Dual Decomposition Method for Reliable Allocations of Wireless Network Resources

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***Abstract*—In the paper, we consider the problem of allocation of network resources in telecommunication networks with respect to both utility and reliability goals. We suggest a solution method based on decomposition and gradient methods for this problem. We present numerical results for the suggested method on test examples.**

***Index Terms*—Telecommunication networks; allocation of links; decomposition method; gradient method.**

I. Introduction

The current development of information technologies and telecommunications gives rise to new control problems related to efficient transmission of information and allocation of limited network resources. All these problems are determined on distributed systems where the spatial location of elements is taken into account. Due to strong variability and increasing demand of different wireless telecommunication services, fixed allocation rules usually lead to serious congestion effects and inefficient utilization of network resources despite the presence of very powerful processing and transmission devices. This situation forces one to replace the fixed allocation rules with more flexible mechanisms, which are based on proper mathematical models; see e.g., [1]– [3]. For example, solution methods for network resource allocation based on optimization formulations of network manager problems and decomposition techniques were presented in [4] [5]. In these problems, the goal function is the total network profit obtained from the total income of users payments and the implementation costs of the network. Otherwise, the total network users utility can serve as a goal function.

At the same time, wireless networks should be reliable with respect to various attacks. The most commonly seen in wireless networks are eavesdropping in which attackers aim at acquiring important/private information of users, jamming and Distributed Denial of Service (DDoS) attacks, which attempt to interfere and disrupt network operations by exhausting the resources available to legitimate systems and users. These attacks may lead to degrading the network performance and Quality of Service (QoS), as well as losing important data, reputations, and revenue; see e.g. [6]–[9]. Hence, the network resource allocation problem should take into account reliability estimates.

In [10], we considered a problem of telecommunication network link resources allocation among users under reliability control of network connections with the pre-defined non-reliability level. For this problem, it was were suggested a penalty method. This method attained a solution, but its convergence does not allow one to attain high accuracy of solutions.

In this paper, we consider also the problem of allocation of link resources in telecommunication networks with respect to both utility and reliability goals. However, unlike [10], we do not indicate any pre-defined non-reliability level. Our cost function is a difference of the total network utility and non-reliability. By using the dual Lagrangian method with respect to the balance constraint, we replace the initial problem with an unconstrained optimization problem, where calculation of the cost function value leads to independent solution of single-dimensional problems. We present results of computational experiments which confirm the applicability of the new methods.

In Section 2 we describe the link resources allocation problem in telecommunication networks with respect to both utility and reliability constraints. In Section 3 we describe how to apply the dual Lagrange method for solving the original optimization problem. Finally, in Section 4 we give computational results which confirm rather stable performance of the method.

II. Problem Description

We first take the optimal link distribution problem in computer and telecommunication data transmission networks, which was suggested in [11]. This model describes a network that contains a set of transmission links (arcs) L and accomplishes some submitted data transmission requirements from a set of selected pairs of origin-destination vertices I within a fixed time period. Denote by x_i and α_i the current and maximal value of data transmission for pair demand i, respectively, and by c_l the capacity of link l. Each pair demand is associated with a unique data transmission path, hence each link l is associated uniquely with the set I_l of pairs of origin-destination vertices, whose transmission paths contain this link. For each pair demand x_i, we denote by $u_i(x_i)$ the

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utility value at this data transmission volume. Then, we can write the network utility maximization problem as follows:

$$\max \to \sum_{i\in I} u_i(x_i)$$

subject to

$$\sum_{i\in I_l} x_i \le c_l,\ l \in L;$$
$$0 \le x_i \le \alpha_i,\ i \in I.$$

If the functions $u_i(x_i)$ are concave, this is a convex optimization problem.

Let us now consider the same telecommunication network where the reliability factor should be taken into account. Namely, we associate the reliability to each arc flow and determine $\mu_l(f_l)$ as the non-reliability of the l-th arc having the flow f_l for $l \in L$. Then $\sum_{l\in L}\mu_l(f_l)$ is the total network non-reliability and we formulate the network manager problem as follows:

$$\max \to \sum_{i\in I} u_i(x_i) - \sum_{l\in L}\mu_l(f_l), \tag{1}$$

subject to

$$\sum_{i\in I_l} x_i = f_l,\ l \in L; \tag{2}$$
$$0 \le f_l \le c_l,\ l \in L; \tag{3}$$
$$0 \le x_i \le \alpha_i,\ i \in I. \tag{4}$$

If the functions $u_i(x_i)$ and $-\mu_l(f_l)$ are concave, this is a convex optimization problem with the polyhedral feasible set. However, solution of problem (1)–(4) is not so easy due to large dimensionality and inexact data. In this paper we consider the case where the functions $u_i(x_i)$ and $-\mu_l(f_l)$ are strictly concave. Then, we can apply the known dual decomposition technique.

III. Dual Decomposition Method

Let us define the Lagrange function of problem (1)–(4) as follows:

$$L(x,f,y) = \sum_{i\in I} u_i(x_i) - \sum_{l\in L}\mu_l(f_l) + \sum_{l\in L} y_l\left(\sum_{i\in I_l} x_i - f_l\right)$$

for

$$x \in X = \prod_{i\in I}[0,\alpha_i] \text{ and } f \in F = \prod_{l\in L}[0,c_l]\,.$$

By duality, we can replace problem (1)–(4) with the dual unconstrained optimization problem:

$$\min \to \varphi(y), \tag{5}$$

where

$$\varphi(y) = \max_{x\in X, f\in F} L(x,f,y). \tag{6}$$

Clearly, the dual cost function φ is convex. Moreover, under the strict convexity of the functions μ_l and the strict concavity of the functions u_i it is differentiable. Calculation of its value and its gradient is rather simple and decomposed into independent solution of single-dimensional problems. Denote by L_i the set of links belonging to the path associated with the origin-destination pair i. By definition,

$$\begin{aligned}
\varphi(y) &= \max_{x\in X, f\in F} L(x,f,y)\\
&= \sum_{i\in I}\max_{0\le x_i\le\alpha_i}\left\{u_i(x_i) + x_i\sum_{l\in L_i} y_l\right\}\\
&- \sum_{l\in L}\min_{0\le f_l\le c_l}\{\mu_l(f_l) + y_l f_l\}\\
&= \sum_{i\in I}\left\{u_i(x_i(y)) + x_i(y)\sum_{l\in L_i} y_l\right\}\\
&- \sum_{l\in L}\{\mu_l(f_l(y_l)) + y_l f_l(y_l)\}\,,
\end{aligned}$$

where $x_i(y)$ and $f_l(y_l)$ are unique solutions of the single-dimensional optimization problems

$$\max_{0\le x_i\le\alpha_i}\left\{u_i(x_i) + x_i\sum_{l\in L_i} y_l\right\}$$

and

$$\min_{0\le f_l\le c_l}\{\mu_l(f_l) + y_l f_l\}\,,$$

respectively. Next, we obtain

$$\frac{\partial\varphi(y)}{\partial y_l} = \sum_{i\in I_l} x_i(y) - f_l(y_l),\ l \in L.$$

These properties enable us to apply the usual Uzawa gradient method to find a solution of the dual problem (5):

$$y^{k+1} = y^k - \lambda_k\varphi'(y^k), \lambda_k > 0.$$

IV. Computational Experiments

As part of the work, a numerical study of the suggested method was carried out. The method was implemented in C++ with a PC with the following facilities: Intel(R) Core(TM) i7-4500, CPU 1.80 GHz, RAM 6 Gb.

In the experiments, we used quadratic functions of utility of origin-destination pairs (QuadC)

$$u_i(x_i) = u_{1,i}x_i^2 + u_{0,i}x_i,\ u_{1,i} < 0, u_{0,i} > 0, i \in I,$$

quadratic functions of non-reliability of arcs (QuadA)

$$\mu_l(f_l) = \mu_{1,l}f_l^2 + \mu_{0,l}f_l,\ \mu_{1,l},\mu_{0,l} > 0, l \in L,$$

logarithmic functions of utility of origin-destination pairs (LogC)

$$u_i(x_i) = u_{2,i}\ln(u_{0,i} + u_{1,i}x_i),\ u_{j,i} > 0, j = 0,\ldots,2, i \in I,$$

and logarithmic functions of non-reliability of arcs (LogA)

$$\mu_i(x_i) = \mu_{0,l}f_l - \ln(1 + \mu_{1,l}f_l),\ \mu_{0,l},\mu_{1,l} > 0, l \in L.$$

All the arcs and origin-destination pairs were indexed as $l = 0,\ldots,|L|-1$ ($|L|$ is the cardinality of L) and $i = 0,\ldots,|I|-1$ ($|I|$ is the cardinality I), respectively.

The coefficients $\mu_{1,l}$, $\mu_{0,l}$, $u_{0,i}$, $u_{1,i}$, and $u_{2,i}$ were formed on the basis of trigonometric functions:

(i) for the functions QuadC

$$u_{0,i} = 2|\sin(2i+2)|+2,\ u_{1,i} = -|\cos(2i+1)|-1,$$

(ii) for the functions QuadA

$$\mu_{0,l} = |\cos(l+1)|+3,\ \mu_{1,l} = 2|\sin(2l+2)|+1,$$

(iii) for the functions LogC

$$\begin{aligned} u_{0,i} &= 2|\sin(2i+2)|+1,\ u_{1,i} = |\sin(i+2)|+1, \\ u_{2,i} &= 3|\sin(2i+2)|+1, \end{aligned}$$

(iv) for the functions LogA

$$\mu_{0,l} = 10|\cos(l+1)|+10,\ \mu_{1,l} = 2|\cos(2l+2)|+1.$$

The maximal arc flow capacity c_l was selected in [1] [10] as follows:

$$c_l = 10|\cos(l+3)|+1.$$

The maximal path flow capacity α_i associated with a origin-destination pair was selected in [1] [7] as follows:

$$\alpha_i = 7|\sin(i)|+1.$$

The stepsize parameter λ_k in the dual gradient method was fixed and equal to 0.6.

In our tests, we used the following combinations of functions: QuadA-LogC, QuadA-QuadC, LogA-LogC. The distribution of the available arcs across the origin-destination pairs was carried out either uniformly or according to the normal distribution law. In the gradient method, we used two different initial points: the vector e of units and vector $100e$.

We now introduce additional notations:

1) ε is the accuracy of finding solution of the problem,
2) $T_{\varepsilon,1}$ and $T_{\varepsilon,100}$ are the time (in seconds) of the method with the starting point e and $100e$, respectively,
3) $I_{\varepsilon,1}$ and $I_{\varepsilon,100}$ are the numbers of iterations spent searching for a solution to the problem with the starting point e and $100e$, respectively.

The gradient method was stopped if the norm $\|\varphi'(y^k)\|$ appeared less than ε. In Tables I–IV we give the results of finding a solution of the problem with QuadA-LogC combination of functions. In Table I, we give the results for the case where $|I| = 620$, $|L| = 310$ and for different values ε. In Table II, we give the results for the case where $|I| = 310$, $|L| = 620$ and for different values ε. In Table III, we give the results for the case where $\varepsilon = 10^{-2}$, $|L| = 310$ and for different values $|I|$. In Table IV, we give the results for the case where $\varepsilon = 10^{-2}$, $|I| = 310$ and for different values $|L|$. In this series of numerical experiments the solution time was less than one second. The experiments for the cases QuadA-QuadC and LogA-LogC gave similar results, which are given in Tables V–VIII and IX–XII, respectively.

V. Conclusions

We presented a general problem of allocation of network resources in telecommunication networks using both utility and reliability factors. We suggest to apply the dual decomposition method to this problem. The results of computational experiments confirmed rather stable performance of the method.

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References

[1] C. Courcoubetis and R. Weber, Pricing Communication Networks: Economics, Technology and Modelling. Chichester: John Wiley & Sons, 2003.

[2] S. Stanczak, M. Wiczanowski and H. Boche, Resource Allocation in Wireless Networks. Theory and Algorithms. Berlin: Springer, 2006.

[3] A. M. Wyglinski, M. Nekovee and Y. T. Hou (eds.), Cognitive Radio Communications and Networks: Principles and Practice. Amsterdam: Elsevier, 2010.

[4] I. Konnov, A. Kashuba and E. Laitinen, ”Dual methods for optimal allocation of telecommunication network resources with several classes of users,” Math. Comput. Appl., vol. 23, no 31, pp. 1–14, 2018.

[5] I. V. Konnov, A. Yu. Kashuba, ”Application of the conditional gradient method to a network resource allocation problem with several classes of users,” Journal of Physics: Conference Series. 1158, no 032015, pp. 1–8, 2019.

[6] S. T. Zargar, J. Joshi and D. Tipper, ”A survey of defense mechanisms against distributed denial of service (ddos) flooding attacks,” IEEE Communications Surveys & Tutorials, vol. 15, no 4, pp. 2046–2069, 2013.

[7] M. H. Manshaei, Q. Zhu, T. Alpcan, T. Bacsar and J.-P. Hubaux, ”Game theory meets network security and privacy,” ACM Computing Surveys (CSUR), vol. 45, no 3, pp. 25–64, 2013.

[8] S. Vadlamani, B. Eksioglu, H. Medal and A. Nandi, ”Jamming attacks on wireless networks: A taxonomic survey,” International Journal of Production Economics, vol 172, pp. 76–94, 2016.

[9] N. C. Luong, D. T. Hoang, P. Wang, D. Niyato, Z. Han, ”Applications of economic and pricing models for wireless network security: a survey,” IEEE Communications Surveys & Tutorials, vol. 19, no 4, pp. 2735–2767, 2017.

[10] I. Konnov, A. Kashuba, E. Laitinen, ”Penalty method for reliable allocations of wireless network resources,” CEUR Workshop Proceedings, no 2642, 2020.

[11] F. P. Kelly, A. Maulloo and D. Tan, ”Rate control for communication networks: shadow prices, proportional fairness and stability,” J. Oper. Res. Soc., vol. 49, no 3, 237–252, 1998.

TABLE I
Computations for $|I| = 620$, $|L| = 310$ (QuadA-LogC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	50	0.078	200
10^{-2}	0.028	57	0.094	220
10^{-3}	0.031	70	0.125	248
10^{-4}	0.047	91	0.172	407

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TABLE II
COMPUTATIONS FOR $|I| = 310$, $|L| = 620$ (QUADA-LOGC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	35	0.235	466
10^{-2}	0.027	65	0.344	629
10^{-3}	0.032	67	0.375	703
10^{-4}	0.035	77	0.377	733

TABLE III
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|L| = 310$ (QUADA-LOGC)

$\|I\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.028	57	0.094	220
930	0.047	84	0.078	157
1240	0.078	84	0.109	139

TABLE IV
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|I| = 310$ (QUADA-LOGC)

$\|L\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.027	65	0.344	629
930	0.047	63	0.563	716
1240	0.062	70	0.750	783

TABLE V
COMPUTATIONS FOR $|I| = 620$, $|L| = 310$ (QUADA-QUADC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	24	0.094	198
10^{-2}	0.023	51	0.093	229
10^{-3}	0.031	77	0.109	268
10^{-4}	0.047	124	0.172	408

TABLE VI
COMPUTATIONS FOR $|I| = 310$, $|L| = 620$ (QUADA-QUADC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.015	32	0.234	467
10^{-2}	0.031	58	0.365	691
10^{-3}	0.032	61	0.370	770
10^{-4}	0.047	97	0.374	793

TABLE VII
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|L| = 310$ (QUADA-QUADC)

$\|I\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.023	51	0.093	229
930	0.032	48	0.094	159
1240	0.031	51	0.094	150

TABLE VIII
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|I| = 310$ (QUADA-QUADC)

$\|L\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.031	58	0.365	691
930	0.047	53	0.640	815
1240	0.078	66	0.797	856

TABLE IX
COMPUTATIONS FOR $|I| = 620$, $|L| = 310$ (LOGA-LOGC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	31	0.078	161
10^{-2}	0.032	57	0.084	186
10^{-3}	0.047	93	0.125	262
10^{-4}	0.062	129	0.156	334

TABLE X
COMPUTATIONS FOR $|I| = 310$, $|L| = 620$ (LOGA-LOGC)

ε	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
10^{-1}	0.016	28	0.187	265
10^{-2}	0.041	69	0.203	313
10^{-3}	0.047	93	0.265	491
10^{-4}	0.063	118	0.344	649

TABLE XI
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|L| = 310$ (LOGA-LOGC)

$\|I\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.032	57	0.084	186
930	0.047	71	0.094	127
1240	0.063	72	0.125	163

TABLE XII
COMPUTATIONS FOR $\varepsilon = 10^{-2}$, $|I| = 310$ (LOGA-LOGC)

$\|L\|$	$T_{\varepsilon,1}$	$I_{\varepsilon,1}$	$T_{\varepsilon,100}$	$I_{\varepsilon,100}$
620	0.041	69	0.203	313
930	0.047	64	0.312	434
1240	0.094	70	0.328	342

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