# Random Transmission in Cognitive Uplink Network

S. Barman Roy
School of Computer Engineering
Nanyang Technological University
Email: swagato002@ntu.edu.sg

S. N. Merchant
Department of Electrical Engineering
Indian Institute of Technology Bombay
Email: merchant@ee.iitb.ac.in

A. S. Madhukumar
School of Computer Engineering
Nanyang Technological University
Email: asmadhukumar@ntu.edu.sg

***Abstract*—To meet the ever increasing demand for higher data rate, improving spectral efficiency is absolutely essential. Spectrum sharing between several users has been proposed for maximum utilisation of available bandwidth. But whenever multiple users are using the same frequency band and at same time, they are going to interfere with each other resulting in poorer performance as compared to single user scenario. This work proposes a novel scheme for channel capacity improvement in Multiple Access Systems (Uplink Communication) in cognitive radio networks and explores the trade offs involved among the cognitive users and primary user. It is argued that, when the mobile transmitters lack the channel state information, they can't use the broadcast scheduling algorithm to cooperate with each other. Convex-concave properties of the data rate is used to find the appropriate bounds. The corresponding scenario with Broadcast Systems (Downlink Communication) is compared where the transmitter has perfect knowledge of the channel state information.**

***Keywords-Broadcast Scheduling; Cognitive Radio; Interference Channel; MIMO Channel; Multiple Access Interference.***

## I. Introduction

The last decade has seen enormous growth of wireless devices in consumer driven and industrial applications resulting in an exponential demand for data rate through wireless media. Eventually, today's telecommunication infrastructures are severely strained to meet this demand. Particularly, radio frequency is a resource whose availability is limited by physics and hardware technology. Still, a number of survey results, including [1] and [2] have shown that the current spectrum usage is suboptimal from utility point of view. In many cases (paging and amateur radios, for example), the licensed users remain silent rendering the dedicated frequency bands idle. At the same time, the mobile frequency bands are severely overloaded to serve existing users and to meet the increasing demand from new users. To meet the discrepancy between high demand and suboptimal usage, cognitive networking has been gaining popularity in current research.

Two major approaches proposed in cognitive networking (both aiming to improve spectral efficiency) are spectrum *sensing* and spectrum *sharing*. A practical system can use any of the approaches or a combination thereof. In the context, a licensed user is referred as a primary user who got priority access over a spectrum. In addition to the primary user there may be one or more secondary (cognitive) users who will use the spectrum opportunistically.

A spectrum sensing network operates based upon burst nature of primary user transmissions. To ensure that no spectrum band remains idle, many experts (see [3] for example) have advocated dynamic spectrum access where the secondary users will continuously scan for free spectral bands, known as *White Space* and use them for transmission. Certainly, it is necessary to ensure retreat of secondary users once the primary users resume their transmission. Needless to say, sensing the spectral range for a White Space and making correct decision about temporary presence/absence of primary user plays the most important role.

In the paradigm of spectrum sharing, along with the primary user, the secondary users will use the spectrum for their own communication so as to cause minimum degree of interference to each other. The primary user will definitely not have an exclusive right over the spectrum, but cognitive users has to ensure that their harmful interference is kept below a certain threshold so that, in an ideal scenario, the primary user is not even aware of their existence. The description is an extremely generic one and quantitative analysis of interference between users will be dependent upon the specific system itself. In this work, the focus will be on a cognitive uplink network described in Section III.

### A. Organisation

This work is organised as follows. Section II gives the background with the current state of the art literature. Section III describes the system model. In Section IV, the proposed technique of random transmission is analysed and the simulation results are given in Section V. Then, Section VI discusses implications of the results and possible applications.

### B. Notations

Capital boldface letters stand for matrices and lowercase boldface for vectors. $||\mathbf{v}||$ and $||\mathbf{v}||_1$ give the Euclidean and $\mathcal{L}_1$ norm of a vector $\mathbf{v}$ respectively. $\mathbf{A}^{\mathrm{H}}$ and $\mathbf{A}^{\mathrm{T}}$ respectively denote conjugate transpose and transpose of matrix $\mathbf{A}$. $\mathbf{u} \succ \mathbf{v}$ indicates tuple wise inequality between two vectors valid for each tuple. $\mathbb{R}$ and $\mathbb{C}$ denote the fields of real numbers and complex numbers respectively. $\mathbb{E}(X)$ gives expected value of

 ISBN: 978-1-61208-313-1

the random number $X$ and $\mathbb{P}(\mathcal{A})$ gives probability of an event $\mathcal{A}$.

## II. Background Work

Multiple Input Multiple Output (MIMO) systems have long been proposed as a way to improve capacity of systems. Effects of multiple antennae (in terms of power allocation and diversity) have been extensively studied in the general context of wireless network. In [4], it is shown that capacity increases linearly with min$\{M, N\}$ where $M$ and $N$ are respectively number of transmit and receive antennae. The flexibility offered by MIMO systems makes it an ideal candidate to meet the challenges of cognitive interference network and various system models have been evaluated in literature. For example, the simplest case of two transmit one receive antenna (a MISO system) has been studied in [5]. A more general approach of user scheduling in a broadcast channel with an objective of throughput maximisation has been undertaken in [6]. The present work concerns with an uplink system model. It will be shown afterwards, the fundamental differences between uplink/downlink models in terms of joint versus distributed receive strategies or centralised power control for downlink will have important implications on performance.

## III. System Model

When several users spread throughout a coverage area transmit to a base station, it is called a multiple access channel (MAC). A standard multiple access system model with a

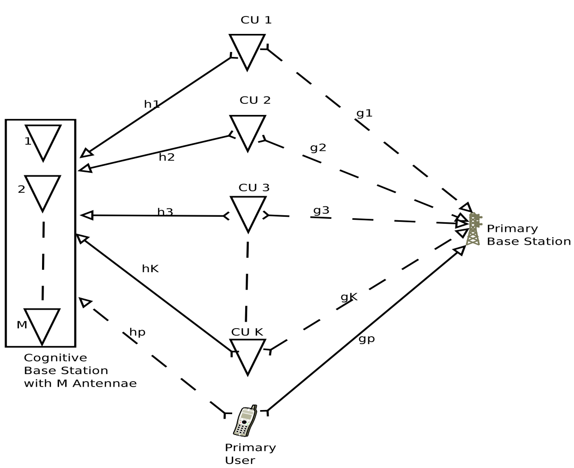


Fig. 1. Multiple Access System Model in Presence of Primary User

cognitive base station (having with $M$ antennae), a single antenna primary base station, $K$ cognitive users and a primary user (each having a single antenna) is shown in Figure 1. The $M \times 1$ channel vector from cognitive user $k$ to the cognitive base station is given by $\mathbf{h}_k, \quad \forall 1 \le k \le K$, the channel from primary user to cognitive base station is another $M \times 1$ channel vector $\mathbf{h}_p$. The scalar channels from cognitive user $k$ and primary user to primary base station are given by $g_k$ and $g_p$ respectively. Throughout this work, our assumption is $M \ll K$, implying number of user is much more than number of antenna. Each cognitive user is transmitting a scalar symbol $s_k \in \mathbb{C}$ and the primary user is transmitting a symbol $s_p \in \mathbb{C}$. It is obvious that power usage of cognitive user $k$ is given by $P_k = \frac{|s_k|^2}{T}$ and that of primary user is $P_p = \frac{|s_p|^2}{T}$, where $T$ is the symbol duration. We assume coherent detection with same symbol rates at the receivers. Since the users are transmitting at same frequency and time, there will be interference between them. Additionally, since the symbol rates are same, the symbol itself serves as a measure of power ignoring a constant. So, we will take $P_k = |s_k|^2$ and $P_p = |s_p|^2$ for brevity.

### A. Multiuser Decoding

With the defined notations, the $\mathbb{C}^{M\times 1}$ vector received at cognitive base station is

$$\mathbf{y} = \sum_{k=1}^{K} \mathbf{h}_k s_k + \mathbf{h}_p s_p + \boldsymbol{\eta} \quad \in \mathbb{C}^M \tag{1}$$

and the scalar received at primary base station

$$y_p = \sum_{k=1}^{K} g_k s_k + g_p s_p + \eta \quad \in \mathbb{C} \tag{2}$$

where $\boldsymbol{\eta} = [\eta_1, \eta_2, \ldots, \eta_M]^{\mathrm{T}}$ is the noise vector with independent identically distributed components and $\eta$ is the scalar noise at the primary receiver.

For brevity again, $\boldsymbol{\eta}$ can be assumed to have a covariance matrix of identity and $\eta$ also has unit variance. So $\mathbb{E}(\boldsymbol{\eta}\boldsymbol{\eta}^{\mathrm{H}}) = \mathbf{I}_M$ (the $M \times M$ identity matrix) and $\mathbb{E}(\eta\eta^*) = 1$. In fact, if the variances are anything apart from unity, the entire expressions of 1 and 2 can be divided by the corresponding factor. That will give rise to the same expression and the constants absorbed in the channel co-efficients.

For the cognitive base station, which receives the vector $\mathbf{y}$, since all the antennae are connected to the same radio front end, joint decoding is possible for each user with a set of $K$ receive strategies. Optimal receive strategies for each of $K$ users can be selected independently [7] with an objective to maximise the individual SINRs. Usually the receive strategy of user $k$ is just to multiply the received signal with a row vector $\mathbf{f}_k$ and the decision statistic is the product $\mathbf{f}_k\mathbf{y}$.

With all the users transmitting simultaneously, the optimal receive strategy of user $k$ can be found using Wirtinger derivative of real functions(see [8]) as $\mathbf{f}_k = \mathbf{A}_k^{-1}\mathbf{h}_k$ where

$$\mathbf{A}_k = \mathbf{I}_M + \mathbf{h}_p\mathbf{h}_p^{\mathrm{H}} p_p + \sum_{i=1, i\neq k}^{K} \mathbf{h}_i\mathbf{h}_i^{\mathrm{H}} p_i \tag{3}$$

Since the primary base station has a single antenna and not connected to the cognitive bases station, its decision statistic is the received scalar $y_p$. After multiplying with the receive strategies, we get the following SINR expressions for cognitive users.

$$\gamma_k = \frac{|\mathbf{f}_k^{\mathrm{H}}\mathbf{h}_k|^2 P_k}{\mathbf{f}_k^{\mathrm{H}}(\mathbf{I}_M + \mathbf{h}_p\mathbf{h}_p^{\mathrm{H}} P_p + \sum_{i=1, i\neq k}^{K} \mathbf{h}_i\mathbf{h}_i^{\mathrm{H}} P_i)\mathbf{f}_k} \quad \forall k \tag{4}$$

 ISBN: 978-1-61208-313-1 

and the primary user SINR is given by

$$\gamma_p = \frac{|g_p|^2 P_p}{1 + \sum_{k=1}^{K} |g_k|^2 P_k} \tag{5}$$

Corresponding maximum data rates (channel capacities normalised by the bandwidth) for all the users users are given by

$$r = \log(1 + \gamma)$$

Albeit the capacity itself is chiefly of theoretical interest, still the logarithmic variation of data rate with SINR is an important measure of performance. In Rayleigh fading AWGN channels, the throughput is closely related to this expression [9]. Also, it intuitively indicates the diminishing marginal return from pumping in more power at high SINR low noise regime and inspires the use of water filling optimisation with power constraint.

### B. Comparison with Broadcast Channel

An apparently similar system model has been analysed in [6]. Although the systems are similar in essence, all the communication links are reversed in direction. For the SISO primary user system, this is of relatively little consequence. For the cognitive users, if they have full knowledge of channels (their owns and also other cognitive users), the orthogonal user selection algorithm [6] selects the same set of users with same power to maximise the throughput. The well known rate duality principle [10], [11] dictates that it is possible to achieve the same throughput here.

In practice, however, the channel state information is usually not available at the mobile transmitters. Even if they are available, limited processing capability at the cognitive terminals makes it difficult to schedule the transmissions. If we assume no inter-user point to point communication, the users have absolutely no way to coordinate their behaviours and schedule their own transmissions keeping the broad interest of whole cognitive user group in mind. In Section IV, a novel protocol has been proposed and analysed to address the multiple access channel.

## IV. Random Transmission

When there is no coordination among the users, in particular, each user is not aware of others' channel condition, throughput demand or power availability, it is not possible to schedule transmission from a centralised control. Then, if all the users transmit continuously, they not only end up spending a lot of power, at the same time, cause harmful interference to others. The proposed approach consists of random transmission from all the cognitive users. Since the primary user has a single antenna, zero forcing from all K cognitive users is not possible. So it has to tolerate interference to some extent. But, usually for the licensed primary users, the requirement is that the link quality (expressed in terms of SINR) must be above a certain limit. So, it is possible to set up a scheme where every cognitive user will transmit in a particular symbol interval with a probability $p \in (0, 1]$ and stay silent with probability $1 - p$.

To put the scenario in more mathematical form, $K$ independent identically distributed Bernoulli variables $\{b_k\}_{k=1}^{K}$ are introduced and in a particular symbol interval

$$b_k = 1 \Leftrightarrow \text{Cognitive user } k \text{ has transmitted}$$

Since every user is transmitting on a random basis with probability $p$ it is obvious that $\mathbb{P}[b_k = 1] = p$ and $\mathbb{P}[b_k = 0] = 1 - p \quad \forall k \in \{1, 2, \ldots, K\}$.

It is assumed that the primary user is transmitting and using its own spectrum range on a continuous basis and there is no question of probabilistic transmission.

### A. Performance Criteria

Since the protocol proposed is not deterministic and there is a degree of randomness involved in the transmission procedure, the performance criteria can no longer be evaluated and/or compared deterministically. From the definitions of $\{b_k\}_{k=1}^{K}$ we can conclude that from the point of view of $k$-th cognitive user, the interference power vector, received at the base station is

$$\mathbf{I}_k^{\text{cognitive}} = \mathbf{h}_p \mathbf{h}_p^{\mathrm{H}} P_p + \sum_{i=1, i \neq k}^{K} b_i P_i \mathbf{h}_i \mathbf{h}_i^{\mathrm{H}} \tag{6}$$

which is easily interpreted as sum of interferences from all other users (primary and cognitive). Similarly, for the primary user, the interference power is

$$I^{\text{primary}} = \sum_{i=1}^{K} b_i P_i |g_i|^2 \tag{7}$$

The corresponding data rates are, for cognitive users

$$r_k = \log\left(1 + \frac{|\mathbf{f}_k^{\mathrm{H}} \mathbf{h}_k|^2 b_k P_k}{\mathbf{f}_k^{\mathrm{H}} \left(\mathbf{I}_M + \mathbf{I}_k^{\text{cognitive}}\right) \mathbf{f}_k}\right) \tag{8}$$

Here the same receive strategy of 4 is used (assuming the extreme case of all the users transmitting) and the interference is obtained from 6. For primary user, the rate is

$$r_p = \log\left(1 + \frac{|g_p|^2 P_p}{1 + I^{\text{primary}}}\right) \tag{9}$$

Since all the rates are random variables (dependent upon $\{b_k\}_{k=1}^{K}$) to find the true performance measures, expectation must be taken over all the independent variables. So for cognitive users,

$$\begin{aligned} &\mathbb{E}\left[r_k\right] \\ &= \mathbb{E}_{\{b_i\}_{i=1}^{K}}\left[\log\left(1 + \frac{|\mathbf{f}_k^{\mathrm{H}} \mathbf{h}_k|^2 P_k b_k}{\mathbf{f}_k^{\mathrm{H}} (\mathbf{I}_M + \mathbf{I}_k^{\text{cognitive}}) \mathbf{f}_k}\right)\right] \end{aligned} \tag{10}$$

For the primary user, there is no randomness involved in its own transmission, but the interference is random and we get,

$$\begin{aligned} &\mathbb{E}[r_{\text{primary}}] \\ &= \mathbb{E}_{\{b_k\}_{k=1}^{K}}\left[\log\left(1 + \frac{|g_p|^2 P_p}{1 + \sum_{k=1}^{K} |g_k|^2 P_k b_k}\right)\right] \end{aligned} \tag{11}$$

### B. Bounds on Rates

For an intuitive idea of how the randomness affects the data rates, properties of the logarithmic function can be used.

Consider the function $f(x, y) = \log\left(1 + \frac{y}{x}\right)$ defined for $x > 0$ and $y \geq 0$. It can be shown to be convex in $x$ when $y$ is constant and concave in $y$ for $x$ constant. So, if $X$ and $Y$ are independent random variables (defined on appropriate supports), applying Jensen's inequality (separately as the convex and concave functions of $X$ and $Y$ respectively), gives the bounds on $\mathbb{E}[f(X,Y)]$ as

$$\begin{aligned} &\mathbb{E}_Y\left[\log\left(1 + \frac{Y}{\mathbb{E}(X)}\right)\right] \\ &\leq \mathbb{E}_{X,Y}\left[\log\left(1 + \frac{Y}{X}\right)\right] \\ &\leq \mathbb{E}_X\left[\log\left(1 + \frac{\mathbb{E}(Y)}{X}\right)\right] \end{aligned} \tag{12}$$

*1) Cognitive User:* To use these inequalities in the context of our multiple access channel, note that, if we set

$$Y = |\mathbf{f}_k^{\mathrm{H}}\mathbf{h}_k|^2 P_k b_k$$

and

$$X = \mathbf{f}_k^{\mathrm{H}}(\mathbf{I}_M + \mathbf{h}_p\mathbf{h}_p^{\mathrm{H}}P_p + \sum_{i=1, i\neq k}^{K} \mathbf{h}_i\mathbf{h}_i^{\mathrm{H}}P_i b_i)\mathbf{f}_k$$

then $\log\left(1 + \frac{Y}{X}\right)$ is the random rate of cognitive user $k$. To find the upper and lower bounds, it suffices to note that according to the above definitions of $X$ and $Y$,

$$\mathbb{E}[X] = \mathbf{f}_k^{\mathrm{H}}(\mathbf{I}_M + \mathbf{h}_p\mathbf{h}_p^{\mathrm{H}}P_p + \sum_{i=1, i\neq k}^{K} \mathbf{h}_i\mathbf{h}_i^{\mathrm{H}}pP_i)\mathbf{f}_k$$

and

$$\mathbb{E}[Y] = |\mathbf{f}_k^{\mathrm{H}}\mathbf{h}_k|^2 pP_k$$

Define the power allocation vector of cognitive users as

$$\mathbf{p} = [P_1, P_2, \ldots, P_K]^{\mathrm{T}}$$

Obviously, $||\mathbf{p}||_1$ gives total power usage by all cognitive users.

Then our assertion is that for $p \in (0, 1]$ and $\mathbf{p} \succ \mathbf{0}$, the following two cognitive uplink systems are identical in terms of power consumption.

- All cognitive users are transmitting continuously with power allocation vector $p\mathbf{p}$.
- Each cognitive user is transmitting with probability $p$ at any symbol interval and the power allocation vector is $\mathbf{p}$.

Based on this assertion, from the first inequality of 12 it is noted that if we replace the random variable $X$ by a deterministic variable $\mathbb{E}[X]$, the rate decreases. In effect, the random variable $X$ corresponds to random transmissions from interfering users and replacing it by $\mathbb{E}[X]$ corresponds to continuous transmissions from interfering users with less power. So, from the inequality itself, it is clear that random transmissions from interfering users is better than continuous transmission so far as tackling the interference goes. But, from the second part of the inequality, it is seen that continuous transmission from user $k$ himself is better so far as its own data rate in concerned. Also, by invoking Jensen's inequality, it can be shown that

$$\begin{aligned} &\mathbb{E}_Y\left[\log\left(1 + \frac{Y}{\mathbb{E}(X)}\right)\right] \\ &\leq \log\left(1 + \frac{\mathbb{E}[Y]}{\mathbb{E}[X]}\right) \\ &\leq \mathbb{E}_X\left[\log\left(1 + \frac{\mathbb{E}(Y)}{X}\right)\right] \end{aligned} \tag{13}$$

A comparison between $\log\left(1 + \frac{\mathbb{E}[Y]}{\mathbb{E}[X]}\right)$ and $\mathbb{E}\left[\log\left(1 + \frac{Y}{X}\right)\right]$ will depend upon specific values and not possible to carry out in general form. Simulation results in Section V show that this depends upon the power of primary user and for high power primary users it is possible to achieve marginally better performance with the proposed scheme.

*2) Primary User:* The same form of expressions for $f(X, Y)$ can be used for the primary user. But in this case, the definitions are

$$X = 1 + \sum_{k=1}^{K} |g_k|^2 P_k b_k$$

and $Y = |g_p|^2 P_p$. To be noted here, since the primary user transmits continuously with its own power, $Y$ is not a random variable anymore (in other words, $Y = \mathbb{E}[Y]$). So the only previous inequality for lower bound reduces to

$$\mathbb{E}_X\left[\log\left(1 + \frac{Y}{X}\right)\right] \geq \log\left(1 + \frac{Y}{\mathbb{E}(X)}\right) \tag{14}$$

So, the primary user is having a clear advantage in tackling the interference from the cognitive users. In light of the simulation results in Section V it will be shown that this advantage can be turned in the favor of cognitive users themselves.

## V. Simulation Results

For the simulation, a cognitive system of three antennas at the base station and ten cognitive users is considered. Rayleigh fading is considered with $h_{kj} \sim \mathcal{CN}(0, 1)$, $\forall 1 \leq k \leq K$, $\forall 1 \leq j \leq M$ i.e., $h_{kj}$ is a circularly symmetric complex normal variable. With a fixed cognitive users power allocation vector $\mathbf{p}$ and primary user power $P_p$ various data rates have been plotted against variation of probability $p$ (varying from $0$ to $1$). For comparison, another system is considered where cognitive users transmit continuously with power allocation vector $p\mathbf{p}$. As per the assertion made in Section IV-B1 these two systems are similar in terms of power consumption and the probability $p$ gives the measure of total cognitive user power apart from a constant factor. Figures 2 and 3 show the average rates of all cognitive users, first with a comparatively low primary user power ($P_p$), and then with a high $P_p$. Figure 4 gives the corresponding variation of primary user data rate with probability $p$. From Figures 2 and 3 it

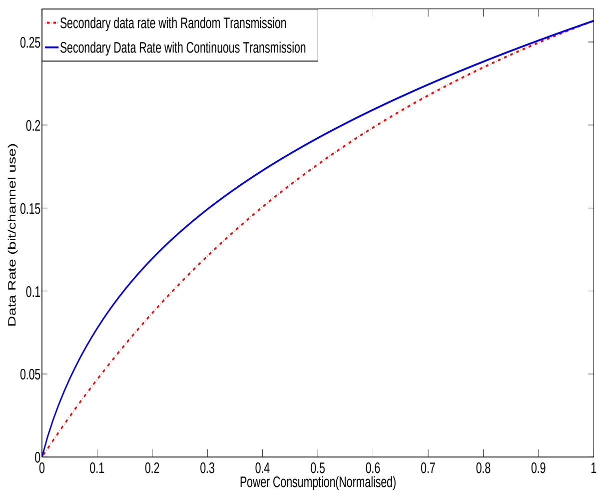


Fig. 2. Average Data Rates of Cognitive Users with Low Primary User Power

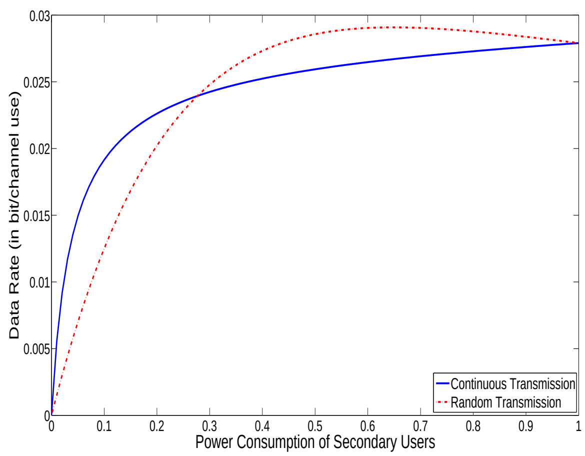


Fig. 3. Average Data Rates of Cognitive Users with High Primary User Power

is obvious that cognitive users gain in the random transmission scheme if the primary user power is high compared to cognitive users. This assumption is often valid, particularly in wireless sensor networks where sensor motes operating at low power are used for short range communication. As Figure 4 suggests and already shown in Section IV-B2, primary user always gains in terms of data rate.

## VI. Conclusion

The random transmission scheme is able to outperform the continuous transmission scheme for primary user and possibly the cognitive users as well for certain cases. As the figures demonstrate, tuning the secondary user parameters can

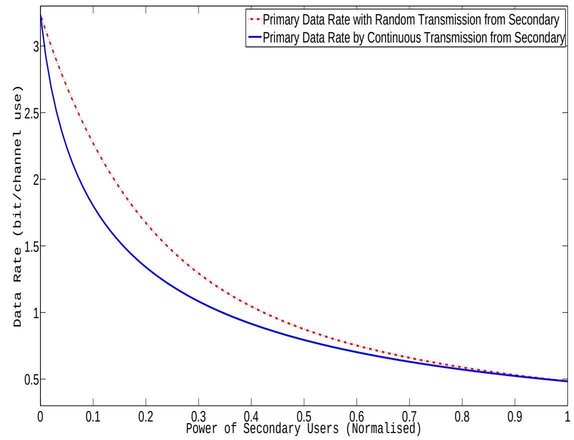


Fig. 4. Primary User Data Rates with Continuous and Discrete Transmissions

be used to limit the data rate of primary users. In certain conditions, higher data rate for primary users may be necessary (with obvious trade-off for cognitive users) and in some other conditions, the primary user can tolerate higher interference from cognitive users. It is shown that the cognitive users can respond to constraints imposed by primary users either by adjusting the actual power or by adjusting the transmission rate $p$. From the design point of view, controlling the probability $p$ is an easier way than to control the battery power.

## References


[1] M. Calabrese, "The End of Spectrum 'Scarcity' : Building on the TV Bands Database to Access Unused Public Airwaves," *New America Foundation*, 2009.

[2] M. Marcus, J. Burtle, N. Mcneil, A. Lahjouji, and B. Franca, "Report of the Unlicensed Devices and Experimental Licenses Working Group," Federal Communications Commission Spectrum Policy Task Force, Tech. Rep., 2002.

[3] S. Geirhofer and L. Tong, "Dynamic Spectrum Access in the Time Domain : Modeling and Exploiting White Space," *IEEE Communications Magazine*, vol. 45, no. 5, pp. 66–72, 2007.

[4] E. Telatar, "Capacity of Multi-antenna Gaussian Channels," *European Transactions on Telecommunications*, vol. 10, no. 6, pp. 585–595, 1999.

[5] H. Huang, Z. Zhang, P. Cheng, G. Yu, and P. Qiu, "Throughput Analysis of Cognitive MIMO System," in *First International Workshop on Cross Layer Design*, Jinan, China, Sep. 2007, pp. 45–49.

[6] K. Hamdi, W. Zhang, and K. B. Letaief, "Opportunistic Spectrum Sharing in Cognitive MIMO Wireless Networks," *IEEE Transactions on Wireless Communications*, vol. 8, no. 8, pp. 4098–4109, 2009.

[7] M. Schubert and H. Boche, "A Generic Approach to QoS-Based Transceiver Optimization," *IEEE Transactions on Communications*, vol. 55, no. 8, pp. 1557–1566, 2007.

[8] R. Remmert, *Theory of Complex Functions*, ser. Graduate Texts in Mathematics. Springer-Verlag, 1991.

[9] G. Dimic and N. D. Sidiropoulos, "On Downlink Beamforming with Greedy User Selection: Performance Analysis and a Simple New Algorithm," *IEEE Transactions on Signal Processing*, vol. 53, no. 10, pp. 3857–3868, 2005.

[10] M. Joham, "MIMO Systems," Associate Institute for Signal Processing, Technological University of Munich, Germany, Tech. Rep., 2009.

[11] S. Vishwanath, N. Jindal, and A. Goldsmith, "Duality, Achievable Rates, and Sum-Rate Capacity of Gaussian MIMO Broadcast Channels," *IEEE Transactions on Information Theory*, vol. 49, no. 10, pp. 2658–2668, Oct. 2003.