

A Robust and Fast Gesture Recognition Method for Wearable Sensing Garments

Ali Boyali

Department of Computing
Macquarie University
Sydney, Australia
ali.boyali@mq.edu.au

Manolya Kavakli

Department of Computing
Macquarie University
Sydney, Australia
manolya.kavakli@mq.edu.au

Abstract— There is an increasing demand for motion capture and full body gesture recognition due to fast paced ubiquitous computing developments and their requirements for natural input modalities. Analysis and synthesis of human body movements are significant issues in bio-medical applications for rehabilitation and identification purposes as in post-stroke patient rehabilitation and gait recognition studies. This paper presents the development of a robust gesture and posture recognition algorithm based on an emerging research field Compressed Sensing (CS) and Sparse Representation (SR), in signal processing for the wearable sensing garment which consists of a sensor network having piezo-resistive properties. The gesture recognition algorithm presented in this study is highly accurate regardless of the signal acquisition method used and gives excellent results even for high dimensional signals and large gesture dictionaries. Our findings state that gestures can be recognized with over 99% accuracy rate using Sparse Representation-based Classification (SRC) algorithm. We tested the algorithm using 3 different gesture dictionaries acquired in 3 different gesture domains with user dependent and user independent test gesture and dictionaries. The system gives 100% recognition accuracy for the gestures performed by sensing t-shirt with two different gesture sets.

Keywords— *Gesture recognition; wearable sensing garments; compressed sensing; sparse signal classification*

I. INTRODUCTION

Among the wide variety of motion capture tools in Human Computer Interaction (HCI) applications, Motion Capture (MoCap) suits are the most complex ones due to their high dimensional sensor networks producing complex data and the large amount of computations needed to interpret a complex data set.

Motion capture in HCI applications has two aspects. The first aspect is the acquisition of motion information, extraction of parameters for reconstruction of motion in a virtual environment [1], and analysis of motion parameters. The second is the synthesis of captured motion information to extract meaningful context as in gesture recognition studies. Although these two aspects serve for distinct aims the latter cannot be implemented without capturing the data.

The structure of motion capture suits is defined by the type of sensor signals, e.g. inertial, acoustic [2], optical, and magnetic or hybrid signal types which makes use of several

signal domains. Every signal domain has pros and cons over other domains. Magnetic motion tracking systems suffer from distortion in their magnetic field [3], whereas optical and accelerometer based systems suffer from occlusion and inherent drift respectively [4, 5]. Other motion capture suits that employ mechanical connections are obtrusive and are not suitable for motion analysis [6, 7] in most cases. In this study, our goal is to develop a robust gesture recognition system for a wearable sensing garment, namely The Sensor T-shirt developed by the research teams at the Electronic Engineering Department of University Pisa, Italy [6, 7]. The sensor t-shirt consists of piezo-resistive sensor threads smeared on an elastic fabric substrate which allows the user to perform motions without any constraint. This feature of the sensor t-shirt makes it a perfect candidate for analysis and synthesis of the motion and many other possible studies. The Sensor T-shirt can be used to aid quadriplegic people to control a wheelchair using their available muscles [9]; or to assist in gesture analysis [10].

The proposed gesture recognition system is based on a new research field, Compressed Sensing (CS) which brings a new insight into signal acquisition and recovery. CS and dimensionality reduction methods such as random projections have been studied intensively in pattern recognition studies. One of the most successful applications of CS and sparse signal recovery is implemented by Wright et al. [11] for face recognition under varying illumination and occlusion. The team simply benefit from the discriminative nature of sparse signal recovery to classify the faces and name their method Sparse Representation-based Classification (SRC).

In this study, we show that gestures can be recognized with an accuracy rate of over 99% using the SRC algorithm without introducing an additional operator in the measurement domain. The adaptation of the SRC method is an advantageous approach in gesture recognition studies.

This paper brings about following advantages in gesture recognition.

- Multi-dimensional gestures can be reshaped (multi dimensional readings are put into vector form) and represented as a one dimensional vector as in the study face recognition implemented by Wright et al. [11] and have a high recognition accuracy.

- No prior clustering algorithm is necessary, however, pre-classification algorithms can be used to reduce the computation time, and there is no upper and lower bound for the number of classes and the number of gesture classes.
- It can be applied in any signal acquisition environment.
- The algorithm achieves high accuracy for rich gesture databases.
- The feature selection is random.

The rest of the paper is organized as follows. Section II gives brief description of the previous sensor garment and gesture recognition and CS and SRC based signal classification studies. Section III presents the contemplated gesture sets for wearable sensor t-shirt and the application of SRC for gesture recognition by sensor garments. The paper ends with conclusion section.

II. PREVIOUS STUDIES

The sensor t-shirt (Fig. 1) consists of a new class of strain sensors network developed at the university of Pisa in Italy [7] which satisfies the user requirements such as wearability, comfort and long term monitoring. The first study on mapping from sensor readings to position and posture domain are conducted by Tognetelli et al in the study [7] who developed and used the sensor garment equipping an elastomeric fabric with piezzo-resistive graphite stripes by smearing them on the garment.

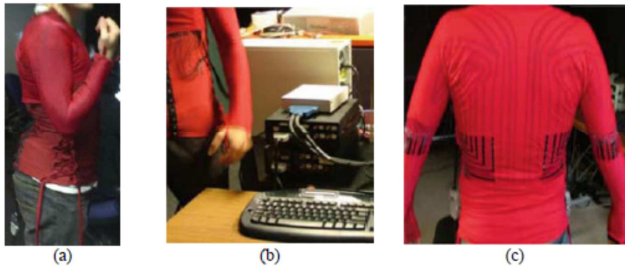


Figure 1. Sensor Shirts and Data Acquisition System Used in Experiments (b-c [20])

Tognetelli et al [7] define posture a geometrical model of body kinematics. According to the definition, when the sensor t-shirt is worn by a user and a posture is performed, the sensor network produces electrical signals strictly related to the posture. The non-linear signal behaviour of the network is modelled by the linear combination of exponential functions. The construct (a mapping function F) between the sensor space and kinematic configuration readings, and corresponding measurements by using a goniometer are stored in a database, and sensor readings are mapped using multi-variate piece-wise interpolation and function inversions. The proposed method in [7] is considered to be time-consuming by the authors as a high number of matrix inversions are necessary.

There are numerous gesture recognition methods available in the literature. Some of these methods require

feature extraction and clusterization algorithms peculiar to gestures that are designed for only their gesture domain and cannot be generalized unlike our gesture recognition algorithm which gives highly accurate results for different measurement and signal domains. On the other hand, stochastic Hidden Markov Models, Dynamic Time Warping and Neural Network based classification methods are common and the accuracy rates vary depending on the application. Although we tested our SRC based gesture recognition method for different measurement domains – with a touchpad, an IR camera and piezo-resistive signal measurements, we only focus on gesture recognition performed by using the sensor jacket in this study. Therefore, we only review the research studies using the sensor jacket, CS, and SRC based classification in gesture recognition.

CS and SR methods were first used in the study [12] by Akl and Valee for gesture recognition with a complementary algorithm Affinity Propagation (AP) proposed by Frey and Dueck [13] which clusters the data by message passing between the points. In their study, the gestural data which consists of accelerometer signals in x, y and z coordinates acquired by a Wiimote are clustered using the AP algorithm into gesture classes using the Dynamic Time Warping (DTW) similarity measure. Then, the gesture to be recognized is compared with candidate exemplars determined by AP. Final classification is carried out converting the classification problem to SR type by introducing a pre-processor. The CS solution is applied to the finalist exemplars hence recovering the gesture class with a high recognition rate attained.

We tested the proposed algorithm by [12 and 19] for three different rich gesture sets. The critical issue is the pre-classification algorithm which will be detailed after outlining the principles of CS and SRC.

Compressed Sensing (CS) and Sparse Representation (SR) of signals is a new research field which allows signals to be recovered with a few number of samples -much below the well-known Nyquist sampling rate from random/non-adaptive measurements- as long as the signal is sufficiently sparse in measurement domain [13-17].

Mathematically, given a sufficiently k -sparse signal $x \in R^n$ whose members consists of a few nonzero k -elements and zeros in a measurement domain with an orthonormal basis Ψ (such as Fourier, Direct Cosine Transformation or wavelet bases), the whole content of x can be recovered by sampling via a random matrix $U \in R^{m \times n}$ which satisfies the Restricted Isometry Property (RIP) by preserving the lengths of k -sparse elements with the condition that $m \ll n$. The resulting equation $y = U\Psi x$ is then solved by the linear programming method ℓ_1 minimization.

The SRC algorithm uses a dictionary matrix consisting of training samples. In the algorithm, first, training samples of k classes and the test image are converted to a column vector and projected on a lower dimensional space using a generated random measurement matrix. Then training vectors are stacked into a matrix to construct the dictionary. The resulting equation is solved to recover the sparse signal x by ℓ_1 minimization methods after the columns of the

dictionary and test vector are normalized. The SRC algorithm assumes the test image vector lie in the linear span of training samples (1) associated with the same class of object where the signal $x=[0, 0, 0, 0, \alpha_1, \alpha_2, \dots, \alpha_n, 0, 0, 0, 0]$.

$$y = \alpha_{i,1} \vartheta_{i,1} + \alpha_{i,2} \vartheta_{i,2} + \dots + \alpha_{i,n_i} \vartheta_{i,n_i} \quad (1)$$

The pseudo code for the algorithm is as follows:

- Construct the dictionary matrix $A=[\vartheta_{i,1}, \vartheta_{i,2}, \dots, \vartheta_{i,k}] \in \mathbb{R}^{m \times n}$ for k classes by reducing the dimension using a random matrix having RIP and converting the samples, and test image vector to one dimensional vectors ϑ_i and y
- Normalize the columns vector of reduced \tilde{A} and the reduced test image vector \tilde{y} ,
- Solve the ℓ_1 minimization problem $\hat{x} = \text{argmin}_x \|x\|_1$ subject to $\tilde{A}x = \tilde{y}$
- Compute the residuals $ri(y) = \|\tilde{y} - \tilde{A}\delta_i(x^*)\|_2$ where δ_i is a selection operator for x^* corresponding the i th class span in A
- Identify y by finding the minimum of $ri(y)$

In the study by Akl and Valee [12], 3 axis gesture traces are divided into the acceleration components of corresponding axes R_x, R_y and R_z and the

$$\bar{y}_x = \bar{R}_x x + \varepsilon_x \quad (2)$$

where \bar{y}_x is the randomly sampled x component of the gesture to be recognized and \bar{R}_x is the classes of the remaining gesture traces after narrowing the AP results.

They convert the recognition problem to CS and SR recognition problem as shown below.

$$Q_x = \text{Orth}(\bar{R}_x^T)^T \quad (3)$$

$$W_x = Q_x \bar{R}_x^\dagger \quad (4)$$

Where Q_x is the orthonormal basis for \bar{R}_x and \bar{R}_x^\dagger is the pseudo-inverse of \bar{R}_x thus the gesture recognition problem has a new form of

$$h_x = W_x \bar{y}_x = Q_x x + \varepsilon'_x \quad (5)$$

Equation (5) is solved for all axis components to identify the gesture class by computing (6) and taking the minimum of $\hat{x}_{eq}(i)$.

$$\hat{x}_{eq} = \hat{x}_x^2 + \hat{x}_y^2 + \hat{x}_z^2 \quad (6)$$

The algorithm gives higher accuracy rates when the AP is applied with this method. The CS based classification with the introduction of the pre-processor operator only gives





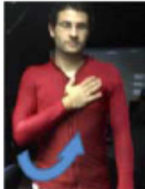
efficient results for a few gesture classes. If the AP is eliminated with our gesture sets.

III. THE SENSOR JACKET SYSTEM AND SRC FOR GESTURE RECOGNITION

The sensor t-shirt has 52 individual piezzo resistive sensor strips which are located from wrist to shoulders on the right and left side of the t-shirt. The data is acquired by the National Instrument Data Acquisition Unit (Fig. 1.).

There are three gesture classes to be recognized by wearing the sensor t-shirt in the first gesture set. These are; moving the arm from relaxed to front at shoulder level, from relaxed position to side at shoulder level and from side to front keeping the arm straight moving horizontally at shoulder level. However we further expands our gesture recognition study by designing a second gesture set to verify and repeat the study. The second gesture set includes 5 gesture classes (Table 1.).

TABLE I. SECOND GESTURE SET

	Right arm up: Right arm is moved from rest to shoulder level straight, hand points ahead
	Right arm up at side: Right arm is moved from rest to shoulder level towards right straightly
	Forearm is moved upper from the rest.
	Right hand is put on head from the relaxed position
	Right hand is put on heart level from the rest

Each sensor reading (Fig. 2.) is sparse in Discrete Cosine Transform (DCT) domain. Before using sensor readings we apply a light Gaussian smoother to the readings to eliminate jitter on the data .

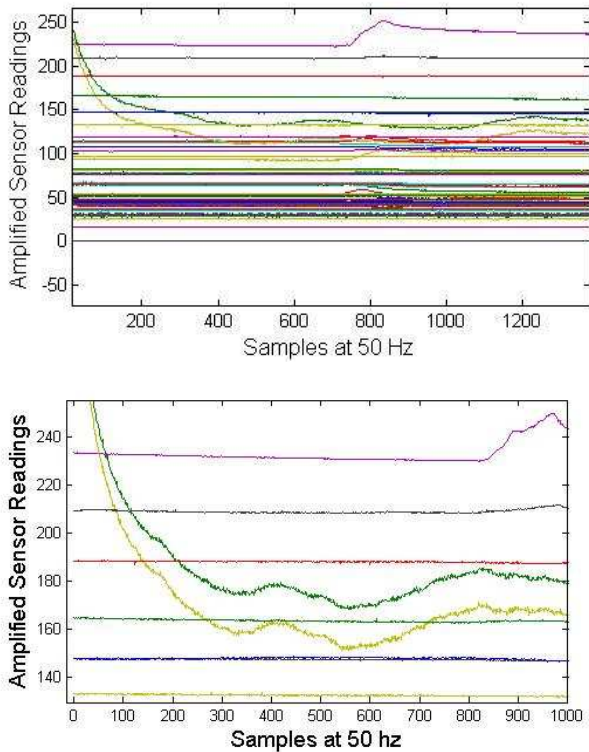


Figure 2. Sensor Data at 50 Hz (Readings from 52 Sensor Threads and a Local View)

All sensor readings are normalized, giving a zero mean and a variance of one. All inactive sensor signals are eliminated defining an absolute total variation threshold that is the ratio of any value in a thread signal to the absolute value of the difference between maximum and minimum values. If this ratio exceeds 10%, it is assumed to be there is a considerable variation in thread readings which contributes to the identification of the activity and gestures (Fig 2).

The remaining sensor readings are concatenated in a vector for an individual gesture. A gesture dictionary is constituted from three gesture classes by stacking the gesture traces as columns of the dictionary. The rest of the solution is to design a measurement matrix and apply ℓ_1 minimization.

CS theory states that if the signal is sparse in any domain, signals can be recovered with an overwhelming probability by random measurement matrices having The Restricted Isometry Property (RIP) condition. Random Gaussian or Achlioptas' matrix [18] can be used for both dimensionality reduction and measurements, since the values having RIP properties preserve distances in the embedding space. We use Achlioptas' matrix since $2/3^{\text{th}}$ of the matrix is sparse, making it easier to construct than a Gaussian matrix thus saving computation time.

Achlioptas' matrix is defined as

$$U_{ij}\sqrt{3} \begin{cases} +1 & \text{with probability } 1/6 \\ 0 & \text{with probability } 2/3 \\ -1 & \text{with probability } 1/6 \end{cases} \quad (7)$$

The pseudo code for the gesture recognition algorithm are as follows.

- Normalize the data, apply a Gaussian smoother and reshape all the gestures and the test gesture, and take DCT of each
- Find the longest length of gestures (lh_{max}) and make the other gestures of the same length by zero padding, and stack the training gestures into training matrix $A_G \in R^{lh_{max} \times n}$

$$A_G = \begin{bmatrix} G_{11} & G_{21} & \dots & G_{k1} \\ G_{12} & G_{22} & \dots & G_{k2} \\ \vdots & \vdots & & \vdots \\ G_{1n1} & G_{2n2} & \dots & G_{knk} \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (8)$$

- Take m measurement from both the test gesture vector and the training matrix with the designed random measurement matrix $U^{m \times n}$ to form the reduced equations $\tilde{y} = \tilde{A}_G x$, where $x = [0, 0, 0, 0, \alpha_1, \alpha_2, \dots, \alpha_p, 0, 0, 0, 0]$ consists of a few nonzero coefficients corresponding to gesture class.
- Solve the ℓ_1 minimization problem $\hat{x} = \text{argmin}_x \|x\|_1$ subject to $\tilde{A}_G x = \tilde{y}$
- Compute the residuals $r_i(y) = \|\tilde{y} - \tilde{A}_G \delta_i(x^*)\|_2$ where δ_i is a selection operator for x^* corresponding the i^{th} class span in \tilde{A}_G
- Identify y by finding the minimum of $r_i(y)$

There are 10 gesture traces collected for each gesture class using sensor jacket for our gesture recognition study. We construct gesture dictionary by randomly choosing 6 gestures from the database, the remaining 4 gestures from each class are used for testing purpose. The proposed algorithm gives 100% recognition rate for the sensor jacket gesture recognition. This paper gives only a brief presentation of the recognition algorithm for the initial studies of gesture recognition with a sensing garment. However, we used the same method for other two gesture databases.

The first database consists of 23 gestures (Fig. 3.) which are 2D gestures captured by the IR camera of a Wiimote. 15 gestures from each gesture class, 10 for dictionary and 5 for testing are captured from 3 subjects. In total, we have 1035 gesture traces from 3 subjects stored in xml format. These gesture files are then converted to .mat files which are then used as input for the SCR gesture recognition algorithm.

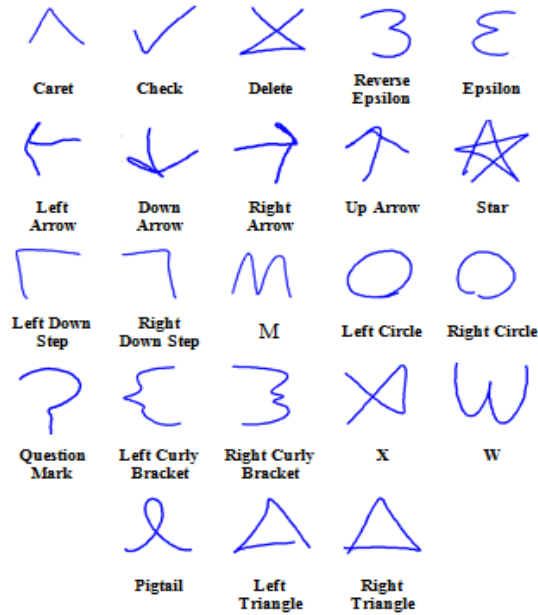


Figure 3. Mixed Gesture Set for Wiimote Gesture Recognition

Our system gives 100% user-dependent recognition accuracy for each person with a full dictionary matrix which consists of 10 gestures for each gesture class including 230 columns in total. To be able to provide a user independent gesture recognition system, the dictionary is built by arbitrarily chosen gestures from each subject with the same number of gestures from each class. Then, we used the remaining gestures for testing purposes. The system misclassified only 2 gestures out of 300 test samples, corresponding to 99.33% recognition accuracy for 20 gestures. 3 gestures were removed from the database, since one gesture class (star shown in Fig 3) was not collected from one of the subjects, and 2 gesture classes which are check mark and left arrow are performed in very different manner.

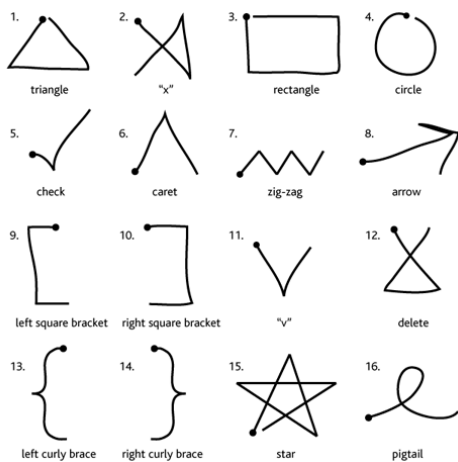


Figure 4. \$1 Gesture Set [19]

We tested our system also with the gesture sets of the \$1 gesture recognition study [19]. The set consists of 16 different gestures collected from 11 subjects (Fig. 4).

Each gesture is repeated 10 times in a 3 speed profile (fast, slow and medium speed) by each subject. The gesture dictionary is built by choosing two random gestures. The random gestures belong to any speed profile from the gesture folder of each of 5 subjects, so that every gesture class consists of 10 gestures. The gestures tested are chosen from the remaining subjects' folder randomly. The SRC gesture recognition algorithm misclassified only 2 gestures out of 80 test gestures with 97.5% accuracy. When the two misclassified gestures are analysed (Fig. 5).

It is seen that the algorithm confuses the circle and rectangle gesture since both of them is unclosed and even may be confused by human brain.

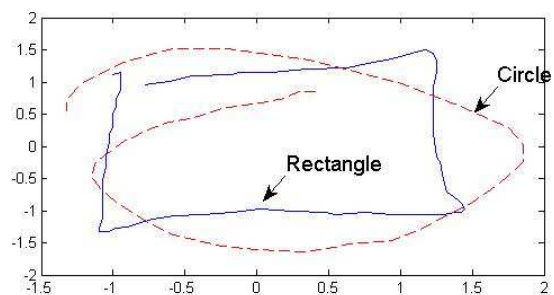


Figure 5. Confused Gestures in \$1 Gesture Set

IV. CONCLUSION

In this paper, we presented a robust gesture and posture recognition algorithm based on an emerging research field CS and SR, in signal processing for the wearable sensing garment which consists of a sensor network having piezo-resistive properties. The gesture recognition algorithm we presented is highly accurate regardless of the signal acquisition method used, and gives excellent results even for high dimensional signals and large gesture dictionaries. Our findings state that gestures can be recognized with over 99% accuracy rate using the SRC algorithm.

Gesture and posture recognition studies in which sensing garments are used have been studied in literature both theoretically and experimentally [6-7, 20]. Various algorithms were proposed in those specific applications for gesture and posture recognition. Our algorithm outperforms in the gesture recognition studies realized by using the sensor jacket with an accuracy level 100% in mapping the sensor readings into gesture domain.

This study can be extended for detection of postures in sensing garment based studies. There are several optimization algorithms proposed for the solution of convex optimization problems. We utilized GPRS (Gradient Projection for Sparse Reconstruction) method proposed by Mario et al. (2008) for the ℓ_1 linear programming problem, as it solves the reconstruction problem in a significantly shorter time [21].

Solution of the equations for the sensor jacket gesture recognition study takes less than 0.1 second with a AMD

Turion 2x2.2Hz processor. This time period can be regarded sufficient for real time applications. The gesture recognition method given in this paper is promising and can provide solutions to high dimensional gesture recognition problems. Gesture spotting is the second fundamental problem in this research field. We focus on the development of a new algorithms which make use of recent developments for low-rank and sparse matrix separation methods for robust posture recognition and gesture spotting.

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