally, we can perform the Operational validation. Note that in this stage, if we have all the hypotheses tested (we previously have been performed an Events and Compare with other models tests) we can argue that our model have all the hypotheses validated, hence the Operational validation is done. However, since the validation process never assure that we have a model correct (the validation can only assure that we have an invalid model) we can perform here some of the tests that works with all the hypotheses, like the Turing tests to improve our confidence in the model. Remark that since not all the tests are focused on the same typology of hypotheses we can argue that it is interesting to test first the model with tests that are focused on certain hypotheses in order to detect possible mistakes. It is more difficult to find an error in our model if we test all the hypotheses using

Black box test that if we are testing only the Structural hypotheses using the Extreme conditions test.

VI. USING NOT VALIDATED HYPOTHESES

Validate a simulation model is a time demanding task, and often we need to work with models that have some of the hypotheses not validated (as an example to analyze extreme conditions or to validate Systemic Structural hypotheses). Since not all the hypotheses have the same effect on the model, we can select what are the more interesting tests to be performed first in order to validate the more critical hypotheses first, always depending on the purpose on the model. Again, remark that we are looking for a useful model, often we can assume to work with non-validated hypotheses. In the table III we show if usually is desirable or not working with no validated hypotheses because the effects that this can imply to the model, again regarding to achieve a specific result. *Wanted* means the desired state of the hypotheses, *Useful* states means that, although it is not a desirable state, can be useful for the model construction, as an example to perform the validation of some model hypotheses, or to obtain some values from a hypothetical data. It is interesting to remark here that the use of Simplification hypotheses can be *Useful*, but never is desirable. The final objective of a simulation model is to work without simplifications. We must note that the simplification hypotheses are always false. That means that we know for sure that the reality is more complex that the structure that we are depicting on the model. Lastly *Unwanted* means that this state of the hypotheses is undesired for any purpose of the simulation model.

TABLE III. EFFECTS OF USING NO VALIDATED HYPOTHESES.

	Systemic / Structural	Systemic / Data	Simplification
Validated	Wanted	Wanted	Useful
Non validated	Unwanted	Useful	Unwanted

As we said previously, Systemic Structural hypotheses depict the relations between the different elements that compose the model. If these relations are not well defined, the model is not correct. For that, using no validated systemic structural hypotheses is an unwanted state, since we need to incorporate the knowledge of the client, and the client must assume that the relations depicted in the model are the relations that exist in the system, assuming the Simplification hypotheses as true.

On the case of Systemic Data hypotheses, using no validated data can be useful for testing purposes. As we said previously, Systemic Data hypotheses are those related with the data assumptions, which often define the different probability distributions that rule the behavior of the model elements. In some cases it is needed to use no validated data in order to analyze the behavior of the model in some specific circumstances, or as we see in Table II to test that the Systemic Structural hypotheses are correct. For this, using no validated Systemic Data hypotheses can be useful.

Finally, if we are using validated Simplification hypotheses on our model, we are assuming that they are useful in order to achieve our expected result with the project constrains (technology, time, resources, knowledge, etc.). Like in the case of the Systemic Structural hypotheses, if these simplification hypotheses are not validated, often implies that we are using some simplifications in or model that the client maybe cannot assume. This is dangerous for the project, and often reflects a bad communication with the client. As is stated in [1], the communication with the client from the beginning of the project, and the definition of a good hypotheses document is a key element for the success of a simulation project. Again, note that the desired state (all in wanted) implies to avoid the use of simplification hypotheses.

VII. WORKING WITH THIS TAXONOMY, WRITING THE HYPOTHESES DOCUMENT

As is stated on [1], the hypotheses document is a key element in the success of a real simulation project. Starting with some initial meetings, it is needed to start the redaction of this document that describes in detail the model assumptions and main objectives. This document is simple but clear, and we propose to use the template shown next. In this template we categorize, for each one of the different elements of the model the hypotheses used. Also, since we need to describe the Systemic Structural hypotheses we can use a formal language to describe the structure and the behavior of the model in a complete and unambiguous manner. A formal language like SDL [10] [11], DEVS [12] or Petri Nets [13] [14] [15] among others, becomes a powerful tool to represent the Systemic Structural hypotheses. In the diagrams of the model, we show the elements we are going to represent and the relation between all the elements. Remember that using a formal language to represent the model allows using some static methods to validate the correctness of the Structural Systemic hypotheses [2]. The proposed outline of the document has the next sections:

1. Description of the system.
2. Purpose of the model.
3. Simplification hypotheses for the external view of the model. Showing for each one if has been validated by the client or not.
4. Systemic Data hypotheses for the external view of the model, again showing if have been validated each one of them by the client.
5. Systemic Structural hypotheses for the external view of the model, using a formal language. This helps to the understanding of what are the key elements of the model that we are going to simulate.
6. For each one of the different elements of the model we detail its hypotheses. Again the Systemic Structural hypotheses can be represented (and we support this) using a formal language.

In our projects, we write in red the hypotheses that have not been validated. This simplifies the understanding by the client and by the modeler teams of the need to validate the

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hypotheses in order to achieve the desired result, and clearly shows what the state of the model construction is. In an iterative construction of a simulation model, once all the hypotheses of the document have been validated by the client and by the modelers, we have a simulation model that can be used to take decisions and can be prepared for its final step; believe in the model, the accreditation.

VIII. CONCLUSIONS AND FUTURE WORKS

The hypotheses are the key element that rules the definition of the model. However, not all the hypotheses used in a simulation model have the same effect on the model definition. Also the tests used to prove the validity of a simulation project not are focused on the same typology of hypotheses, for that is needed a taxonomy in order to focus our efforts in a selected subset of the tests that validates those hypotheses. In our taxonomy, three classes of hypotheses exist, Systemic Structural hypotheses, Systemic Data hypotheses and Simplification hypotheses. Regarding the data, we note that two aspects must be validated, the nature, that means that the data will be correct during the life of the simulation model, and the structure, that means the usual validation process for the data (for example, test if the inputs follow an exponential distribution). The Systemic Data hypotheses are focused on the structure of the data, since the nature can be assured if an institution take care of this data or we have the knowledge that the nature of the data do not change during the life of our simulation model, this is usual in an industrial simulation model, but unusual in an environmental model where the climatic data can change day to day and we need an institution that take care of this data.

We showed in this paper how this taxonomy can help in the validation process of a simulation model, thanks to improve the selection mechanism of the tests in order to achieve a complete (if needed) validation of the model.

Also we show the implications of work with no validated hypotheses. Sometimes it could be desirable to work with no validated Systemic Data hypotheses in order to validate the Systemic Structural hypotheses, or to obtain data related to extreme conditions situations. From this taxonomy, we can clearly understand that the Systemic hypotheses must grow in order to represent better and with more detail the relations and the data assumptions of the system, and the simplification hypotheses must decrease in order to represent the deeper understanding of the system.

The improvement on the perception of the system, or the improvement on the tools we can use to implement the model can modify the hypotheses. Often an improvement on the tools imply the use of less Simplification hypotheses, but an improvement on the system knowledge implies the use of more Systemic hypotheses, implying a detailed description of the model. This taxonomy helps to understand the implications on the modification in the system knowledge, or on the tools used to implement the model or in our needs, in order to define faster a new model and perform a new implementation.

The future work is focused in develop a methodology to systematize not only the validation but also the verification of the hypotheses, combining some existing methods to define the appropriate tools to implement a simulation model [16]. This can help us to understand the limitations of our simulation model, due to the hypotheses used, before any implementation.

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