

## A Helpful Positioning Method with Two GNSS Satellites in Urban Area

Hiroyuki Hatano  
Graduate School of Engineering,  
Utsunomiya University  
Tochigi, Japan

e-mail: hatano@is.utsunomiya-u.ac.jp

Tomoya Kitani  
Graduate School of Informatics,  
Shizuoka University.  
Shizuoka, Japan

e-mail: t-kitani@ieee.org

Masahiro Fujii, Yu Watanabe  
Graduate School of Engineering,  
Utsunomiya University  
Tochigi, Japan

e-mail: {fujii, yu}@is.utsunomiya-u.ac.jp

Hironobu Onishi  
Graduate School of Engineering,  
Shizuoka University  
Shizuoka, Japan

e-mail: onishi@hatanolab.eng.shizuoka.ac.jp

**Abstract**—For estimating user’s location, Global Positioning System (GPS) is very popular and important technique. The GPS estimation uses satellites which fly in the air. The estimator needs the open sky and multiple satellites (usually 4 and more satellites). However, there are many buildings in urban area. The receiver tends to receive bad signals, called as non-line-of-sight (NLOS) or multipath signals. The problem is that the receiver cannot get direct path signals from adequate number of the satellites coinstantaneously. This case leads to degrade estimation quality. So, we introduce the novel estimation algorithm, which can estimate own position with as low number of satellites as possible.

**Keywords**—GPS; Global Positioning System; GNSS; Localization; Multipath; NLOS; Shielded signals

### I. INTRODUCTION

For next generation, a plan of Intelligent Transportation System (ITS) is very attractive. ITS can resolve problems such as traffic accidents and jam. By realizing ITS, convenient and safety life will be turn up. As a hot topic, electrical vehicles and hybrid vehicles are famous. Compared to fossil fuel, the electrical vehicles can miniaturize vehicle’s size. Then, personal mobility, such as “Segway” and “Winglet”, can be provided [1].

In development of high performance vehicles, own location is one of important information. The most popular technique is Global Positioning System (GPS). GPS can estimate user’s location by receiving signals from GPS satellites. In our life, we have been getting benefits of GPS already. Route guidance by car navigation system or smart phones is very useful. However, the estimation by GPS still has problems [2], [3]. Especially, we cannot get own position precisely in urban area. In urban area, there are a lot of structures such as buildings. So, GPS receivers receive the signals from the satellites via multipath propagation. Also, the receivers cannot watch the satellites in line-of-sight (LOS) when the satellites are shield by the structures. That is the case called as non-line-of-sight (NLOS). In case of riding the personal mobility and walking on street, users exist near building. The harmful effect such as receiving multipath or shielded signals often occurs. The quality of positioning becomes worse. So, it is important to realize robust and accurate positioning even if the above bad environment. Ideally, it is important to be able to estimate own position everywhere. The above robust and accurate positioning leads to high functional drivers assistant systems, such as supporting every each of road lanes or automatic drivng [4]–[6]. Moreover,

the application to probe car systems for collection of various data, road-to-vehicle / vehicle-to-vehicle communication will be realized effectively [7]–[9]. In our research, we focus on the positioning algorithm under such a bad environment of receiving multipath or NLOS signals.

In GPS positioning, the receiver estimates the own position from the propagated distance between the LOS satellites and the receiver [10]. The distance can be derived from both a transmitted time at the satellite and a received time at the receiver. So, the estimated parameters are four parameters, that is 3 dimension position and time clock error between the satellite and receiver. The estimator needs four and more satellites. In urban area, the number of satellites which can observe in LOS fluctuates. Unfortunately, there are the places where we cannot observe four satellites frequently. In this paper, we show the novel positioning method which can estimate own position with as low number of satellites as possible. Our estimation uses two LOS satellites. Also, we assume a moving vehicle. So, we use a travelling distance as sensor data too. In our method, we can estimate user’s position even if the number of LOS satellites is reducing.

This paper is organized as follows. In Section II, we introduce the related works. And, we confirm the problem of the urban positioning. In Section III, we introduce the coordinate systems which is required in our positioning algorithm. Then, in Section IV, we introduce the our positioning algorithm and performance. Finally, Section V summarizes the paper.

### II. RELATED WORKS AND CURRENT PROBLEMS

In this section, we will introduce the related works. There are a lot of works which can improve the positioning performance in urban area. After the introduction, the simple experiment, which we confirm the problem of the urban positioning, will be shown.

#### A. Related works

The propagation environment in urban area is multipath propagation. There are a lot of works which can mitigate the multipath interference. For example, techniques which focus on signal tracking on a receiver or channel estimation by multiple correlators have been shown [11], [12]. Moreover, the mitigation methods of Direction-of-Arrival (DoA) estimation by array antenna have also been shown [13], [14]. The purpose of these methods is to prevent the degradation of the positioning performance in case of the multipath propagation with LOS satellites.

In our research, we focus on the multipath environment with the NLOS satellites. That is, the direct signal is shielded by some structures such as buildings. This situation may often occur in urban area. One of the researches under assumption of NLOS is [15]. Meguro et al. [15] uses a camera which watches the sky. The camera decides if there is the structure or not.

If the receiver exists at static position, observed satellites are always the same satellites. However, in case of a moving receiver in urban area, the observed satellites are changed frequently. The changing the satellites leads to the fluctuation of the positioning accuracy. Irie's work [16] is to prevent such a problem of the accuracy degradation. Kawamura and Tanaka's work [17] is also to keep the accuracy good by calculating weightings to the satellites.

When the number of satellites which can be observed is reduced, the positioning accuracy becomes worse. In the worst case, the receiver cannot estimate own position at all. This is because the number of observed satellites is few. In Fan et al.'s work [18], the receiver adds the virtual satellites by using the reflected signals at the ground.

As other approaches, the estimator can keep the positioning accuracy fine by using other information and devices. Map matching techniques may be most famous. The devices such as camera, direction, gyro or travelling distance sensors can improve the positioning performance. The vehicle-to-vehicle or vehicle-to-road communication can be also used for improvement. The above researches are shown in [4], [8], [9], [19].

In Japan, in order to improve the positioning accuracy in urban area and the environment decreasing the number of observable LOS satellites, the project of Quasi-Zenith Satellite System (QZSS) is under way [20]–[22]. In September 2010, the first QZSS "MICHIBIKI" was launched. The satellite of QZSS moves on quasi-zenith orbit of Japan. So, the receiver can observe QZS at high angle of elevation. Even if there are a lot of buildings, the QZS can tend to become the LOS satellite.

### B. Checking current problem and our approach

In urban area, there are a lot of structures such as buildings. It leads to the bad effect of both reduction of the observable LOS satellites and generation of multipath propagation. This bad effect generates large positioning error. In order to confirm the positioning performance and its problem, we tested simple experiments of the positioning in urban area. We used GPS logger (model: m-241, made by HOLUX), which is popular and we can buy at stores easily. The frequency of records is 1 Hz. We tried estimation a static position in 10 minutes. The receiver's position and estimated positions are summarized in Figure 1. The positions are plotted to Google Map by a useful plotting tool "Wadachi Ver.3.44".

The location in the experiment is near Hamamatsu Station in Shizuoka, Japan. There are a lot of buildings. We set the logger on the edge of the street, that is by the side of the building wall. From Figure 1, the estimated positions have about 20-30m error compared to the true position of the logger. We can confirm that the positioning in urban area has large errors. The reason may be NLOS or multipath environment.

The bad satellites, which have NLOS path or multipath should not be used to estimate the receiver's position. However, the estimate algorithm needs four or more satellites. So, conventionally, in order to get adequate number of the satellites, the estimator needs to use the bad satellites too.

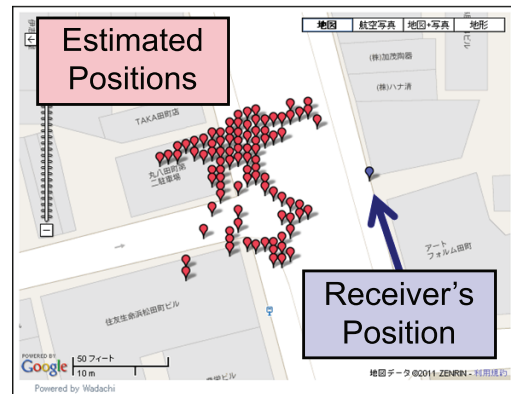


Figure 1. Positioning results in urban area (front of building)

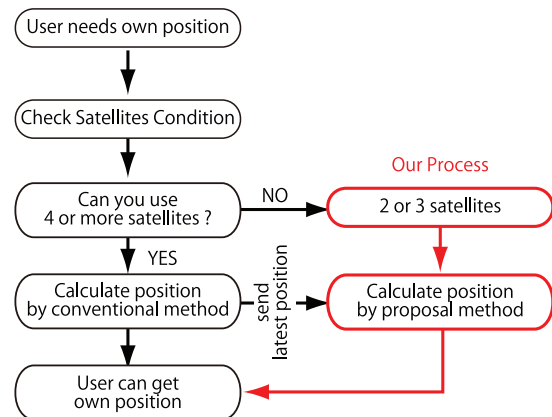


Figure 2. Process flow in positioning

In our approach, we avoid using these bad satellites. So, we consider another algorithm which can estimate own position with less than four satellites. In this paper, we introduce the estimation algorithm which uses two satellites and the information of the receiver's movement. We assume the positioning of a moving vehicle. So, the receiver can get the information of the travelling distance.

The flow of the usage is illustrated in Figure 2. Usually, the receiver estimates own positions by conventional algorithm, that is with four and more satellites. In case of bad environment, we continue the positioning by selecting just two satellites which have high elevation angle. Our approach may help in the robust positioning everywhere.

### III. COORDINATE SYSTEM (ECEF, ENU COORDINATE)

In this section, before the introduction to our positioning algorithm, the coordinate systems which are used in the paper will be shown. The conversion equation will be also shown. The coordinate systems are both ECEF and ENU. Both coordinates illustrate as Figure 3. Our algorithm, which is presented in Section IV, use the assumption "the altitude of the receiver do not change in short time". For applying the assumption, the conversion of the coordinate is needed.

ECEF stands for Earth-Centered Earth-Fixed. The origin of ECEF is the center of the earth, that is the point  $O_{enu}$  in

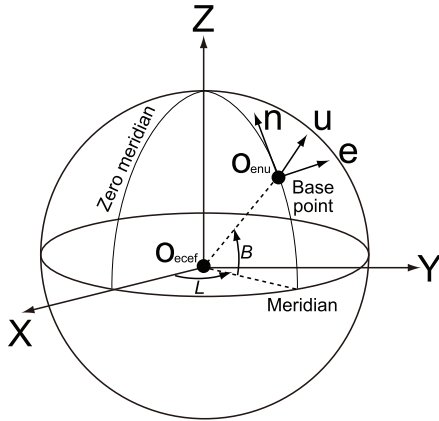


Figure 3. ECEF and ENU coordinate system

Figure 3. ECEF is  $x - y - z$  orthogonal coordinates on the fixed earth. The unit of  $x, y, z$  is meter.

ENU means East-North-Up. As pointing out in Figure 3, each of coordinates  $e, n, u$  means east-direction, north-direction, vertical(up)-direction respectively. The origin of ENU is defined as a arbitrary base position, such as the position  $O_{enu}$  in Figure 3. The unit of  $x, y, z$  is also meter.

Next, we introduce the conversion equation. We define a ECEF position  $(x, y, z)$  as a vector  $\vec{x}_{ecef}$ , and also define a ENU position  $(e, n, u)$  as  $\vec{x}_{enu}$ . The conversion equation from the ECEF position  $\vec{x}_{ecef}$  to the ENU position  $\vec{x}_{enu}$  is the following:

$$\vec{x}_{enu} = \mathbf{R}(B, L) \cdot [\vec{x}_{ecef} - \vec{x}_{0,ecef}] \quad (1)$$

where the vector  $\vec{x}_{0,ecef}$  means the base position which is expressed in the ECEF coordinate. Generally, the matrix  $\mathbf{R}(B, L)$  is called as rotation matrix. In (1), in order to rotate the coordinate, the matrix  $\mathbf{R}(B, L)$  is used. So the matrix is:

$$\mathbf{R}(B, L) = \begin{pmatrix} -\sin L & \cos L & 0 \\ -\cos L \sin B & -\sin L \sin B & \cos B \\ -\cos L \cos B & \sin L \cos B & \sin B \end{pmatrix} \quad (2)$$

where the parameter  $B$  is the degree of latitude and the parameter  $L$  is the degree of longitude at the base position  $\vec{x}_{0,ecef}$  (Figure 3). By using (1), we can convert positions from ECEF to ENU.

In this paper, we apply World Geodetic System 84 (WGS-84) as a geographical coordinate system. Then, the semi-major axis  $R_e$  of the earth and the oblateness  $f_e$  are:

$$R_e = 6378137.0[m] \quad (3)$$

$$f_e = \frac{1.0}{298.257223563} \quad (4)$$

When we want to convert from ENU to ECEF, the conversion equation is the following:

$$\vec{x}_{ecef} = \mathbf{R}^{-1}(B, L) \cdot \vec{x}_{enu} + \vec{x}_{0,ecef} \quad (5)$$

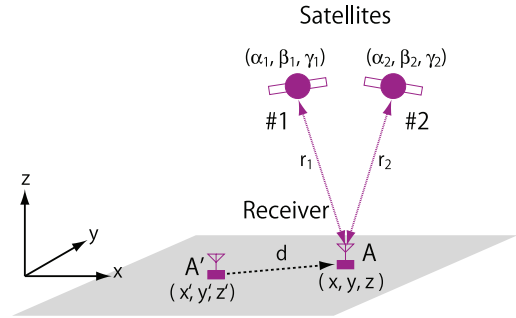


Figure 4. System model (positioning algorithm with two satellites)

where the inverse matrix  $\mathbf{R}^{-1}(B, L)$  is negative rotation to (1) and can be expressed as

$$\mathbf{R}^{-1}(B, L) = \begin{pmatrix} -\sin L & -\cos L \sin B & \cos L \cos B \\ \cos L & -\sin L \sin B & \sin L \cos B \\ 0 & \cos B & \sin B \end{pmatrix} \quad (6)$$

#### IV. POSITIONING ALGORITHM USING TWO SATELLITES

In this section, we present the positioning algorithm which can estimate with the two satellites. Usually, the conventional estimator needs four and more satellites. However, in urban area, the number of satellites which can observe in LOS fluctuates. Moreover, the place where we cannot observe four and more satellites often occurs. In these bad situations, our positioning method can estimate own position even if the number of LOS satellites is reducing.

##### A. System model

Our system model is shown in Figure 4. There are the two LOS satellites in the sky. Their ECEF coordinates are  $(\alpha_1, \beta_1, \gamma_1), (\alpha_2, \beta_2, \gamma_2)[m]$  respectively. The position of the receiver which should be estimated is  $(x, y, z)[m]$  and the position is shown as point  $A$ . It is ECEF coordinate. The parameter  $t$  [sec] means time at the receiver. The previous time is also denoted as the parameter  $t'$  and the relation is  $t = t' + \Delta t$ . The past receiver's position  $A'$  is  $(x', y', z')[m]$ , that is at the time  $t'$ . The travelling distance between the time  $t'$  and  $t$  is  $d[m]$ . The time interval  $\Delta t$  is assumed as short time. So, we assume that the altitude of the receiver at  $t$  and  $t'$  is the same. The distance  $d$  can be gotten from the sensor which measures car speed. The parameter  $r_1[m]$  means the range between the satellite #1 and the receiver. The parameter  $r_2[m]$  also means the range to the satellite #2. These ranges  $r_1, r_2$  are called as pseudorange because the clock error between the satellites and the receiver is added to the true range.

##### B. Algorithm explication

In order to estimate the receiver's position  $A$ , we enumerate the related equations as follows.

$$r_1 = \sqrt{(\alpha_1 - x)^2 + (\beta_1 - y)^2 + (\gamma_1 - z)^2} + s \quad (7)$$

$$r_2 = \sqrt{(\alpha_2 - x)^2 + (\beta_2 - y)^2 + (\gamma_2 - z)^2} + s \quad (8)$$

$$d = \sqrt{(x' - x)^2 + (y' - y)^2 + (z' - z)^2} \quad (9)$$

Equations (7), (8) are the pseudorange to the satellite #1 and #2, respectively. The parameter  $s$  means the error [m] of

the clock difference between the satellites and the receiver. Equation (9) means the travelling distance of the movement of the receiver in the time interval  $\Delta t$ .

We must estimate both the receiver's position  $(x, y, z)$  and the clock error  $s$  from (7)~(9). Equations (7)~(9) are non-linear simultaneous equations. So, we try estimating the parameters by sequential approach as follows.

[Step 1] We set initial values  $x^0, y^0, z^0, s^0$  of the unknown parameters  $x, y, z, s$ .

[Step 2] When the receiver's position and the clock error are  $x^0, y^0, z^0, s^0$ , the pseudoranges and the travelling distance can be denoted as the following:

$$r_1^0 = \sqrt{(\alpha_1 - x^0)^2 + (\beta_1 - y^0)^2 + (\gamma_1 - z^0)^2} + s^0 \quad (10)$$

$$r_2^0 = \sqrt{(\alpha_2 - x^0)^2 + (\beta_2 - y^0)^2 + (\gamma_2 - z^0)^2} + s^0 \quad (11)$$

$$d^0 = \sqrt{(x' - x^0)^2 + (y' - y^0)^2 + (z' - z^0)^2} \quad (12)$$

[Step 3] The residual errors between the observed value and the above  $r_1^0, r_2^0, d^0$  are:

$$\Delta r_1^0 = r_1 - r_1^0 \quad (13)$$

$$\Delta r_2^0 = r_2 - r_2^0 \quad (14)$$

$$\Delta d^0 = d - d^0 \quad (15)$$

Starting from the initial values  $x^0, y^0, z^0, s^0$ , we update the estimating unknown parameters  $x, y, z, s$  as the above residual errors become zeros.

[Step 4] In order to derive the updating values  $\Delta x, \Delta y, \Delta z, \Delta s$ , we approximate (7), (8), (9) to the followings. That is, the non-linear simultaneous equations are transformed to linear simultaneous equations.

$$\Delta r_1 = \frac{\partial r_1}{\partial x} \Delta x + \frac{\partial r_1}{\partial y} \Delta y + \frac{\partial r_1}{\partial z} \Delta z + \frac{\partial r_1}{\partial s} \Delta s \quad (16)$$

$$\Delta r_2 = \frac{\partial r_2}{\partial x} \Delta x + \frac{\partial r_2}{\partial y} \Delta y + \frac{\partial r_2}{\partial z} \Delta z + \frac{\partial r_2}{\partial s} \Delta s \quad (17)$$

$$\Delta d = \frac{\partial d}{\partial x} \Delta x + \frac{\partial d}{\partial y} \Delta y + \frac{\partial d}{\partial z} \Delta z + \frac{\partial d}{\partial s} \Delta s \quad (18)$$

[Step 5] The above simultaneous equations can be represented as matrix forms as below:

$$\Delta \vec{r} = \mathbf{G} \Delta \vec{x} \quad (19)$$

, where

$$\Delta \vec{r} = [\Delta r_1, \Delta r_2, \Delta d]^T \quad (20)$$

$$\Delta \vec{x} = [\Delta x, \Delta y, \Delta z, \Delta s]^T \quad (21)$$

$$\mathbf{G} = \begin{pmatrix} \frac{\partial r_1}{\partial x} & \frac{\partial r_1}{\partial y} & \frac{\partial r_1}{\partial z} & \frac{\partial r_1}{\partial s} \\ \frac{\partial r_2}{\partial x} & \frac{\partial r_2}{\partial y} & \frac{\partial r_2}{\partial z} & \frac{\partial r_2}{\partial s} \\ \frac{\partial d}{\partial x} & \frac{\partial d}{\partial y} & \frac{\partial d}{\partial z} & \frac{\partial d}{\partial s} \end{pmatrix} \quad (22)$$

$$= \begin{pmatrix} \frac{-(\alpha_1 - x)}{r_1 - s} & \frac{-(\beta_1 - y)}{r_1 - s} & \frac{-(\gamma_1 - z)}{r_1 - s} & 1 \\ \frac{-(\alpha_2 - x)}{r_2 - s} & \frac{-(\beta_2 - y)}{r_2 - s} & \frac{-(\gamma_2 - z)}{r_2 - s} & 1 \\ \frac{-(x' - x)}{d} & \frac{-(y' - y)}{d} & \frac{-(z' - z)}{d} & 0 \end{pmatrix} \quad (23)$$

[Step 6] Equation (19) has four unknown parameters. However, the row of the matrix  $\mathbf{G}$  is three. So, we cannot solve the each of the unknown parameters. Then, we assume that the altitude values of the receiver  $A$  and  $A'$  are the same. In order to use the assumption, we try converting the coordinate from ECEF to ENU.

[Step 7] The matrix  $\mathbf{G}$  is presented as ECEF coordinate system. We convert the matrix  $\mathbf{G}$  to ENU coordinate system. The matrix  $\mathbf{G}$  means the amount of change in terms of each direction  $x, y, z$ . For example, in the satellite #1, the value  $\frac{\partial r_1}{\partial x}$  means the change of  $r_1$  in terms of the direction  $x$ . The value  $\frac{\partial r_1}{\partial y}$  also means the change of  $r_1$  in terms of the direction  $y$ . The value  $\frac{\partial r_1}{\partial z}$  of course means the change of  $r_1$  in terms of the direction  $z$ . Then, we just have to convert each of directions to the east, north and vertical (altitude), respectively. So, we represent the matrix  $\mathbf{G}$  in ENU as follows:

$$\mathbf{G}_{\text{enu}} = \begin{pmatrix} g_{11} & g_{12} & g_{13} & 1 \\ g_{21} & g_{22} & g_{23} & 1 \\ g_{31} & g_{32} & g_{33} & 0 \end{pmatrix} \quad (24)$$

By using (1), the each of components means as follows:

$$\begin{pmatrix} g_{k1} \\ g_{k2} \\ g_{k3} \end{pmatrix} = \mathbf{R}(B, L) \begin{pmatrix} \frac{\partial r_k}{\partial x} \\ \frac{\partial r_k}{\partial y} \\ \frac{\partial r_k}{\partial z} \end{pmatrix}, k = 1, 2 \quad (25)$$

$$\begin{pmatrix} g_{31} \\ g_{32} \\ g_{33} \end{pmatrix} = \mathbf{R}(B, L) \begin{pmatrix} \frac{\partial d}{\partial x} \\ \frac{\partial d}{\partial y} \\ \frac{\partial d}{\partial z} \end{pmatrix} \quad (26)$$

The values  $B, L$  are the degrees of latitude and longitude at the position  $(x, y, z)$ .

[Step 8] Because of the matrix  $\mathbf{G}_{\text{enu}}$  is ENU coordinate, the vector  $\Delta \vec{x}$  have to be ENU coordinate. Then, (19) have to be converted to ENU coordinate system.

$$\begin{aligned} \Delta \vec{r} &= \mathbf{G}_{\text{enu}} \Delta \vec{x}_{\text{enu}} \quad (27) \\ &= \begin{pmatrix} g_{11} & g_{12} & g_{13} & 1 \\ g_{21} & g_{22} & g_{23} & 1 \\ g_{31} & g_{32} & g_{33} & 0 \end{pmatrix} \begin{pmatrix} \Delta x_{\text{enu}} \\ \Delta y_{\text{enu}} \\ \Delta z_{\text{enu}} \\ \Delta s \end{pmatrix} \quad (28) \end{aligned}$$

In this paper, the movement of the altitude is assumed as zero. This means that  $\Delta z_{\text{enu}} = 0$ . Equation (28) can be represent as the following.

$$\begin{aligned} \Delta \vec{r} &= \begin{pmatrix} g_{11} & g_{12} & 1 \\ g_{21} & g_{22} & 1 \\ g_{31} & g_{32} & 0 \end{pmatrix} \begin{pmatrix} \Delta x_{\text{enu}} \\ \Delta y_{\text{enu}} \\ \Delta s \end{pmatrix} \\ &\equiv \mathbf{G}_{\text{enu}2} \Delta \vec{x}_{\text{enu}2} \quad (29) \end{aligned}$$

The row of the above matrix is three. The unknown parameters is also three. Then, we can solve these parameters.

[Step 9] We transform (29) to the following, and calculate the updating value  $\Delta \vec{x}_{\text{enu}2}$ .

$$\Delta \vec{x}_{\text{enu}2} = \mathbf{G}_{\text{enu}2}^{-1} \Delta \vec{r} \quad (30)$$

Table I  
SIMULATION PARAMETERS (FOR OUR ALGORITHM)

Satellite Position #1 [m]	$\alpha_1$	-26309844.5749
	$\beta_1$	3237477.5201
	$\gamma_1$	2627019.5575
Satellite Position #2 [m]	$\alpha_2$	5096038.9206
	$\beta_2$	15688974.7606
	$\gamma_2$	21240453.8041
Receiver Position A [m]	$x$	-3460143.2936
	$y$	3657442.8374
	$z$	3616321.2928
Past Receiver Position A' [m]	$x'$	-3760139.5967
	$y'$	3657446.1923
	$z'$	3616321.0131
Pseudorange Error [m]	$4\sigma$	1.0/5.0/10
Distance Sensor Error[m]	$\epsilon_{\max}$	1.25

[Step 10] The calculated updating values  $\Delta x_{\text{enu}}, \Delta y_{\text{enu}}$  are converted to ECEF coordinate.

$$\begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} = \mathbf{R}^{-1}(B, L) \begin{pmatrix} \Delta x_{\text{enu}} \\ \Delta y_{\text{enu}} \\ \Delta z_{\text{enu}} \end{pmatrix} \quad (31)$$

, where  $\Delta z_{\text{enu}} = 0$ .

[Step 11] Based on the updating values in ECEF, we update the estimating parameters as follows.

$$\begin{aligned} x^1 &= x^0 + \Delta x, & y^1 &= y^0 + \Delta y, \\ z^1 &= z^0 + \Delta z, & s^1 &= s^0 + \Delta s \end{aligned} \quad (32)$$

After that, we return back to the step 2 with the updated values  $x^1, y^1, z^1, s^1$  instead of the initial value  $x^0, y^0, z^0, s^0$ . We update the estimating unknown parameters  $x, y, z, s$  again and again as the residual errors  $\Delta r_1, \Delta r_2, \Delta d$  become small adequately. Finally we get the solution of  $x, y, z, s$ .

By the above steps, we can estimate own position with only two satellites. Our algorithm uses also the travelling distance. In case of vehicles, we should apply the distance to the positioning effectively because we can get the distance easily. As mentioned before, in urban area, the number of satellites which can observe in LOS fluctuates. Moreover, the place where we cannot observe four and more satellites often occurs. Our algorithm can continue the positioning by selecting just two satellites which have high elevation angle. Our approach may achieve the robust positioning everywhere.

### C. Performance evaluation

In this section, we show the examples of our estimation performance. The simulation parameters are summarized in Table I. The positions of the two satellites and the receiver are decided from the real data which was recorded at the fourth order triangulation point (reference code: TR45235161301, Housono, Kyoto Japan). Especially, we set the position of the satellites as the decoded position at 17:50:4.801, December 27, 2012. The simulated satellite #1, #2 is set as the real GPS satellites whose PRN is #5, #18, respectively. The distance  $d$  is 5 [m].

The pseudorange includes some errors such as troposphere and ionosphere delay error, clock error, multipath error and some noise. The experiment was done under the open sky. So, we can ignore the multipath error. The troposphere and ionosphere delay error can be modeled and the satellites send the modeled delay in their messages. So, we can subtract the above modeled delay from the pseudoranges. The remained error is sum of model error and clock error

Table II  
SIMULATION PARAMETERS (ADDING FOR CONVENTIONAL)

Satellite Position #3 [m]	$\alpha_3$	-15787358.0601
	$\beta_3$	20079908.0255
	$\gamma_3$	7249534.1559
Satellite Position #4 [m]	$\alpha_4$	-14284027.2837
	$\beta_4$	-12788411.1086
	$\gamma_4$	19047366.2733

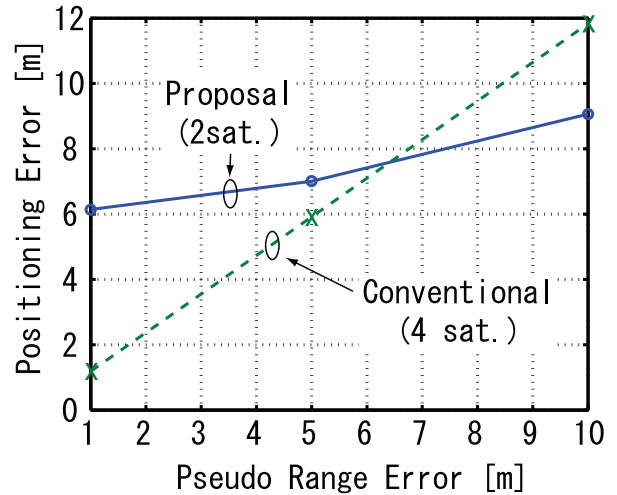


Figure 5. Positioning error (Our algorithm and Conventional)

and some noise. In this simulation, in order to confirm robust convergence, we assume the remained error as a normal distribution with standard variation  $\sigma$ . By using the positions of both the satellites and receiver in Table I, we calculated the true range. And we prepared the pseudorange  $r_1, r_2$  by adding the above remained error to the true range. In the simulation, the standard variation of the error is set as 3 cases, 1.0m, 5.0m, 10.0m. These values are affected in terms of the elevation angle to the satellites. We also modeled the error of the travelling sensor as uniform distribution whose range is  $\epsilon_{\max} = \pm 1.25$ [m]. The range is decided as a outer perimeter of a tire of a personal vehicle.

For comparison, we also estimated the receiver's position by using the conventional algorithm with four satellites. We added more two satellites #3, #4 in Table II to the satellites #1, #2 in Table I. The PRN of the added satellites #3, #4 is #24 and #28 respectively.

As performance measure, we evaluate the positioning error which means the Euclidean distance between the true position and the estimated position. The trial times is 100,000. The final positioning error is defined as the mean value of each of trials. The results are summarized in Figure 5. The horizontal axis is the pseudorange error and the vertical axis is the final positioning error.

From Figure 5, our algorithm has large positioning error when the pseudorange error is small. This is because the error of sensor affects the positioning error as a dominant factor. On the other hand, our algorithm becomes better when the pseudorange error is large. This is because the error of sensor is small compared to the pseudorange error, can suppress the positioning error. Our algorithm has some errors, but noteworthy big advantage. We note our algorithm can estimate the position though the conventional cannot

estimate with only two satellites.

#### V. CONCLUSION

In this paper, we presented the novel positioning algorithm in urban area where propagation environment of multipath or NLOS occurs. Especially, our algorithm can estimate own positions even if the number of LOS satellites is reduced because of the structures such as buildings. The conventional algorithm needs four and more LOS satellites. Our algorithm can estimate with just two LOS satellites by using the sensor data of the travelling distance. In the computer simulations, our algorithm achieved better positioning performance than that of the conventional in case that the pseudorange errors of the satellites were large.

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