Noninvasive Monitoring of Temperature Distribution in Liquids with Ultrasound by Locally Resolved Measurement of Sound Velocity

M. Wolf, E. Kühnicke
Solid State electronics Laboratory
Technical University of Dresden
Dresden, Germany
Mario.Wolf@tu-dresden.de, Elfgard.Kuehnicke@tu-dresden.de

Abstract **— A new method to measure sound velocity and distance simultaneously and locally resolved is applied in media with continuously changing properties. Instead of using reflectors at known positions the echoes of moving scattering particles are analysed to determine the focus position of an annular array. Systematic deviation between measurements and sound field simulations show that sound propagation has to be described with a modified wave equation. A new approach determining Green's functions for a half space with continuously changing properties is presented. It combines the high frequency approximation with an integral transform method.**

Keywords - Ultrasound: measurement of sound velocity; locally resolved; annular arrays; high frequency approximation; integral transform method

I. INTRODUCTION

A locally resolved monitoring of sound velocity allows estimating locally physical quantities like concentration or temperature or material properties like density or elasticity. This facilitates investigating and optimising many industrial processes, like mixing or chemical reactions, as well as medical therapy like hyperthermia for cancer treatment.

In this contribution, a measurement technique is applied, which allows measuring sound velocity locally resolved using an ultrasound annular array with concentric rings. In contrast to conventional tomographic techniques, it works without any reflectors at known positions. Instead of evaluating different propagation paths, the focusing of the array is varied and the focus position and the sound velocity at the focus point are determined simultaneously by analysing the echoes of moving scattering particles. This is possible because the focus position depends on the sound velocity and the parameters of the used transducer. Therefore, the time of flight to the focus, which is equal to the maximum of the averaged amplitude of the echo signals, is used with calibration curves for the simultaneous determination of sound velocity and focus position.

The examined medium has to contain scattering particles, much smaller than wavelength and in a sufficient low concentration, so that the properties of the propagation medium are not influenced. The emitted wave is reflected at each particle in which the amplitude of the reflected signal is proportional to the amplitude of the incident wave. Therefore the echoes from particles within the focus area are strongest. As particles are in movement it is possible to consider a uniform distribution of particles in average time. So the averaged echo signal amplitude becomes maximal for that time for flight to the focus and back. In preliminary work, media with constant sound velocity were investigated in-depth and sound velocity was determined with a deviance of less than 1‰ [1] [2]. This was possible because the sound field simulations based on GREEN's functions [3] represented the sound emission from the used array very well. To measure a sound velocity profile a static temperature gradient (heating at the top and cooling at the bottom) was generated in water because the dependence of temperature and sound velocity is well known [4] and can be generated in a stable state. To predict the focus position, the sound propagation was modelled with Fermat's principle calculating curved propagation paths by calculus of variation. But a significant deviation of measurements and simulation had been observed [2].

This contribution introduces a new method for modelling sound propagation in media with continuously changing material parameters. The modified wave equation caused by non-constant material parameters is solved for a point source with an integral transform method and a high frequency approximation. At the current state of this work the equation is solved for a point source in the transformed domain.

This paper is divided in 4 sections. The sound propagation in media with non-constant material properties is discussed theoretically in Section II. Section III shows some measurement results for a linear temperature gradient in water. Section IV gives a summary and perspectives.

II. SOUND PROPAGATION IN MEDIA WITH NON-CONSTANT PROPERTIES

A. Derivation of the wave equation

The sound propagation in liquids is based on two fundamental equations, the equations of motion and elasticity:

$$-\nabla p = \rho \dot{\vec{v}}\ , \qquad \chi \dot{p} = \nabla \vec{v}\ , \tag{1}$$

with pressure p, particle velocity v, and the material parameters mass density ρ_0 and elasticity χ. If the material parameters are constant, differentiating these equations with respect to location and time leads to the well known wave equation. If these parameters are functions of location an additional term appears in the derivative of (1) with respect to location. This additional term appears also in the wave equation:

$$\Delta p + \rho_0 \nabla\left(\frac{1}{\rho_0}\right) \cdot \nabla p - \rho_0 \chi \ddot{p} = 0 . \qquad (2)$$

Also the definition of potential Φ has to be modified:

$$\vec{v} = \frac{1}{\rho} grad \Phi . \qquad (3)$$

The following examination shall be done for a one-dimensional dependence of the material properties in direction z.

B. One-dimensional solution

Considering a plane wave propagating in the direction of the gradient of the material properties the wave equation for the potential is obtained

$$\frac{d^2}{dz^2}\Phi + \rho_0 \frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{d}{dz}\Phi - \rho_0 \chi \frac{d^2}{dt^2}\Phi = 0 . \qquad (4)$$

If the mass density depends linearly on z this equation has a solution in the form of a generalized power series [5]:

$$\Phi = \sum_{n=1}^{\infty} c_n \left(z - z_0\right)^{n+K} . \qquad (5)$$

However, there are already numerical problems evaluating the coefficients and the solution with respect to the convergence for this one-dimensional case. So it seems not to be feasible finding an exact solution for a two- or three-dimensional problem.

C. High frequency approximation and integral transform

High frequency approximation had been developed in geophysics and is applied in techniques like ray tracing [6]. In this contribution, harmonic GREEN's functions shall be derived with this approach in combination with an integral transform method. This allows calculating a transfer function for a point source for a specific geometry. Just the axially symmetric problem is solved because it is much easier manageable than the general problem. The wave equation in cylindrical coordinates (r, φ, z) is used:

$$0 = \left[\frac{1}{r}\left(\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right)\right) + \frac{\partial^2}{\partial z^2}\right]\Phi + \rho_0 \frac{\partial}{\partial z}\left(\frac{1}{\rho_0}\right)\frac{\partial}{\partial z}\Phi - \rho_0 \chi \frac{\partial^2}{\partial t^2}\Phi \qquad (6)$$

The single derivative to r in the second term vanishes due to its scalar multiplication with the gradient of the mass density, having only a component in z direction. Applying a Hankel transform as described in [7] with respect to r

$$\Phi^{H0}(\xi, z, t) = \int_0^{\infty} \Phi(r, z, t) J_0(j\omega \xi r) r dr \qquad (7)$$

leads to an one-dimensional wave equation in the transformed domain, which is denoted by the index $H0$:

$$0 = \frac{d^2}{dz^2}\Phi^{H0} + \rho_0 \frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{d}{dz}\Phi^{H0} + \xi^2 \omega^2 \Phi^{H0} + \rho_0 \chi \frac{\partial^2}{\partial t^2}\Phi^{H0} \qquad (8)$$

The high frequency approximation assumes that (8) can be solved with the following ansatz

$$\Phi = A(x, z) e^{j\omega(t - T(x,z))} . \qquad (9)$$

Now, Φ is replaced with this ansatz and the terms are arranged according to its powers of ω.

$$\begin{aligned} 0 = A\left[\omega^2\left[\rho_0 \chi - \left[\frac{dT}{dz}\right]^2 - \xi^2\right]\right] - \ldots \\ j\omega\left[\frac{d^2 T}{dz^2} + 2\frac{1}{A}\frac{dA}{dz}\frac{dT}{dz} + \rho_0 \frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{dT}{dz}\right]\ldots \\ \ldots + \rho_0 \frac{d}{dz}\left(\frac{1}{\rho_0}\right)\frac{dA}{dz} + \frac{d^2 A}{dz^2} \end{aligned} \qquad (10)$$

Assuming that ω is very high the first two lines are equated to zero independently and the frequency-independent third line is neglected. So T can be determined directly from the first line and with this solution A is determined from the second line

$$T(\xi, z) = \pm \int_{z_0}^{z} \sqrt{\rho_0(z)\chi(z) + \xi^2}\, dz . \qquad (11)$$

$$A(\xi, z) = A_0 \sqrt{\frac{\left(\rho_0(z_0)\chi(z_0) - \xi^2\right)\rho_0(z)}{\left(\rho_0(z)\chi(z) - \xi^2\right)\rho_0(z_0)}} . \qquad (12)$$

with the solely free parameter A_0. Note that this leads to the solution for a homogeneous medium if ρ and χ are constants. All methods of generalized ray theory explained in [7] like the derivation of source functions for a point source acting on an interface considering the boundary conditions can be applied to this solution. Finally, the inverse transformation has to be done:

$$\Phi(r, z) = -\omega^2 \int_0^{\infty} A(\xi, z) e^{j\omega(t - T(\xi,z))} J_0(j\omega \xi r) \xi d\xi \cdot \qquad (13)$$

Current work is on an evaluation of this integral with a steepest descent approximation as it is described in [3]. The approximation facilitates the integral into a solvable form. This causes a neglect of surface waves being not of interest for the presented application. However, the method is complicated because of the integral expression of T in the exponent. Though, the integral can be evaluated by a finite series expansion resulting in additional terms containing higher powers of ξ.

III. MEASUREMENT RESULTS

An evident way to achieve a sound velocity gradient is to generate a temperature gradient with water, because the sound velocity as a function of temperature is well known for water [4] and it can be generated in a stable state. Figure 1 shows the general set-up. Water in a basin located at the bottom is kept at a temperature of 6°C. A second smaller basin is placed above. It contains a metal plate at its bottom for good thermal conduction and a heat source at its top. This generates a vertical layered arrangement of warm water above cooler water, whereby a fluid flow is avoided. The temperature is measured by an array of temperature sensors to determine the sound velocity profile in the experimental set-up. The sound field for this gradient and the time of flight to the focus point were calculated applying Fermat's Principle [2].

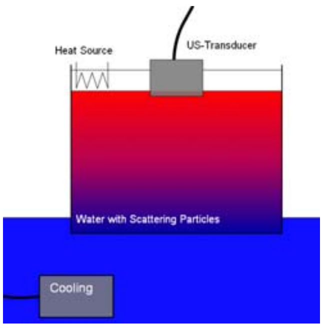

Figure 1: Experimental set-up for a sound velocity gradient

Figure 2 shows the comparison of calculated and measured times of flight to the focus as a function of used set of delay times, corresponding with a focus point (*Fok*) in the calibration medium water of 20 °C (Sound velocity gradient in water from 40 °C at the transducer to 6 °C at a distance of 50 mm).

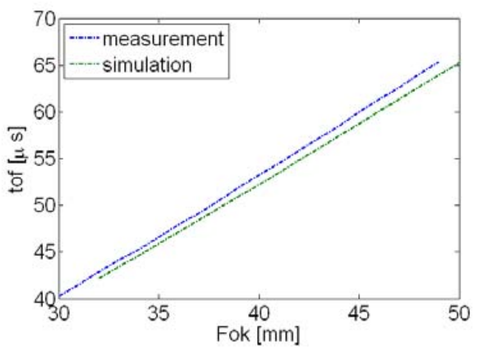

Figure 2: Comparison of measured and calculated times of flight as a function of used set of delay times

Although the notable difference is just in the range of one microsecond this would cause an error of more than 100 m/s determining the sound velocity.

For additional examinations, the sound velocity was measured conventionally via measuring the time of flight to a reflector at a known position. Moving the reflector along the acoustic axis allows a stepwise reconstruction of the sound velocity profile. Additionally sound velocity was determined from temperature sensors again. Figure 3 shows a comparison of two conventional methods to determine the mean sound velocity between the probe and a reflector at distance z. First the time of flight for different reflector distances is measured (blue line). Second the temperatures are measured at different locations and converted to a sound velocity according to [4] and averaged over the propagation path (green line). The systematic deviation can be seen here, too.

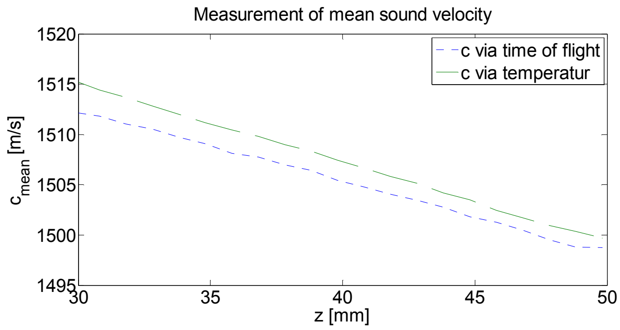

Figure 3: Comparison of two conventional methods to determine the mean sound velocity between the probe and a reflector at distance z

Both deviations (see Figures 2 and 3) result from the deficient assumption that wave propagation can be described with the wave equation for homogeneous media.

IV. SUMMARY AND PERSPECTIVE

This contribution discusses a method to measure the sound velocity along the acoustic axis of the used annular array. The three-dimensional distribution of sound velocity is obtained by scanning.

It has been shown that the continuous change of material properties has to be taken into account for the modelling of sound propagation. The potential of a point source has been calculated in the Hankel transformed domain. The inverse transform is actually realized and will allow calculating GREEN's functions for media with continuously changing properties.

Due to the assumed change of material properties in only one dimension a change of these properties in other dimensions would cause a lateral deviation of the focus position. This effect has to be examined in further works.

ACKNOWLEDGMENT

The authors would like to thank Deutsche Forschungsgemeinschaft (DFG) for their financial support of the ongoing research project KU1075/17-1.

REFERENCES

[1] M. Lenz, M. Bock, E. Kühnicke, J. Pal, and A. Cramer, Measurement of the sound velocity in fluids using the echo signals from scattering particles, Ultrasonics, ULTRAS-D-11-00034.

[2] M. Wolf, E. Kühnicke, and M. Lenz, "Modelling of sound propagation in media with continuously changing properties towards a locally resolved measurement of sound velocity," Ultrasonics Symposium (IUS), IEEE International 21-25 July 2013, vol., no., pp.1045-1048.

[3] E. Kühnicke, "Plane arrays – Fundamental investigations for correct steering by means of sound

field calculations", Wave Motion, Volume 44, Issue 4, March 2007, pp. 248-261, ISSN 0165-2125.

[4] W. Marczak, "Water as a standard in the measurements of speed of sound in liquids." J. Acoust. Soc. Am. 102(5) 1997, pp. 2776–2779.

[5] L. Fuchs: "Zur Theorie der linearen Differentialgleichungen mit veränderlichen Coefficienten." In: Journal für die reine und angewandte Mathematik 66 (1866), p. 121.

[6] V. Cerveny, Seismic Ray Theory, Cambridge University Press, 2005, p. 724.

[7] Y.H. Pao and R.R. Gajewski: The generalized ray theory and transient responses of layered elastic solids, Physical acoustics, 1977 - Academic, New York.

 ISBN: 978-1-61208-375-9