Deduction System for Decision Logic based on Partial Semantics

Yotaro Nakayama* ‡, Seiki Akama† and Tetsuya Murai‡
*Nihon Unisys, Ltd., 1-1-1, Toyosu, Koto-ku, Tokyo, 135-8560, Japan
Email: yotaro.nakayama@unisys.co.jp
†C-Republic, Inc., 1-20-1 Higashi-Yurigaoka, Asao-ku, Kawasaki-shi, 215-0012, Japan
Email: akama@jcom.home.ne.jp
‡Chitose Institute of Science and Technology,
758-65 Bibi, Chitose, Hokkaido, 066-865, Japan
Email: t-murai@photon.chitose.ac.jp

Abstract—**Rough set theory has been extensively used both as a mathematical foundation of granularity and vagueness in information systems and in a large number of applications. However, the decision logic for rough sets is based on classical bivalent logic; therefore, it would be desirable to develop decision logic for uncertain or ambiguous objects. In this study, a deduction system based on partial semantics is proposed for decision logic. Three-valued logics based on Gentzen sequent calculi are adopted. A deductive system based on three-valued framework is intuitively adequate for the structure of positive, negative, and boundary regions of rough sets, and has already been studied. In this study, consequence relations based on partial semantics for decision logic are defined, and systemization by Gentzen's sequent calculi is attempted. Three-valued logics of different structures are investigated as the deductive system of decision logic. The interpretation of decision logic is extended using partial semantics, and extended decision logic based on three-valued logics is proposed.**

Keywords–rough set; decision logic; consequence relation; three-valued logic; sequent calculi.

I. INTRODUCTION

Pawlak introduced the theory of rough sets for handling rough (coarse) information [1]. Rough set theory is now used as a mathematical foundation of granularity and vagueness in information systems and is applied to a variety of problems. In applying rough set theory, decision logic was proposed for interpreting information extracted from data tables. However, decision logic adopts the classical two-valued logic semantics. It is known that classical logic is not adequate for reasoning with indefinite and inconsistent information. Moreover, the paradoxes of material implication of classical logic are counterintuitive.

Rough set theory can handle the concept of approximation by the indiscernibility relation, which is a central concept in rough set theory. It is an equivalence relation, where all identical objects of sets are considered elementary. Rough set theory is concerned with the lower and the upper approximation of object sets. This approximation divides sets into three regions, namely, the positive, negative, and boundary regions. Thus, Pawlak rough sets have often been studied in a three-valued logic framework because the third value is thought to correspond to the boundary region of rough sets [2][3].

In this study, non-deterministic features are considered the characteristic of partial semantics. The formalization of three-valued logic is carried out using a consequence relation based on partial semantics. The basic logic for decision logic is assumed to be many-valued, in particular, three-valued and some of its alternatives [4]. If such three-valued logics are used as a basic deduction system for decision logic, it can be enhanced to a more useful method for data analysis and information processing. The decision logic of rough set theory will be axiomatized using Gentzen sequent calculi and three-valued semantic relation as basic theory. To introduce three-valued logic to decision logic, consequence relations based on partial interpretation are investigated, and sequent calculi of three-valued logic based on them are constructed. Subsequently, three-valued logics with different structure are considered for the deduction system of decision logic.

The deductive system of decision logic has been studied from the granule computing perspective, and in [5], an extension of decision logic was proposed for handling uncertain data tables by fuzzy and probabilistic methods. In [6], a natural deduction system based on classical logic was proposed for decision logic in granule computing. In [2], Gentzen-type three-valued sequent calculi were proposed for rough set theory based on non-deterministic matrices for semantic interpretation.

The paper is organized as follows. In Section II, an overview of rough sets and decision logic is presented. In Section III, the relationship between decision logic and three-valued semantics based on partial semantics is discussed. In Section IV, an axiomatization using Gentzen sequent calculus is presented, according to a consequence relation based on the previously discussed partial semantics. In Section V, an extension of decision logic is discussed, based on three-valued sequent calculus as partial logic. Finally, in Section VI, a summary of the study and possible directions for future work are provided.

II. OVERVIEW OF ROUGH SETS AND DECISION LOGIC

Rough set theory, proposed by Pawlak [1], provides a theoretical basis of sets based on approximation concepts. A rough set can be seen as an approximation of a set. It is denoted by a pair of sets, called the lower and upper approximation of the set. Rough sets are used for imprecise data handling. For the upper and lower approximations, any subset X of U can be in any of three states, according to the membership relation of objects in U. If the positive and negative regions on a rough set are considered to correspond to the truth value of a logical form, then the boundary region corresponds to ambiguity in deciding truth or falsity. Thus, it is natural to adopt a three-valued logic.

 ISBN: 978-1-61208-600-2

Rough set theory is outlined below. Let U be a non-empty finite set, called a universe of objects. If R is an equivalence relation on U, then U/R denotes the family of all equivalence classes of R, and the pair (U, R) is called a Pawlak approximation space. A knowledge base K is defined as follows:

Definition 1. A knowledge base K is a pair $K = (U, R)$, where U is a universe of objects and **R** is a set of equivalence relations on objects in U.

Definition 2. Let $R \in \mathbf{R}$ be an equivalence relation of the knowledge base $K = (U, R)$, and X any subset of U. Then, the lower and upper approximations of X for R are defined as follows:

$$\underline{R}X = \bigcup\{Y \in U/R \mid Y \subseteq X\} = \{x \in U \mid [x]_{\mathrm{R}} \subseteq X\}$$
$$\overline{R}X = \bigcup\{Y \in U/R \mid Y \cap X \neq 0\} = \{x \in U \mid [x]_{\mathrm{R}} \cap X \neq \emptyset\ \}$$

Definition 3. If K = (U,R), R ∈ R, and X ⊆ U, then the R-positive, R-negative, and R-boundary regions of X with respect to R are defined respectively as follows:

$$POS_R(X) = \underline{R}X$$
$$NEG_R(X) = U - \overline{R}X$$
$$BN_R(X) = \overline{R}X - \underline{R}X$$

Let C and D be subsets of an attribute A, denoted as $C, D \subseteq A$. Moreover, it is assumed that C is a conditional attribute and D a decision attribute. Then, the decision table T is denoted by $T = (U, A, C, D)$.

The function $s_x : A \rightarrow V$ (for simplicity, the subscript x will be omitted) is defined where $\forall x \in U$, and $\forall a \in C \cup D$.

Language of Decision Logic: A decision logic language (DL-language) L is now introduced [1]. The set of attribute constants is defined as $a \in A$, and the set of attribute value constants is $V = \bigcup V_a$. The propositional variables are φ and ψ, and the propositional connectives are $\bot,\ \sim,\ \wedge,\ \vee\ and\ \rightarrow$.

Definition 4. The set of formulas of the decision logic language (DL-language) L is the smallest set satisfying the following conditions:

1) (a, v), or in short a_v, is an atomic formula of L.
2) If $\varphi\ and\ \psi$ are formulas of the DL-language, then $\sim\varphi,\ \varphi \wedge \psi,\ \varphi \vee \psi\ and\ \varphi \rightarrow \psi$ are formulas.

The interpretation of the DL-language L is performed using the universe U in $S = (U, A)$ of the Knowledge Representation System ($KR{-}system$) and the assignment function, mapping from U to objects of formulas. Formulas of the DL-language are interpreted as subsets of objects consisting of a value v and an attribute a.

Atomic formulas (a, v) describe objects that have a value v for the attribute a. $S \models_s \varphi$ denotes that the object $x \in U$ satisfies the formula φ of $S = (U, A)$. The semantics of DL-language is defined as follows:

$$S \models_s (a, v)\ \textit{iff}\ a(x) = v$$
$$S \models_s \sim\varphi\ \textit{iff}\ S \not\models_s \varphi$$
$$S \models_s \varphi \vee \psi\ \textit{iff}\ S \models_s \varphi\ or\ S \models_s \psi$$
$$S \models_s \varphi \wedge \psi\ \textit{iff}\ S \models_s \varphi\ and\ S \models_s \psi$$
$$S \models_s \varphi \rightarrow \psi\ \textit{iff}\ S \models_s \sim\varphi \vee \psi$$

Let φ be an atomic formula of the DL-language, $R \in C \cup D$ an equivalence relation, and X any subset of U. Then, the truth value of φ is defined as follows:

$$||\varphi||_s = \begin{cases} \mathbf{t}\ \textit{if}\ |\varphi|_s \subseteq POS_R(U/X) \\ \mathbf{f}\ \textit{if}\ |\varphi|_s \subseteq NEG_R(U/X) \end{cases}$$

This shows that decision logic is based on bivalent logic. In the next section, an interpretation of decision logic based on three-valued logics will be discussed.

III. Relationship with Three-valued Semantics

Partial semantics for classical logic has been studied by van Benthem in the context of the *semantic tableaux* [7][8]. In this section, the application of partial semantics to decision logic is investigated. As the proposed approach can replace the base (bivalent) logic of decision logic, alternative versions of decision logic based on three-valued logics are obtained.

The model $\mathcal{S}$ of decision logic based on three-valued semantics consists of a universe U for the language L and an assignment function s that provides an interpretation for L.

For the domain $|\mathcal{S}|$ of the model $\mathcal{S}$, a subset is defined by $S = \langle S^+, S^- \rangle$. The first term of the ordered pair denotes the set of n-tuples of elements of the universe that $verify$ the relation S, whereas the second term denotes the set of n-tuples that $falsify$ the relation. The interpretation of propositional variables of L for the model $\mathcal{S}$ is given by $S_{\mathcal{S}} = \langle (S)^+_{\mathcal{S}}, (S)^-_{\mathcal{S}} \rangle$. Let $T = \{t, f, u\}$ be the truth value for the three-valued semantics of L, where each value is defined as true, false, or undefined (or indeterminate). Then, the truth value of φ on $S = (U, A)$ is defined as follows:

$$||\varphi||_s = \begin{cases} \mathbf{t}\ \textit{if}\ |\varphi|_s \subseteq POS_R(U/X) \\ \mathbf{f}\ \textit{if}\ |\varphi|_s \subseteq NEG_R(U/X) \\ \mathbf{u}\ \textit{if}\ |\varphi|_s \subseteq BN_R(U/X) \end{cases}$$

A semantic relation for the model $\mathcal{S}$ is defined following [7][9][10]. The truth and the falsehood of a formula of the DL-language are defined in a model $\mathcal{S}$. The truth (denoted by $\models^+_s$) and the falsehood (denoted by $\models^-_s$) of the formulas of the decision logic in $\mathcal{S}$ are defined inductively:

Definition 5. Semantic relation of $\mathcal{S} \models^+_s \varphi$ and $\mathcal{S} \models^-_s \varphi$ are defined as follows:

$$\mathcal{S} \models^+_s \varphi\ \textit{iff}\ \varphi \in \mathcal{S}^+$$
$$\mathcal{S} \models^-_s \varphi\ \textit{iff}\ \varphi \in \mathcal{S}^-$$
$$\mathcal{S} \models^+_s \sim\varphi\ \textit{iff}\ \mathcal{S} \models^-_s \varphi$$
$$\mathcal{S} \models^-_s \sim\varphi\ \textit{iff}\ \mathcal{S} \models^+_s \varphi$$
$$\mathcal{S} \models^+_s \varphi \vee \psi\ \textit{iff}\ \mathcal{S} \models^+_s \varphi\ or\ \mathcal{S} \models^+_s \psi$$
$$\mathcal{S} \models^-_s \varphi \vee \psi\ \textit{iff}\ \mathcal{S} \models^-_s \varphi\ and\ \mathcal{S} \models^-_s \psi$$
$$\mathcal{S} \models^+_s \varphi \wedge \psi\ \textit{iff}\ \mathcal{S} \models^+_s \varphi\ and\ \mathcal{S} \models^+_s \psi$$
$$\mathcal{S} \models^-_s \varphi \wedge \psi\ \textit{iff}\ \mathcal{S} \models^-_s \varphi\ or\ \mathcal{S} \models^-_s \psi$$
$$\mathcal{S} \models^+_s \varphi \rightarrow \psi\ \textit{iff}\ \mathcal{S} \models^-_s \varphi\ or\ \mathcal{S} \models^+_s \psi$$
$$\mathcal{S} \models^-_s \varphi \rightarrow \psi\ \textit{iff}\ \mathcal{S} \models^+_s \varphi\ and\ \mathcal{S} \models^-_s \psi$$

$\models^+$ denotes $confirmation$ and $\models^-$ *refutation*. The model $\mathcal{S}$ is *consistent* if and only if $S^+ \cap S^- = \emptyset$. The symbol $\sim$ denotes strong negation, in which $\sim$ is interpreted as true if the proposition is false.

Theorem 1. For every model $\mathcal{S}$, DL-language L, and formula φ, it is not the case that $\mathcal{S} \models^+_s \varphi$ and $\mathcal{S} \models^-_s \varphi$ hold.

 ISBN: 978-1-61208-600-2

Proof: Only the proof for $\sim$ and $\wedge$ will be provided. It can be carried out by induction on the complexity of the formula. The condition of *consistent* implies that it is not the case that $\varphi \in \mathcal{S}^+$ and $\varphi \in \mathcal{S}^-$. Then, it is not the case that $\mathcal{S} \models_s^+ \varphi$ and $\mathcal{S} \models_s^- \varphi$.

$\sim$: We assume that $\mathcal{S} \models_s^+ \sim\varphi$ and $\mathcal{S} \models_s^- \sim\varphi$ hold. Then, it follows that $\mathcal{S} \models_s^+ \varphi$ and $\mathcal{S} \models_s^- \varphi$. This is a contradiction.

$\wedge$: We assume that $\mathcal{S} \models_s^- \varphi \wedge \psi$ and $\mathcal{S} \models_s^+ \varphi \wedge \psi$ hold. Then, it follows that $\mathcal{S} \models_s^+ \varphi$ *and* $\mathcal{S} \models_s^+ \psi$ and $\mathcal{S} \models_s^- \varphi$ *or* $\mathcal{S} \models_s^- \psi$. This is a contradiction. ■

Example. We assume the decision table below, where the condition and decision attributes are not considered.

$U = \{x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8\}$
Attribute: $C = \{c1, c2, c3, c4\}$
$c_1 = \{x_1, x_4, x_8\}, c_2 = \{x_2, x_5, x_7\}, c_3 = \{x_3\}$,
$c_4 = \{x_6\}$
$U/C = c_1 \cup c_2 \cup c_3 \cup c_4$
Any subset $X = \{x_3, x_6, x_8\}$
$POS_C(X) = c_3 \cup c_4 = \{x_3, x_6\}$
$BN_C(X) = c_1 = \{x_1, x_4, x_8\}$
$NEG_C(X) = c_2 = \{x_2, x_5, x_7\}$
Evaluation of truth value of formulas as follows:
If $|C_{c3}| \subseteq POS_C(X)$ *then* $||C_{c3}||_s = \mathbf{t}$
If $|C_{c1}| \subseteq BN_C(X)$ *then* $||C_{c1}||_s = \mathbf{u}$
If $|C_{c2}| \subseteq NEG_C(X)$ *then* $||C_{c2}||_s = \mathbf{f}$

IV. Consequence relation and Sequent calculus

Partial semantics in classical logic is closely related to the interpretation of the Beth tableau [8]. Van Benthem [7] suggested the relationship of the consequence relation to Gentzen sequent calculus. Thus, the application of the consequence relation for partial semantics to decision logic will be discussed, as well as the structure of three-valued logic that is based on partial semantics and replaces the basic (bivalent) logic of decision logic.

To prove $X \to Y$ by the Beth tableau, a counterexample, such as $X \& \sim Y$, is constructed. Here, let X be Γ and Y be Δ (set of formulas), and let A and B be formulas.

Axiom: $A \Rightarrow A \ \ (ID)$

Sequent rule:

$$\frac{\Gamma \Rightarrow \Delta}{A, \Gamma \Rightarrow \Delta, A}\ (Weakening) \qquad \frac{\Gamma, A \Rightarrow \Delta \quad \Gamma \Rightarrow A, \Delta}{\Gamma \Rightarrow \Delta}\ (Cut)$$

$$\frac{A, \Gamma \Rightarrow \Delta}{\Gamma \Rightarrow \Delta, \sim A}\ (\sim R) \qquad \frac{\Gamma \Rightarrow \Delta, A}{\sim A, \Gamma \Rightarrow \Delta}\ (\sim L)$$

$$\frac{\Gamma \Rightarrow \Delta, A \quad \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \wedge B}\ (\wedge R) \qquad \frac{A, B, \Gamma \Rightarrow \Delta}{A \wedge B, \Gamma \Rightarrow \Delta}\ (\wedge L)$$

$$\frac{\Gamma \Rightarrow \Delta, A, B}{\Gamma \Rightarrow \Delta, A \vee B}\ (\vee R) \qquad \frac{A, \Gamma \Rightarrow \Delta \quad B, \Gamma \Rightarrow \Delta}{A \vee B, \Gamma \Rightarrow \Delta}\ (\vee L)$$

$$\frac{A, \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \to B}\ (\to R) \qquad \frac{\Gamma \Rightarrow \Delta, A \quad B, \Gamma \Rightarrow \Delta}{A \to B, \Gamma \Rightarrow \Delta}\ (\to L)$$

This axiomatization is based on the sequent calculus for classical logic LK (logistischer klassischer Kalkül) originally introduced by Gentzen in 1935 [11]. Decision logic is a predicate logic; however, in this study, the focus is on propositional logic without quantifiers and predicate symbols. This LK is extended to other deductive systems for partial semantics based on a different consequence relation. For example, the three-valued logic by Kleene has no tautology. Thus, to define a consequence relation, a logical system for three-valued logic is formalized. In the Beth tableau, to interpret the consequence relation for partial semantics, an atomic formula A with left open branch is evaluated as $V(A) = 1$, and an atomic formula B with right open branch as $V(B) = 0$. This can be interpreted according to sequent calculus. It is assumed that V is a partial assignment function assigning to an atomic formula the values 0 or 1. Then, the consequence relation is defined as follows:

(C1) *for all* V, *if* $V(Pre) = 1$ *then* $V(Cons) = 1$,

(C2) *for all* V, *if* $V(Pre) = 1$ *then* $V(Cons) \neq 0$.

Pre and $Cons$ represent sequent premise and conclusion, respectively. In classical logic, (C1) and (C2) can be interpreted as equivalent; however, they are not equivalent in partial logic based on partial semantics.

Sequent calculi G1 for (C1) can be obtained by adding the following rules to LK$\backslash\{(\sim R)\}$, where, "$\backslash$" implies that the rule following "$\backslash$" is excluded.

$$\frac{\Gamma \Rightarrow \Delta, A}{\Gamma \Rightarrow \Delta, \sim\sim A}\ (\sim\sim R) \qquad \frac{A, \Gamma \Rightarrow \Delta}{\sim\sim A, \Gamma \Rightarrow \Delta}\ (\sim\sim L)$$

$$\frac{\Gamma \Rightarrow \Delta, \sim A, \sim B}{\Gamma \Rightarrow \Delta, \sim (A \wedge B)}\ (\sim \wedge R)$$

$$\frac{\sim A, \Gamma \Rightarrow \Delta \quad \sim B, \Gamma \Rightarrow \Delta}{\sim (A \wedge B), \Gamma \Rightarrow \Delta}\ (\sim \wedge L)$$

$$\frac{\Gamma \Rightarrow \Delta, \sim A \quad \Gamma \Rightarrow \Delta, \sim B}{\Gamma \Rightarrow \Delta, \sim (A \vee B)}\ (\sim \vee R)$$

$$\frac{\sim A, \sim B, \Gamma \Rightarrow \Delta}{\sim (A \vee B), \Gamma \Rightarrow \Delta}\ (\sim \vee L)$$

These Gentzen-type sequent calculi axiomatize (C1) [12][7].

We are now in a position to define GC1. For GC1, (A1) defined below is added to G1$\backslash\{(\sim L)\}$.

(A1) $A, \sim A \Rightarrow$

GC1:=$\{(ID)$, $(Weakening)$, (Cut), $(A1)$, $(\wedge R)$, $(\wedge L)$, $(\vee R)$, $(\vee L)$, $(\to R)$, $(\to L)$, $(\sim\sim R)$, $(\sim\sim L)$, $(\sim \wedge R)$, $(\sim \wedge L)$, $(\sim \vee R)$, $(\sim \vee L)\}$

For the rule $(\sim L)$ obtained from (A1), GC1 and G1 are equivalent.

Theorem 2. GC1 = G1.

Proof: (A1) can be considered as $(\sim L)$, then double negation and de Morgan laws in GC1 are obtained. ■

The semantic relation of the implication of $\mathcal{S}$ for GC1 is defined in Definition 5.

Then, rule (C2) for the Gentzen system is axiomatized as GC2. GC2 is obtained by replacing axiom (A1) from GC1 to (A2) below.

(A2) $\Rightarrow A, \sim A$

By exclusion of the restriction in Theorem 1, the definition of the semantic relation for the implication of GC2 is obtained as follows:

$\mathcal{S} \models_s^+ \varphi \to \psi$ *iff* $\mathcal{S} \not\models_s^+ \varphi$ *or* $\mathcal{S} \not\models_s^- \psi$ or
$(\mathcal{S} \models_s^+ \varphi$ *and* $\mathcal{S} \models_s^- \varphi$ *and* $\mathcal{S} \models_s^+ \psi$ *and* $\mathcal{S} \models_s^- \psi)$
$\mathcal{S} \models_s^- \varphi \to \psi$ *iff* $\mathcal{S} \models_s^+ \varphi$ *and* $\mathcal{S} \models_s^- \psi$

Theorem 3. C2 is axiomatized by GC2.

 ISBN: 978-1-61208-600-2

Proof: GC2 is an axiomatization which is obtained from GC1 by replacing (A1) with (A2). ∎

There are some possible options to define consequence relation. For our purposes, (C3) below is proposed as alternative definition.

(C3) *for all* V, *if* $V(Pre) = 1$ *then* $V(Cons) = 1$,
if $V(Cons) = 0$ *then* $V(Pre) = 0$.

The Gentzen system GC3 for (C3) is obtained by replacing (A1) of GC1 with the following (A3):

(A3) $A, \sim A \Rightarrow B, \sim B$

V. Relationship Partial Logic

In this section, the relationship between the sequent calculi system based on partial semantics and three-valued logic is discussed. The three-valued logic is extended by defining the weak negation $\neg$. $\sim$ is treated as the strong or classical negation. Weak negation represents the lack of truth. In partial semantics, it allows an interpretation whereby $\neg$ is true if a proposition is not true, that is false or undefined. The semantic relation for weak negation is as follows:

$\mathcal{S} \models_s^+ \neg\varphi$ *iff* $\mathcal{S} \not\models_s^+ \varphi$

$\mathcal{S} \models_s^- \neg\varphi$ *iff* $\mathcal{S} \models_s^+ \varphi$

The truth value of weak negation is defined as follows:

$$||\neg\varphi||_s = \begin{cases} \mathbf{t} \ \textit{if} \ ||\varphi||_s = \mathbf{f} \ \textit{or} \ \mathbf{u} \\ \mathbf{f} \ \textit{if} \ ||\varphi||_s = \mathbf{t} \end{cases}$$

By introducing weak negation, the representation of deduction for uncertain concepts may be handled; however, this is beyond the scope of this study. Moreover, weak implication may be defined using weak negation as follows:

$A \rightarrow_w B =_{def} \neg A \vee B$

The following rules for weak negation and weak implication are now presented.

$$\frac{\Gamma \Rightarrow A, \Delta}{\Gamma \Rightarrow \neg A, \Delta}\ (\neg R) \qquad \frac{\Gamma, A \Rightarrow \Delta}{\Gamma, \neg A \Rightarrow \Delta}\ (\neg L)$$

$$\frac{A, \Gamma \Rightarrow \Delta, B}{\Gamma \Rightarrow \Delta, A \rightarrow_w B}\ (\rightarrow_w R) \qquad \frac{B, \Gamma \Rightarrow \Delta \quad \Gamma \Rightarrow \Delta, A}{A \rightarrow_w B, \Gamma \Rightarrow \Delta}\ (\rightarrow_w)$$

Three extended decision logics (EDLs) based on three-valued logic are subsequently presented. They are adapted to handle ambiguity and uncertainty. GC1, which was discussed above, is interpreted as a strong Kleene three-valued logic. It is first assumed that GC1 is the basic deduction system for decision logic. Then, the inference rules of weak negation and weak implication are added. This logic is the extended decision logic EDL1. Its semantic relation is denoted by $\models_{EDL1}$.

The axioms and rules of EDL1 are as follows:

EDL1 := $\{(ID), (Weakening), (Cut), (A1), (\wedge R), (\wedge L),$
$(\vee R),\ (\vee L),\ (\rightarrow R),\ (\rightarrow L),\ (\sim\sim R),\ (\sim\sim L),$
$(\sim \wedge R),\ (\sim \wedge L),\ (\sim \vee R),\ (\sim \vee L),$
$(\neg R),\ (\neg L),\ (\rightarrow_w R),\ (\rightarrow_w L)\}$

The concept of a proposition that is neither true nor false is possible in EDL1. If the designated value of three-valued logic of GC2 is defined as {t, u}, then this system is a paraconsistent logic. Paraconsistent logic does not hold for the principle of explosion (ex falso quodlibet); therefore, it is possible to interpret the consequence relation by (C2). The semantic relation of EDL2 is obtained from EDL1 by replacing (A1) with (A2).

EDL2 := EDL1\{(A1)} + {(A2)}

The semantic relation of EDL3 is obtained from EDL1 replacing (A1) with (A3).

EDL3 := EDL1\{(A1)} + {(A3)}

EDL3 is interpreted as both paracomplete and paraconsistent. This prevents the paradox of material implication of classical logic. In decision logic, the decision rule is interpreted as follows: If the premise is valid, then the conclusion is also valid. If the conclusion is not valid, then the premise is not valid either.

VI. Conclusion and Future Work

It was proposed that a partial semantics interpretation of the consequence relation may serve as a foundation for decision logic. A three-valued logic system based on a consequence relation that is defined by partial semantics was investigated, and the relationship between them was studied. By adopting three-valued logic as basic logic for decision logic, its deductive system can be enhanced. Moreover, this allows the extension of the scope of its application.

In future work, the semantic relationship between decision logic and partial semantics should be investigated in detail. Furthermore, soundness and completeness results should be derived for extended decision logic. This is required for the foundation of a logical system for decision logic. Finally, the application of decision logic based on three-valued logic should be investigated.

Acknowledgment

We thank anonymous referees for valuable feedback on an earlier version of this paper.

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