Stability and Delay of Network-Diversity Multiple Access with Backlog Retransmission Control

Ramiro Sámano-Robles

Research Centre in Real-time and Embedded Computing Systems
Intituto Politécnico do Porto, Porto, Portugal
Email: `rasro@isep.ipp.pt`

Abstract—This paper proposes two strategies for retransmission control of backlog traffic in the family of algorithms known as Network Diversity Multiple Access (NDMA). This type of algorithm has been shown to achieve (in ideal conditions) the following aspects: 1) collision-free performance for contention-based traffic, 2) low latency values, and 3) reduced feedback complexity. These features match the machine-type traffic, real-time, and dense object connectivity requirements in 5G. This makes NDMA a candidate for contention traffic support in 5G systems. However, existing analysis ignores the effects of backlog traffic generated by the imperfect detection conditions that arise in settings with finite Signal-to-Noise Ratio (SNR). This paper aims to partially fill this gap, by providing analytic expressions for the performance of symmetrical training-based NDMA protocols with two different types of backlog traffic retransmission schemes. In the first strategy, all terminals involved in an unsuccessful resolution period retransmit immediately in the subsequent resolution periods or epoch slots. This procedure is repeated continuously (*persistent retransmission*) by inducing the same collision event under different channel outcomes until all the contending signals are correctly detected and received. In the second retransmission strategy, the terminals in backlog state retransmit at a randomly selected time-slot with a probability that that is assumed (for simplicity) to match the transmission rate of the system. In both strategies, expressions are here obtained of the maximum stable throughput and the average delay experienced by any packet to be correctly received by the destination. This allows us to determine the capabilities of NDMA for achieving low-latency, reduced feedback complexity, as well as highly stable and real-time throughout performance. The results shown here suggest that NDMA can achieve attractive low latency and high throughput figures mainly at high SNR values and moderate traffic loads.

Keywords–Multiple Access in 5G; Retransmission Diversity; Signal Processing; Multi-User Detection; Cross-layer Design; Random Access; Multi-Packet Reception.

I. INTRODUCTION

Some of the main requirements in future 5G systems are the following: 1) low latency for real-time and machine-type communications, 2) increased spectral efficiency and throughput performance for broadband high speed and industrial applications, and 3) reduced signalling load and feedback complexity to cope with the large number of objects and terminals that will be competing for access to network resources. Network Diversity Multiple Access or NDMA is the family of signal-processing-based random access algorithms originally proposed in [1] that represent a good candidate to achieve the main goals of contention-based access in 5G networks. In NDMA, adaptive retransmissions are used to resolve collisions of variable size. For example, if a collision of

K terminals occurs, then the system attempts to induce enough diversity via retransmissions to create K or more degrees of freedom. These degrees of freedom or sources of diversity will allow the system to recover the signals of the colliding terminals via multi-user detection tools. NDMA is the perfect example of cross-layer interactions: PHY (PHYsical) layer diversity is created explicitly by retransmissions induced by MAC (Medium Access Control) layer processes.

Training-based NDMA protocols have been proposed in [1] and [2] for non-dispersive and dispersive channels, respectively. Blind versions based on rotational invariance techniques and Independent Component Analysis (ICA) were proposed in [3] and [4], respectively. More recently, a combination of NDMA, Multi-Packet Reception (MPR) and Successive Interference Cancellation (SIC) was shown in [7] to potentially break the barrier of M packets per time-slot, where M is the number of antennas at the receiver. This is the highest throughput potentially found in random access theory. Performance of NDMA combined with Automatic Repeat reQuest (ARQ) has been investigated in [8]. NDMA is expected to be an attractive solution for future 5G systems, mainly because it achieves (in ideal conditions) an almost collision-free throughput performance with low values of latency or access delay and reduced signalling feedback complexity.

There are several open issues that need to be solved in the design of NDMA systems, particularly related to stability. Stability of asymmetrical NDMA systems under perfect detection and reception conditions using the Foster-Lyapunov criterion and the Loynes' theorem were presented in [5]. Stability of symmetrical NDMA protocols with finite SNR using a Markov model for backlog states of the system was presented in [6]. *Stability* is loosely defined here as the ability of a network to deal with the traffic requests of all the terminals within a finite period of time. Stability is often more important but also more difficult to investigate than throughput or delay. Exact stability boundaries of NDMA in the case of imperfect collision multiplicity estimation have not yet been obtained in the literature.

This paper attempts to partially fill this gap, by obtaining more accurate predictions of the stable throughput of conventional NDMA systems considering backlog traffic with two different types of retransmission strategy. In the first strategy, also called *persistent retransmission strategy*, the backlogged terminals immediately engage in a new resolution period in an attempt to correctly decode the colliding packets. This procedure is repeated until the conflict has been successfully resolved. In the second retransmission strategy, backlogged terminals retransmit *randomly* in future time slots assuming, for

convenience, a retransmission probability that exactly matches the transmission rate of the system. For both cases, the stability condition is evaluated by means of a balance traffic equation which is the result of a particular application of Loynes' theorem of stability in queuing systems. Delay is evaluated by means of the M/G/1 queue analytical framework, which is commonly used in the literature of NDMA to estimate average delay. The results show that NDMA is capable to surpass by a significant margin its ALOHA counterparts, and when the SNR is high enough and with moderate traffic loads, low latency values and high throughput performance could be attractive for future 5G networks.

The organisation of this paper is as follows. Section II presents the system assumptions and definitions. Section III presents the details of the first retransmission strategy (also called here persistent retransmission strategy) where back-logged terminals continuously retransmit information until the BS correctly resolves the collision. Section IV details the random retransmission strategy. Results of the retransmission schemes are displayed and discussed in Section V. The conclusions of the paper are then presented in Section VI.

Notation: $E[\cdot]$ is the statistical average operator, $E[x|y]$ is the average of random variable x conditional on a particular instance of random variable y, $E_x[\cdot]$ is the statistical average operation over the probability space of random variable x, $\overline{(\cdot)} = 1 - (\cdot)$ is the complement to one operator, $\binom{N_1}{N_2} = \frac{N_1!}{(N1-N_2)!N_2!}$ is the combinatorial number of N_1 elements (objects) in N_2 positions.

II. SYSTEM MODEL AND ASSUMPTIONS

This section deals with the system model and the assumptions used throughout the paper. The subsections are organised as follows: Subsection II-A describes the scenario and the steps of the original NDMA protocol. Subsection II-B describes the backlog retransmission strategies. Subsection II-C defines the types of collision resolution periods or epoch-slots, and finally Subsection II-D provides illustrative examples of the two proposed retransmission strategies.

A. Scenario description and NDMA protocol operation

Consider the wireless random access network depicted in Figure 1 with one base station (BS) and J terminals. All network elements have only one antenna. Each terminal is assumed to have a buffer experiencing a packet arrival process with Poisson statistics described by the parameter λ. The transmission probability of any terminal at the beginning of any resolution period is denoted by p. All channels are considered non-dispersive, flat and block fading with Rayleigh statistics.

Whenever the terminals are allowed to transmit a packet, they do so at the beginning of a new collision resolution period or epoch-slot. At the beginning of every epoch slot, the BS proceeds to obtain an estimate of the identity of the contending terminals by means of signal processing tools (details can be found in [1]). Each terminal uses as packet header a unique orthogonal code previously assigned. The BS exploits this header using a matched filter receiver and energy detection processing to estimate the presence of each terminal in the collision event. Since this process is prone to errors due to fading and noise, the probability of detection conditional

on the terminal having transmitted a packet in the current time-slot is given by P_D (probability of correct presence detection). On the other hand, the probability of presence detection conditional on the terminal not having engaged in transmission is given by P_F or probability of false alarm. In Rayleigh fading channels, it has been shown in [1] that the Receiver Operational Characteristic (ROC) of the terminal presence detector is given by $P_D = P_F^{\frac{1}{1+\gamma}}$, where γ is the average post-detection Signal-to-Noise Ratio (SNR).

SCHEME 1

Superepoch	e=1		e=2		e=3			e=4	
Contending terminals	1,3,8	1,3,8	1,3,8	1,3,8	1,3,8	1,3,8	1,3,8	1,7	1,7
Detected terminals	3,8	3,8	1,5	1,5	1,3,8	1,3,8	1,3,8	1,7	1,7
Feedback flag 1	1	0	1	0	1	1	0	1	0
Feedback flag 2		0		0		1			1

SCHEME 2

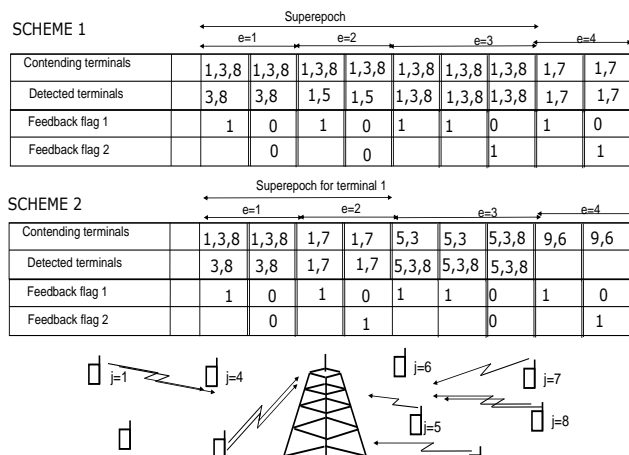| Superepoch for terminal 1	e=1		e=2		e=3			e=4	
Contending terminals	1,3,8	1,3,8	1,7	1,7	5,3	5,3	5,3,8	9,6	9,6
Detected terminals	3,8	3,8	1,7	1,7	5,3,8	5,3,8	5,3,8		
Feedback flag 1	1	0	1	0	1	1	0	1	0
Feedback flag 2		0		1			0		1

Figure 1. NDMA operation with the two backlog retransmission algorithms. Scheme 1 is also called persistent retransmission scheme. Scheme 2 is also called random retransmission scheme

The detection of the presence of the different active terminals provides the BS with an estimation of the collision size. Based on this information, the BS proceeds to request retransmissions from the contending terminals so as to construct a virtual Multiple-Input Multiple-Output (MIMO) system with the convenient rank conditions that will ensure that the collision can be resolved via multi-user detection.

B. Backlog retransmission schemes

In NDMA, it is conventionally assumed that any detection error at the BS side yields the loss of all packets involved in the collision. Conversely, the collision is successfully resolved only when all the terminals are correctly detected (both active and idle terminals). This paper proposes two backlog retransmission schemes to deal with the packets that were involved in an unsuccessful resolution period or epoch-slot. The first scheme (also called *persistent*) allows the contending terminals to engage immediately in a new resolution period. The BS indicates to the terminals that the previous resolution process did not succeed, and therefore the same contending terminals are induced to collide again at the beginning of the new resolution period. This procedure is repeated until all the packets involved in the collision are correctly decoded by the destination. In the second retransmission strategy, the backlogged terminal retransmits randomly in a future epoch slot with probability p, which is exactly the same probability as the overall system attempt rate. This scheme is also called *random retransmission scheme*.

C. Epoch-slot definition and feedback model

The collision multiplicity at the beginning of any epoch slot will be denoted by the random variable K. The length

of a simple collision resolution period will be denoted by the random variable l. The period of time used for a packet to be correctly decoded by the destination will be denoted by L, and it will also be called *super-epoch*. Two types of epoch and super-epoch are further defined: *relevant*, where a particular terminal under analysis is always present, and *irrelevant*, where such incumbent terminal is idle.

The BS has two feedback flags that are considered to be ideal and instantaneous. One flag is used to indicate to the colliding terminals that retransmission is needed in the next time slot for purposes of diversity. The second feedback flag occurs at the end of a collision resolution period and indicates whether the epoch was successful or not. Based on this information, the colliding terminals decide to enter in one of the backlog retransmission schemes presented in this paper.

D. Examples

To further illustrate the proposed algorithms, Figure 1 shows the realization of the two retransmission strategies over 4 epoch-slots. In the first epoch ($e = 1$) of scheme 1, three terminals collide at the beginning of the epoch slot ($\{1,3,8\}$). However, only two of them were detected correctly as active ($\{3,8\}$). The system has requested only one more retransmission when indeed it was necessary to collect two more retransmissions to resolve the collision. This means this epoch $e = 1$ is unsuccessful. The terminals are now in backlog state and retransmit immediately in the next resolution period. Once again, the detection process was incorrect, by miss-detecting two of the contending terminals ($\{3,8\}$) and estimating one of the idle terminals ($\{5\}$) as active (false alarm). The backlogged terminals proceed then to retransmit again in a third consecutive epoch slot ($e = 3$). This time all terminals were correctly detected and the collision is conveniently resolved. The fourth epoch ($e = 4$) allows new terminals to transmit, and it can be observed that this case was a successful epoch. Note that the first collision took three epoch slots to be correctly resolved with total length of $L = 7$. This set of epoch slots that a collision experiences to be resolved is called *super epoch*.

In the second retransmission strategy, the three contending terminals involved in the first resolution period ($\{1,3,8\}$) become backlogged. However, they start retransmission randomly over the next epoch slots. In the second epoch slot, terminal $j = 1$ retransmits the backlogged packet and this time the resolution is successful. By contrast, the third epoch sees terminal $j = 3$ to experience again an incorrect detection with one case of false alarm. The last epoch shows that two non-backlogged terminals experience a successful collision resolution. Note that the super-epoch for terminal $j = 1$ is given by the first and second resolution periods with a total length of $L = 4$.

III. Persistent Retransmission Strategy

In the first retransmission strategy, all the terminals involved in an incorrect resolution period are forced to retransmit immediately in the next resolution period(s). This process is repeated until the collision is correctly resolved. The steps of the persistent retransmission strategy are described in Algorithm 1. Stability will be investigated here by using a modified traffic balance equation. This equation has been used before in [5] for stability analysis of NDMA. The expression states

the balance between the incoming and outgoing traffic in the NDMA system. It is a modification of the Loynes' theorem of stability in queuing systems, and it can be written, in our context, as follows:

$$p = \lambda E[L], \qquad (1)$$

which states the balance between the transmission attempt rate p and the incoming traffic rate per super-epoch-slot. In conventional NDMA, correct resolution occurs when all terminals are correctly detected. This occurs when all K contending terminals have been correctly detected with probability P_D^K and all $J - K$ idle terminals are not incorrectly detected as active with probability $\bar{P}_F^{J-K}$, where $\bar{P}_F = 1 - P_F$.

Consider now a collision of K out of J terminals. The probability of correct resolution is equal to the joint probability of correct detection of all terminals (active and idle), which can be written as follows:

$$P_{c,K} = P_D^K \bar{P}_F^{J-K}. \qquad (2)$$

To obtain the expression for the average length of a super epoch $E[L]$ we consider that the resolution of any collision of size K takes a random number of attempts described by a geometric distribution with parameter $\bar{P}_{c,K}$ from (2) and with average number of attempts given by $1/P_{c,K}$. Therefore the average length of a super epoch conditional on the collision size is given by:

$$E[L|K] = \frac{E[l|K]}{P_{c,K}},$$

where

$$E[l|K] = K P_D + (J - K) P_F + \bar{P}_D^K \bar{P}_F^{J-K}. \qquad (3)$$

For details of the derivation of the previous expression please see the Appendix. Averaging over the probability space of all potential collision sizes we obtain:

$$E[L] = \sum_{K=1}^{J} \binom{J}{K} p^K \bar{p}^{J-K} \frac{E[l|K]}{P_{c,K}} \qquad (4)$$

The access delay for NDMA is usually approximated by the formula of delay for an M/G/1 queue with vacations [1]:

$$D = E[L_r] + \frac{\lambda E[L_r^2]}{2(1 - \lambda E[L_r])} + \frac{E[L_{ir}^2]}{2E[L_{ir}]}, \qquad (5)$$

where, $E[L_r]$, $E[L_{ir}]$, $E[L_r^2]$, and $E[L_{ir}^2]$ denote, respectively, the first- and second-order moments of the length of a relevant and irrelevant super-epochs. For the particular case of the persistent retransmission scheme we obtain the following:

$$E[L_r] = \sum_{K=1}^{J} \binom{J}{K} p^{K-1} \bar{p}^{J-K} \frac{E[l|K]}{P_{c,K}} \qquad (6)$$

and

$$E[L_{ir}] = \sum_{K=1}^{J-1} \binom{J-1}{K} p^K \bar{p}^{J-1-K} \frac{E[l|K]}{P_{c,K}} \qquad (7)$$

The second order moments of the two types of super-epoch are given by

$$E[L_r^2] = \sum_{K=1}^{J} \binom{J}{K} p^{K-1} \bar{p}^{J-K} E[l^2|K] \frac{2 - P_{c,K}}{P_{c,K}^2} \qquad (8)$$

and

$$E[L_{ir}^2] = \sum_{K=1}^{J-1} \binom{J-1}{K} p^K \bar{p}^{J-1-K} E[l^2|K] \frac{2 - P_{c,K}}{P_{c,K}^2} \quad (9)$$

where

$$E[l^2|K] = KP_D(KP_D + \bar{P}_D) + (J-K)P_F[(J-K)P_F + \bar{P}_F]$$
$$+ 2KP_D(J-K)P_F + \bar{P}_D^K \bar{P}_F^{J-K} \quad (10)$$

For details of the derivation of this last expression please see the Appendix.

1) Generate set of colliding terminals using traffic model.
2) Start super-epoch slot.
3) Start of a conventional epoch-slot of NDMA
4) Detect the presence of contenting terminals
5) Request retransmissions to create a virtual MIMO system
6) Attempt the decoding of the colliding terminals
7) Is the collision resolved? If Yes, then end of a super-epoch and go back to step 1. If not, the same contending terminals restart one more epoch slot. Go back to step 3.

Algorithm 1: Algorithm NDMA with persistent backlog retransmission control.

IV. RANDOM BACKLOG RETRANSMISSION STRATEGY

In the second retransmission strategy, backlogged terminals use a random retransmission scheme with a probability that is forced to match the transmission probability of the system p. This assumption simplifies the derivation of metrics in the system. In the case of different selection of retransmission probability, it is necessary to use a Markov chain model of the system and a two-state model for each terminal in the network (see [6]). Terminals involved in a collision with an unsuccessful first epoch will retransmit at different time slots randomly selected. The steps of the random retransmission scheme are enumerated in Algorithm 2. To investigate this scheme, we will use a modified traffic balance equation written as follows:

$$p = \lambda E[L] = \lambda(pE[L_r] + \bar{p}E[l_{ir}]), \quad (11)$$

where L_r and l_{ir} indicate, respectively, the length of a relevant super-epoch and irrelevant epochs. It is called relevant because it denotes the super-epoch where a given terminal is involved in transmission. In the random retransmission strategy, the average number of attempts is dictated by the probability of success resolution, denoted here by P_c and given by:

$$P_c = P_D(pP_D + \bar{p}\bar{P}_F)^{J-1}$$

The number of attempts has therefore a geometric distribution with parameter $\bar{P}_c$ and with average given by $\frac{\bar{P}_c}{P_c}$. Now, since the retransmission attempt is randomized, there is a number of resolution periods ignored by the backlogged terminal. Another geometric distribution of this inter attempt process is modelled with parameter $\bar{p}$ and average given by $\frac{\bar{p}}{p}$. The final expression is thus given by:

$$E[L_r] = \frac{\bar{P}_c}{P_c}\left(\frac{\bar{p}}{p}E[l_{ir}] + E[l_r]\right) + E[l_r],$$

where the average length of a relevant and an irrelevant epoch can be written, respectively, as follows:

$$E[l_r] = (J-1)P_A + P_D + \bar{P}_D \bar{P}_A^{J-1} \quad (12)$$

and

$$E[l_{ir}] = (J-1)P_A + P_F + \bar{P}_F \bar{P}_A^{J-1}. \quad (13)$$

For details of the derivation of these previous two expressions we refer the reader to the Appendix. The average delay for NDMA is usually approximated by the formula of delay an M/G/1 queue with vacations [1]:

$$D = E[L_r] + \frac{\lambda E[L_r^2]}{2(1 - \lambda E[L_r])} + \frac{E[L_{ir}^2]}{2E[L_{ir}]}, \quad (14)$$

where using the properties of binomial and geometric probability distributions we can obtain:

$$E[L_r^2] = \frac{E[l_r^2]}{P_c^2 p_r^2} \quad (15)$$

$$E[l_r^2] = (J-1)P_A[\bar{P}_A + (J-1)P_A]$$
$$+ 2(J-1)P_A P_D + P_D + \bar{P}_D \bar{P}_A^{J-1} \quad (16)$$

and

$$E[L_{ir}^2] = (J-1)P_A[\bar{P}_A + (J-1)P_A]$$
$$+ 2(J-1)P_A P_F + \bar{P}_F \bar{P}_A^{J-1} \quad (17)$$

For details of the derivations of these expressions we refer the reader to the Appendix.

1) Generate set of colliding terminals using traffic model.
2) Start of a conventional epoch-slot of NDMA
3) Detect the presence of contenting terminals
4) Request retransmissions to create a virtual MIMO system
5) Attempt the decoding of the colliding terminals
6) Is the collision resolved? If Yes, then go back to step 1. If not, terminals backlog randomly the lost packet with probability p. Go back to step 3.

Algorithm 2: Algorithm NDMA with random backlog retransmission control.

V. RESULTS

The results discussed in this section have been obtained with a network configuration with $J = 16$ terminals with an average SNR γ of 7, 10, and 15 dB. The detection threshold has been adjusted to obtain a probability of false alarm of $P_F = 0.01$. Figure 2 shows the stable throughput $T = J\lambda$ versus different traffic load values. Figure 3 shows the delay experienced by the two retransmission schemes. It can be observed that the persistent retransmission scheme only slightly outperforms the random retransmission strategy, particularly at low SNR. Both strategies seem to be able to achieve the maximum throughput previously estimated in [1] for the conventional version of the protocol without backlog traffic. This is a significant result that paves the way for further analysis about the equivalence of stability and throughout metrics of the protocol.

It is worth pointing out that the main virtues of the random retransmission strategy cannot be fully observed in the figures

provided here. The random strategy will be optimum in networks affected by deep and long fades, or with terminals with long term degrading channel conditions. Therefore, the reader should keep in mind that random retransmission will play an important role in particular network situations. Future networks are meant to be more adaptive and cognitive to network and channel conditions, and therefore it is expected that different backlog retransmission strategies can be adopted on the fly to maximize performance. Another aspect to point out is that in comparison with ALOHA solutions, NDMA protocols are capable to adopt persistent retransmission strategies, which in ALOHA is practically impossible. Once a collision event occurs in ALOHA, terminals must engage in random backlog retransmission algorithms, mainly because the repetition of the same collision event (used in persistent retransmission schemes) leads inevitably to unstable performance. This is another proof that NDMA is considerably better in terms of stability than its ALOHA counterparts.

Regarding delay in Figure 4, both algorithms seem to achieve the same performance. Delay is degraded as traffic load reaches the maximum channel transmission rate. The values of delay suggest that NDMA has good performance for real time systems only at moderate traffic loads and at relatively high values of SNR. To further illustrate the difference between the conventional NDMA and the proposed retransmission algorithms, Figure 4 shows the average length of the super epochs of the two algorithms. It can be observed that the super-epochs clearly are larger by several orders of magnitude than the conventional NDMA, which is consequence of the retransmission schemes. However, the reader must remember that the original protocol ignores the effects of backlog traffic, whereas in the present approach we estimate the effects of backlog traffic by inducing further retransmissions so that we can evaluate the performance of the algorithm in such conditions. Note that at high SNR the retransmission schemes are closer in performance to the average length of the epoch in the conventional NDMA protocol.
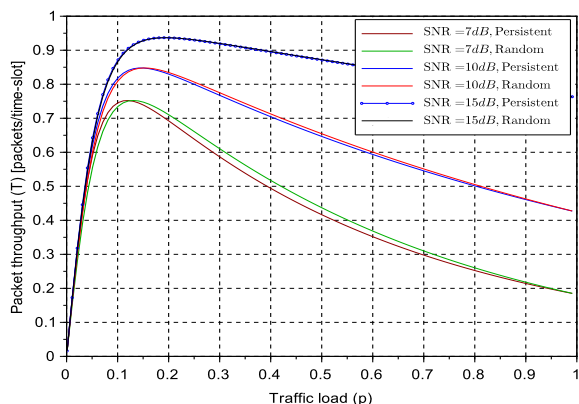

Figure 3. Average Delay (D) vs. transmission probability (p) using the two proposed backlog retransmission schemes for various values of SNR.

Figure 4. Average length of a super-epoch ($E[L]$) vs. transmission probability (p) using the two proposed backlog retransmission schemes for various values of SNR.

Figure 2. Stable throughput ($T = \lambda J$) vs. transmission probability (p) using the two proposed backlog retransmission schemes for various values of SNR.

VI. CONCLUSIONS

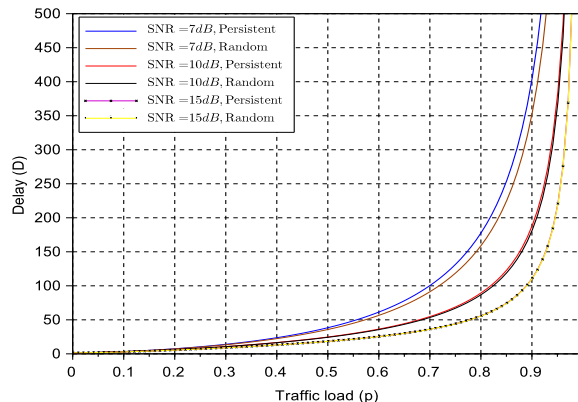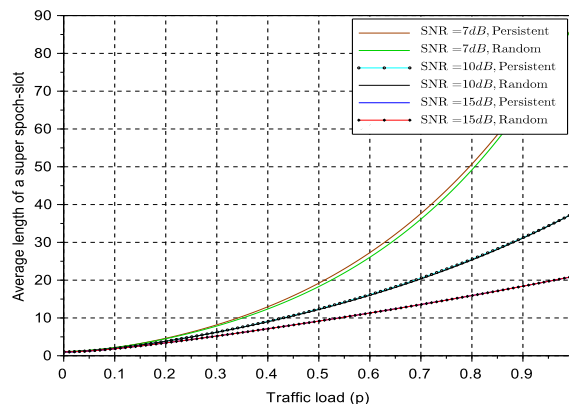This paper has presented two retransmission schemes of backlog traffic for the conventional NDMA protocol in Rayleigh block fading and non-dispersive channels. It has been observed that under these assumptions the persistent retransmission strategy, where terminals involved in an unsuccessful resolution keep retransmitting until the collision is resolved, provides the best results achieving an almost identical value as the throughput without backlog traffic consideration. However, the random retransmission strategy with a retransmission probability equal to the system transmission rate performs almost identically, but it has the further advantage of being suitable for scenarios with deep and long-term fades or with terminals with persistent bad channels conditions. The results show that NDMA considerably outperforms stability of ALOHA solutions. It has been also shown that for high SNR values, the persistent retransmission scheme boils down to the conventional NDMA solution. The importance of the results in this paper is that NDMA shows great potential for handling future low-latency traffic, particularly at high values of SNR and moderate traffic loads. Additionally, we have obtained for the first time a figure of the performance of NDMA with backlog traffic, thus helping in the evaluation of the stability properties of this type of protocol.

REFERENCES

[1] M. K. Tsatsanis, R. Zhang, and S. Banerjee, "Network-Assisted Diversity for Random Access Wireless Networks," *IEEE Trans. on Sig. Proc.*, Vol. 48, No. 3, March 2000, pp. 702-711.

[2] M. K. Tsatsanis, R. Zhang, and S. Banerjee, "Network-Assisted Diversity for Random Access systems in dispersive channels," *IEEE Trans. on Commun.*, Vol. 50, No. 4, 2002, pp. 623-632.

[3] R. Zhang, N. D. Sidiropoulos, and M. K. Tsatsanis, "Collision resolution in packet radio networks using rotational invariance techniques," *IEEE Trans. on Commun.*, Vol. 50, No. 1, 2002, pp. 146-155.

[4] B. Ozgul and H. Delic, "Wireless access with blind collision-multiplicity detection and retransmission diversity for quasi-static channels," *IEEE Trans. on Commun.*, Vol. 54, No. 5, 2006, pp. 858-867.

[5] G. Dimic, N. D. Sidiropoulos, and L. Tassiulas, "Wireless networks with retransmission diversity access mechanisms: stable throughput and delay properties," *IEEE Trans. on Signal Processing*, Vol. 51, No. 8, 2003, pp. 2019-2030.

[6] R. Samano-Robles and A. Gameiro "Stability properties of network diversity multiple access protocols with multiple antenna reception and imperfect collision multiplicity estimation," *Journal of Computer Networks and Communications,* Vol. 2013, No. 984956, pp. 1 - 10, December, 2013.

[7] R. Samano-Robles, D. C. McLernon, and M. Ghogho "A Random Access Protocol incorporating Retransmission Diversity, Multipacket Reception, and Successive Interference Cancellation," *Workshop on multiple access communications MACOM,* Helsinki, Finland, pp. 70-86, September 2015

[8] F. Ganhao, et al., "Performance Analysis of an Hybrid ARQ Adaptation of NDMA Schemes," *IEEE Transactions on Communications,* Vol. 61, no. 8, pp. 3304-3317, August 2013

APPENDIX

A. Derivation of the first- and second-order moments of the length of an epoch-slot in (3) and (10), respectively, conditional on the number of contending terminals K in the first retransmission scheme (persistent retransmission)

The length of an epoch in NDMA can be regarded as the linear combination of the contribution of the following terms: 1) active terminals correctly detected, 2) idle terminals incorrectly considered as active (due to false alarm), and 3) the contribution of only one time slot when no terminal is detected (either active or idle). The first contribution of active terminals conditional on K contending terminals has a binomial distribution with parameter P_D. Using the properties of the binomial distribution, this component has an average given by KP_D and a variance given by $KP_D(KP_D + \bar{P}_D)$. Similarly, the contribution of the remaining $J - K$ terminals has also binomial distribution with parameter P_F, average $(J - K)P_F$ and variance $(J - K)P_F[(J - K)P_F + \bar{P}_F]$. Finally, the probability that no terminals is detected as active is given by $\bar{P}_D{}^K \bar{P}_F^{J-K}$ with one time slot contribution to the average and the variance of the length of the epoch slot.

The expressions for the first order moment is given by the combination of the average lengths:

$$E[l|K] = KP_D + (J - K)P_F + \bar{P}_D{}^K \bar{P}_F^{J-K},$$

while the variance is given by the second-order combination of the three components:

$$E[l^2|K] = KP_D(KP_D + \bar{P}_D) + (J - K)P_F[(J - K)P_F + \bar{P}_F]$$
$$+ 2KP_D(J - K)P_F + \bar{P}_D^K \bar{P}_F^{J-K}$$

B. Derivation of the first- and second-order moments in (12) and (16), respectively of the length of a relevant epoch-slot in the second retransmission scheme

A relevant epoch is defined as the resolution period where a particular terminal under study is assumed to be always present in the collision event. Since the detection of a persistent terminal has two detection cases, the derivation of the average length of a relevant epoch is split into two cases: when the persistent terminal is correctly detected with probability P_D, and when the terminal is incorrectly detected with probability $\bar{P}_D = 1 - P_D$. When the persistent terminal is correctly detected ($j \in \hat{T}_d \cap \mathcal{T}$, where $\mathcal{T}$ is the set of colliding terminals and $\mathcal{T}_d$ is the set of terminals detected as active), the Probability Mass Function (PMF) of the length of an epoch is given by a modified binomial distribution with parameter P_A considering one terminal j is always present in the collision set and is always correctly detected:

$$\Pr\{l = m | j \in \hat{T}_d \cap \mathcal{T}\}$$
$$= \begin{cases} 0, & m = 0 \\ \begin{pmatrix} J - 1 \\ m - 1 \end{pmatrix} P_A^{m-1} \bar{P}_A^{J-m+1} & 0 < m \leq J \end{cases},$$

and in the case the persistent terminal is not detected as active ($j \notin \hat{T}_d; j \in \mathcal{T}$) we obtain:

$$\Pr\{l = m | j \notin \hat{T}_d; j \in \mathcal{T}\} =$$
$$= \begin{pmatrix} J - 1 \\ m \end{pmatrix} P_A^m \bar{P}_A^{J-m}, \quad m = 0, \ldots J - 1.$$

The unconditional average length of a relevant epoch is obtained by averaging over these previous two PMFs weighted by their probability of occurrence as follows:

$$E[l_r] = P_D E[l | j \in \hat{T}_d \cap \mathcal{T}] + \bar{P}_D E[l | j \notin \hat{T}_d; j \in \mathcal{T}],$$

which can be proved, using the properties of binomial distributions, to lead to:

$$E[l_r] = P_D[(J - 1)P_A + 1] + \bar{P}_D[(J - 1)P_A + \bar{P}_A^{J-1}],$$

which after some algebraic operations yields the desired expression in (12). Similarly the second order moment can be computed as follows:

$$E[l_r^2] = P_D E[l^2 | j \in \hat{T}_d \cap \mathcal{T}] + \bar{P}_D E[l^2 | j \notin \hat{T}_d; j \in \mathcal{T}],$$

which yields:

$$E[l_r^2] = P_D\{(J - 1)P_A[\bar{P}_A + (J - 1)P_A] + 2(J - 1)P_A + 1\} +$$
$$\bar{P}_D\{(J - 1)P_A[\bar{P}_A + (J - 1)P_A] + \bar{P}_A^{J-1}]\},$$

which after some algebraic operations yields the desired expression in (16).