Making Medication Prognoses for Prostate Cancer Patients by the Application of Linguistic Approaches

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***Abstract*—Apart from the probabilistic model and the model of 2-tuple linguistic representations, a new extension of the fuzzy set, known as the hesitant fuzzy linguistic term set can be seen as the third representative of linguistic approaches. In this paper, we focus on multi-expert decision-making problems, in which a group of physicians are independently asked for assessing the effectiveness of a set of treatment therapies. Our goal is to rank the effectiveness of treatment modalities from the most recommended to the contraindicated. Two individual prostate cancer patients have been taken into account in the practical studies. For the first patient, the probabilistic model and the model of 2-tuple linguistic representations have been adopted to accomplish the medical application. Whereas, for the second patient, the approach of hesitant fuzzy linguistic term set has been used to make the medication prognoses. Moreover, the continuous fuzzy numbers in the Left-Right representations are used to mathematically express the experts' judgments and *s*-parametric membership functions are designed to represent the fuzzy linguistic terms.**

Keywords-multi-expert decision making; fuzzy group decision making; probabilistic model; 2-tuple linguistic representations model; hesitant fuzzy linguistic term set

I. INTRODUCTION

Prostate cancer is one of the most common oncological diseases in the world. Due to the vide heterogenicity of malignant potential, the prostate cancer treatment is multifactorial. Like in any other oncological disease, the cooperation of health professionals is required to make the consensual treatment decision. One way to facilitate the treatment decision-making process can be a multidisciplinary team meeting (MDT) - an event or a platform where decision makers from various relevant treatment / diagnostics fields meet to discuss further proceeding.

At the Urology Department of Blekinge County Hospital, Karlskrona, the MDT is a forum of health care providers including medical oncologists, urologists, urology sub-specialized nurses, radiologists and pathologists. The aim of the conference is to assess and establish treatment decisions for particular patients with a spectrum of problematic urological conditions that cannot be easily solved by means of available resources. Our long term aim is also to discuss the best and available treatment modalities of all newly diagnosed cases of prostate cancer. Quite often the decision making process is very clear and straight forward, but some cases lay outside the frames of guidelines and recommendations. Obviously, the final choice of treatment is also on discretion of the patient. This modern approach has however two pitfalls. One of them is when there is a discrepancy between forum members and the other one is when the patient is not interested in the treatment modality chosen by the panel. The best solution is to obtain a method for solving discrepancies and simultaneously to find a method that shows panel's results as treatment recommendations ranged from the strongly recommended to the contraindicated. Such approach should be very helpful particularly in such diseases as prostate cancer, which has a broad spectrum of treatment methods that can be tailored to the particular patient's needs and requirements.

Therefore, in view of the physicians' requirement, we wish to extend our earlier research presented in [1] by adopting the approach of hesitant fuzzy linguistic term set (HFLTS) [2] to make the medication prognoses.

In real life, we often are in such situations that we need to evaluate some information that cannot be expressed in numerical values. In such cases the linguistic approach [3] and its extensions [4]-[8] can be seen as good alternatives. Actually, in medical community, the information often is characterized vaguely and imprecisely, which makes it hard to be evaluated by singular numerical values. For example, the expressions such as "very painful", "slightly painful", "medium" and "not very painful" are just some examples of the linguistic evaluations of subjective pain feeling that can be easily formulated by the patient. Also in group decision making cases, when the experts assess the effectiveness of treatment therapies for prostate cancer patients, the semantic terms such as "contraindicated", "doubtful", "acceptable", "possible", "suitable", "recommended" and "strongly recommended" can be used. Comparing to the numerical quantity, the linguistic approach is regarded in [9], [10] as a more realistic, intuitionistic and natural method. Due to the advantages of the linguistic approach, an extensive application has been presented in [11], [12]. Reasonable results have been reported, e.g., the adoption of the probabilistic model and the model of the 2-tuple fuzzy linguistic representations illustrated that the linguistic approaches supplied the physicians with treatment effectiveness ranked from the strongly recommended to the contraindicated [1]. The linguistic approaches also supported investors with the valuable information how the capital can be effectively invested [12], [13].

By applying three models, namely, the probabilistic model [14], the 2-tuple linguistic representations [15] and the hesitant fuzzy linguistic term sets [2], we intend to rank the effectiveness of treatment alternatives from the most recommended to the contraindicated. The entire process will be defined in the linguistic framework.

The construction of this paper is organized as follows. In Section II, the preliminaries are presented. Section III provides two practical studies of the medical applications. Finally, conclusion and discussion are given in Sections IV and V, respectively.

II. Preliminaries

In this section, some preliminary items are presented. We start with the detailed description of the probabilistic model.

In [14], a general property of a multi-expert decision-making problem is considered as the introduction of a finite set of experts denoted by $E = \{e_1, \cdots, e_p\}$ who are asked for selecting assessments stated in another finite set of alternatives $A = \{a_1, \cdots, a_n\}$. The assessments are expressed by semantic words in an order structured linguistic term set $S = \{s_0, \cdots, s_g\}$, such that $s_k < s_l$ if and only if $k < l$. An example of the ordered structured linguistic term set S is given below.

Example 1: Suppose that we determine a linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ consisting of s_0 = "contraindicated" = C, s_1="doubtful"= D, s_2= "acceptable" = A, s_3= "possible" = P, s_4 = "suitable" = S, s_5 = "recommended" = R and s_6= "strongly recommended" = SR.

A. The Probabilistic Model

According to [14], the probability model mainly contains four steps:

- In the first step, all the assessments are collected in a judgment table as shown in Table I. Here each judgment $L_{ij}, i = 1, \cdots, n$ and $j = 1, \cdots, p$ is expressed by the linguistic term selected from the linguistic term set S.

We should emphasize that each linguistic term s_l, $l = 0, \cdots, g$ via a fuzzy number is associated with a general s-parametric membership function [16]-[18] given by (1), where $z = [0,1]$ is a symbolic reference set for all effectiveness terms, $z_{\min} = 0$, and h_z is defined as the distance between of the peaks between two adjacent fuzzy sets.

TABLE I. THE JUDGMENT TABLE OF THE PROBABILISTIC MODEL

Alternatives	**Experts**		
	e_1	⋯	e_p
a_1	L_{11}	⋯	L_{1p}
a_2	L_{21}	⋯	L_{2p}
⋯	⋯	⋯	⋯
a_n	L_{n1}	⋯	L_{np}

If we set $z_{\min}$ and h_z in (1) as fixed values when choosing $l = 0, \cdots, g$, then we will obtain the membership functions for $s_0, \cdots, s_g$ as

$$\mu_{s_l}(z) = \begin{cases} 2\left(\frac{z-((z_{\min}-h_z)+h_z l)}{h_z}\right)^2 \text{ for} \\ (z_{\min}-h_z)+h_z l \le z \le \left(z_{\min}-\frac{h_z}{2}\right)+h_z l, \\ 1-2\left(\frac{z-(z_{\min}+h_z l)}{h_z}\right)^2 \text{ for} \\ \left(z_{\min}-\frac{h_z}{2}\right)+h_z l \le z \le z_{\min}+h_z l, \\ 1-2\left(\frac{z-(z_{\min}+h_z l)}{h_z}\right)^2 \text{ for} \\ z_{\min}+h_z l \le z \le \left(z_{\min}+\frac{h_z}{2}\right)+h_z l, \\ 2\left(\frac{z-((z_{\min}+h_z)+h_z l)}{h_z}\right)^2 \text{ for} \\ \left(z_{\min}+\frac{h_z}{2}\right)+h_z l \le z \le (z_{\min}+h_z)+h_z l. \end{cases} \quad (1)$$

- X_{a_i} is assumed as a random preference value for each alternative $a_i, i = 1, \cdots, n$, with associated probablility distribution P defined by [8] as

$$P\left(X_{a_i} = s_l\right) = P_E\left(\{e_j \in E \mid L_{ij} = s_l\}\right). \quad (2)$$

It is worth highlighting that the statement of random preference X_{a_i} is a crucial procedure in the approach of probability. Since each X_{a_i} is stochastically independent of each other, it will make the comparisons of any two random preferences to be possible.

- The choice value $V(a_i)$ for each alternative $a_i, i = 1, \cdots, n$, is computed by the choice function implemented by

$$V(a_i) = \sum_{i \neq j} P\left(X_{a_i} \geq X_{a_j}\right)$$
$$= \sum_{i \neq j} \sum_{s_l \in S} \left[P\left(X_{a_i} = s_l\right) \sum_{\substack{L_{ij} \in S \\ s_l \geq L_{ij}}} P\left(X_{a_j} = L_{ij}\right) \right], \quad (3)$$

where the quantity $P\left(X_{a_i} \geq X_{a_j}\right)$ could be interpreted as the probability of "the performance of a_i is as least as good as that of a_j".

- Finally, by ranking the choice values obtained by the former step, we can select the optimal one by

$$a_{\text{optimal}} = \max_{a_i \in A}(V(a_i)). \quad (4)$$

B. The Model of 2-tuple Linguistic Representation

In this model, the physicians' judgments of the treatments are represented by the 2-tuples of the form of (s_l, α), where $s_l \in S$ is a semantic word to which a fuzzy set is assigned and $\alpha \in [-0.5, 0.5)$ is defined as a numerical value.

A 2-tuple linguistic representation model presented in [15] composes the following steps:

- Each judgment that is expressed by a semantic word in Table I is changed into a 2-tuple linguistic representation as (s_l, α). If $s_l \in S$, then $(s_l, 0)$ will reflect s_l. Next, $x_{a_i} = \{(s_l, \alpha)\}$ is defined as a finite set that consists of judgments of the 2-tuple linguistic representations for each alternative $a_i, i = 1, \cdots, n$.

- Two transformations are used in this model.

The first transform Δ^1 maps a 2-tuple representation $(s_l, \alpha) \in S \times [-0.5, 0.5)$ of an alternative a_i into a numerical value $\beta_{a_i}^{e_j} \in [0, g], i = 1, \cdots, n, j = 1, \cdots, p$, in which $\beta_{a_i}^{e_j} = l + \alpha$. The action of Δ^1 is formalized by

$$\Delta^1: \quad S \times [-0.5, 0.5) \rightarrow [0, g]$$
$$(s_l, \alpha) \rightarrow \beta_{a_i}^{e_j} = l + \alpha. \quad (5)$$

We explicate the performance of Δ^1 by the following example.

Example 2: Let $S = \{s_0, \cdots, s_6\}$. In Table II the assessment of a_1, given by expert e_3, is expressed by the semantic term s_2 = "acceptable" =A. By the model of 2-tuple linguistic representation we can employ the judgment (A, 0) presented in Table III for s_2 = "acceptable" =A

TABLE II. THE DECISION TABLE OF THE JUDGMENTS FOR EXAMPLE 2

Alternatives	**Experts**			
	e_1	e_2	e_3	e_4
a_1	s_0	s_1	s_2	s_3
a_2	s_2	s_0	s_1	s_4
a_3	s_3	s_4	s_5	s_1
a_4	s_2	s_1	s_2	s_0

TABLE III. THE JUDGMENT TABLE OF THE 2-TUPLE LINGUISTIC REPRESENTATIONS

Experts	**Alternatives**			
	a_1	a_2	a_3	a_4
e_1	$(C, 0)$	$(A, 0)$	$(P, 0)$	$(A, 0)$
e_2	$(D, 0)$	$(C, 0)$	$(S, 0)$	$(D, 0)$
e_3	$(A, 0)$	$(D, 0)$	$(R, 0)$	$(A, 0)$
e_4	$(P, 0)$	$(S, 0)$	$(D, 0)$	$(C, 0)$

and $\alpha = 0$. The 2-tuple linguistic representations for other judgments are aggregated in Table III.

Due to the first transformation, the 2-tuple representation of (A, 0) can be performed as a numerical value $\beta_{a_1}^{e_3} = l + \alpha = 2 + 0 = 2$, which belongs to the interval [0, 6]. Furthermore, $x_{a_1} = \{(C, 0), (D, 0), (A, 0), (P, 0)\}$ consists of the judgments of the 2-tuple linguistic representations for alternative a_1.

In addition, we use the notation, $\overline{\beta_{a_i}}$, to represent the arithmetic mean of the sum of $\beta_{a_i}^{e_j}$, in which $i = 1, \ldots, n$ and $j = 1, \ldots, p$. The computation of $\overline{\beta_{a_i}}$ is given by

$$\overline{\beta_{a_i}} = \frac{1}{p} \sum_{j=1}^{p} \beta_{a_i}^{e_j}. \quad (6)$$

Example 3: From Table III we obtain $x_{a_1} = \{(C, 0), (D, 0), (A, 0), (P, 0)\} = \{(s_0, 0), (s_1, 0), (s_2, 0), (s_3, 0)\}$, which leads to $\beta_{a_1}^{e_1} = 0 + 0 = 0$, $\beta_{a_1}^{e_2} = 1 + 0 = 1$, $\beta_{a_1}^{e_3} = 2 + 0 = 2$ and $\beta_{a_1}^{e_4} = 3 + 0 = 3$. According to (6), the arithmetic mean of $\overline{\beta_{a_1}}$ is equal to $\frac{1}{4}(0 + 1 + 2 + 3) = 1.5$.

The second transformation Δ^2 can be regarded as an inverse of the first one, i.e., it maps the numerical value $\overline{\beta_{a_i}} \in R$ into a 2-tuple (s_l, α) by

$$\Delta^2: \quad \overline{\beta_{a_i}} \rightarrow S \times [-0.5, 0.5)$$
$$\overline{\beta_{a_i}} \rightarrow (s_l, \alpha). \quad (7)$$

Here s_l has the closest index label to $\overline{\beta_{a_i}}$, the interval of $[0, g]$ represents the space consisting of the semantic label indices in the linguistic term set $S = \{s_l\}$, $l = 0, \cdots, g$.

Example 4: Let $S = \{s_0, \cdots, s_6\}$. According to (6), $\overline{\beta_{a_2}} = \frac{1}{4}(2 + 0 + 1 + 4) = 1.75$. Since 1.75 is closer to s_2 than to

s_1, then we choose s_2 as the semantic word. The difference between 1.75 and 2 is 0.25, and 1.75 lies to the left of 2. Therefore, we choose –0.25 to be the value of α. By means of the second transformation, $\Delta^2(1.75) = (s_2, -0.25)$, which is depicted in Figure 1.

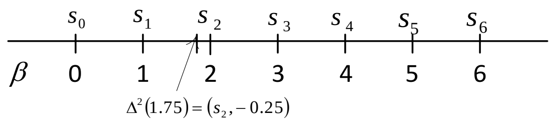

Figure 1. The 2-tuple linguistic representation of $\beta = 1.75$

- The third step contains the computation of the arithmetic mean $\bar{x}^e_{a_i}$ of 2-tuples for each alternative $a_i, i = 1, \cdots, n$, This is formalized by

$$\bar{x}^e_{a_i} = \Delta^2\left(\overline{\beta_{a_i}}\right). \tag{8}$$

Since the arithmetic means, supplied from the previous step, are presented by 2-tuples, a computational technique to compare the arithmetic mean for each alternative proposed in [15] is given as follows.

- Let (s_k, α_1) and (s_l, α_2) be two 2-tuples linguistic representations, with each one representing a counting of information as follows:
  1) if $k < l$, then (s_k, α_1) is smaller than (s_l, α_2).
  2) if $k = l$, we check the following conditions:
     if $\alpha_1 = \alpha_2$, then (s_k, α_1) and (s_l, α_2) represents the same information.
     if $\alpha_1 < \alpha_2$, then (s_k, α_1) is smaller than (s_l, α_2).
     if $\alpha_1 > \alpha_2$, then (s_k, α_1) is greater than (s_l, α_2).

- At last, by comparing the arithmetic values with each other and ranking the alternatives, the optimal alternative(s) will be obtained.

C. The Hesitant fuzzy Linguistic Term Sets

For better understanding of the later application of the hesitant fuzzy linguistic term set (HFLTS) in making medication prognosis for the second prostate cancer patient, we need shortly review the conception of the hesitant fuzzy linguistic term set [2].

Definition 1: Let $S = \{s_0, \dots, s_g\}$ be a linguistic term set. A hesitant fuzzy linguistic term set, denoted by H_s, is an ordered finite subset of the consecutive linguistic terms of S.

The empty HFLTS and the full HFLTS for elements s ∈ S are defined as follows [2]:

- Empty HFLTS: H_s(s) = ∅,
- Full HFLTS: H_s(s) = S.

Example 5: Let us assume that $S = \{s_0, s_1, s_2, s_3, s_4, s_5\}$ is a linguistic term set describing the effectiveness of some treatment schemes for prostate cancer patients. If s_0 = "contraindicated" = "C", s_1 = "acceptable" = "A", s_2 = "possible" = "P", s_3 = "suitable" = "S", s_4 = "recommended" = "R", and s_5 = "strongly recommended"= "SR", then a HFLTS might be $H_s(s) = \{s_2, s_3, s_4\}$ = {P, S, R} = between" P" and "R".

We still suppose that $A = \{a_i\}$, i =1, …, n represents a set including n types of treatment alternatives, $E = \{e_j\}$, j = 1, …, p, denotes a collection of p experts and $S = \{s_l\}$, l = 0, …, g, consists of $g + 1$ linguistic assessments. We use the combination of comparative terms and the words selected from S to express the judgments P_{ij} (the judgments of e_j referring to treatment a_i). Especially, S contains the elements ordered in such a way that $s_q \le s_r$ if and only if $q \le r$, $q, r = 0, \dots, g$ [14]. It is worth highlighting that each s_l is represented by a continuous fuzzy number in the Left-Right form, (*L-R* form) [19]. The aggregated preferences from individual experts are presented in Table IV.

TABLE IV. THE HESITANT JUDGMENT TABLE

Alternatives	Experts		
	e_1	…	e_p
a_1	P_{11}	…	P_{1p}
a_2	P_{21}	…	P_{2p}
…	…	…	…
a_n	P_{n1}	…	P_{np}

Sets H_S^{ij} contain these elements of S which consider the judgments P_{ij}. $H_S^{ij} \subseteq S$,i = 1, …, n, j = 1, …, p. By utilizing the operation of union on sets H_S^{ij} on each row, the new generated HFLTS, U_{a_i}, i = 1, …, n, becomes a subset of the linguistic term set S and obtains all conceivable effectiveness assessments. Subsequently, the union of all the elements in U_{a_i} yields the effectiveness of each alternative denoted by Eff(a_i) = $W(a_i)$. We illustrate this by Example 6.

Example 6: Consider three alternatives $\{a_1, a_2, a_3\} \subset A$ which represent three different kinds of treatment schemes. Three experts $\{e_1, e_2, e_3\} \subset E$ express their preferences about these treatment alternatives by combining comparative terms and words selected from the linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5\}$, in which s_0 = "contraindicated" = "C", s_1 = "acceptable" = "A", s_2 = "possible" = "P", s_3 = "suitable" = "S", s_4 = "recommended" = "R", s_5 = "strongly recommended" = "SR". Table V displays the collection of preferences and Table VI shows the HFLTS's subset of S.

TABLE V. THE HESITANT JUDGMENT TABLE FOR EXAMPLE 6

Alternatives	Experts		
	e_1	e_2	e_3
a_1	$\leq A$	$[A, P]$	R
a_2	$[A, S]$	$[P, S]$	C
a_3	$[S, R]$	$[C, P]$	$[P, S]$

TABLE VI. THE SETS H_S^{ij} AS THE SUBSETS OF S DUE TO EXAMPLE 6

Alternatives	Experts		
	e_1	e_2	e_3
a_1	$\{s_0, s_1\}$	$\{s_1, s_2\}$	$\{s_4\}$
a_2	$\{s_1, s_2, s_3\}$	$\{s_2, s_3\}$	$\{s_4\}$
a_3	$\{s_3, s_4\}$	$\{s_0, s_1, s_2\}$	$\{s_2, s_3\}$

We use the operation of union on the sets H_S^{ij} for each row, denoted by $U_{a_i} = \cup H_S^{ij}$, $i, j = 1, \ldots, 3$, to obtain all possible effectiveness assessments for each alternative. $U_{a_1} = \{C, A\} \cup \{A, P\} \cup \{R\} = \{C, A, P, R\} = \{s_0, s_1, s_2, s_4\}$, $U_{a_2} = \{A, P, S\} \cup \{P, S\} \cup \{C\} = \{s_0, s_1, s_2, s_3\}$ and $U_{a_3} = \{S, R\} \cup \{C, A, P\} \cup \{P, S\} = \{C, A, P, S, R\} = \{s_0, s_1, s_2, s_3, s_4\}$. Hence, the effectiveness assessments of a_1, a_2 and a_3 can be given as follows:

$Eff(a_1) = W_{a_1} = s_0 + s_1 + s_2 + s_4$,

$Eff(a_2) = W_{a_2} = s_0 + s_1 + s_2 + s_3$,

$Eff(a_3) = W_{a_3} = s_0 + s_1 + s_2 + s_3 + s_4$.

In the HFLTS, the judgment expressions based on comparative terms like, e.g., between … and …, greater than … or less than …, [2] will be used to supply the preferences. Symbolically, we denote "between acceptable and possible" as [A, P], "less than possible" as $\leq$ P and "greater than suitable" as $\geq S$. Single words such as "contraindicated" are abbreviated as "C".

After obtaining the sets U_{a_i} containing all conceivable effectiveness assessments, we would like to utilize the algorithm for calculating the sum of fuzzy numbers in the Left-Right (*L-R*) form, and later on to transform the *L-R* form into the interval form [19]. Finally, by adopting the technique of ranking fuzzy numbers in compliance with [20], we hopefully can select the most consensual alternative or alternatives.

We recall the information about fuzzy numbers expressed in the *L-R* form. We suppose that s_q and s_r are two fuzzy numbers in the *L-R* form, in which q, r = 0, …, g. We describe $s_q = \left(m_{s_q},\ \alpha_{s_q}, \beta_{s_q}\right)_{LR}$ and $s_r = \left(m_{s_r},\ \alpha_{s_r}, \beta_{s_r}\right)_{LR}$ in which m_{s_q} and m_{s_r} are called the mean values, α_{s_q} and α_{s_r} are defined as the left spreads, β_{s_q} and β_{s_r} are known as right spreads, respectively. The union of s_q and s_r is calculated by

$$s_q + s_r = \left(m_{s_q} + m_{s_r}, \alpha_{s_q} + \alpha_{s_r}, \beta_{s_q} + \beta_{s_r}\right)_{LR} \tag{9}$$

Being able to rank the fuzzy numbers obtained from (9), we need first transfer them into interval forms.

We review the fuzzy number transformation from the *L-R* form into interval form in [19]. Assume $W_{a_i} = \left(m_{W_{a_i}}, \alpha_{W_{a_i}}, \beta_{W_{a_i}}\right)_{LR}$ is a fuzzy number in the *L-R* form. The interval form of W_{a_i} is given by

$$W_{a_i} = \left[b^-_{W_{a_i}}, m_{W_{a_i}}, b^+_{W_{a_i}}\right]_{int} \tag{10}$$

in which $m_{W_{a_i}}$ is the mean value, $b^-_{W_{a_i}} = m_{W_{a_i}} - \alpha_{W_{a_i}}$ and $b^+_{W_{a_i}} = m_{W_{a_i}} + \beta_{W_{a_i}}$ are defined as the left and the right border, respectively. The membership function associated with the fuzzy number $W_{a_i} = \left[b^-_{W_{a_i}}, m_{W_{a_i}}, b^+_{W_{a_i}}\right]_{int}$ can be given by the following *s*-functions [16]-[18]:

$$y = \mu_{W_{a_i}}(z) = \begin{cases} Left\left(\mu_{W_{a_i}}(z)\right) & \text{for} \quad z \leq m_{W_{a_i}}, \\ Right\left(\mu_{W_{a_i}}(z)\right) & \text{for} \quad z \geq m_{W_{a_i}}, \end{cases} \tag{11}$$

in which

$$Left\left(\mu_{W_{a_i}}(z)\right) = \begin{cases} 2\left(\dfrac{z - b^-_{W_{a_i}}}{m_{W_{a_i}} - b^-_{W_{a_i}}}\right)^2 & \text{for} \quad b^-_{W_{a_i}} \leq z \leq c^1_{W_{a_i}}, \\ 1 - 2\left(\dfrac{z - m_{W_{a_i}}}{m_{W_{a_i}} - b^-_{W_{a_i}}}\right)^2 & \text{for} \quad c^1_{W_{a_i}} \leq z \leq m_{W_{a_i}} \end{cases} \tag{12}$$

and

$$Right\left(\mu_{W_{a_i}}(z)\right) = \begin{cases} 1 - 2\left(\dfrac{z - m_{W_{a_i}}}{b^+_{W_{a_i}} - m_{W_{a_i}}}\right)^2 & \text{for} \quad m_{W_{a_i}} \leq z \leq c^2_{W_{a_i}}, \\ 2\left(\dfrac{z - b^+_{W_{a_i}}}{b^+_{W_{a_i}} - m_{W_{a_i}}}\right)^2 & \text{for} \quad c^2_{W_{a_i}} \leq z \leq b^+_{W_{a_i}}, \end{cases} \tag{13}$$

where $c^1_{W_{a_i}} = \frac{b^-_{W_{a_i}} + m_{W_{a_i}}}{2}$ and $c^2_{W_{a_i}} = \frac{b^+_{W_{a_i}} + m_{W_{a_i}}}{2}$ are arithmetic mean values.

Ranking fuzzy numbers in a decision-making environment is a very important and complex procedure. So far, the approaches to ranking fuzzy numbers have been proposed in [21]-[25]. Some of them are difficult to perform and others lead to different outcomes for a same problem. Therefore, a revised approach, based on [26] was explicated by Wang and Lee in [20]. In [20], the authors argued that "multipying the value on the horizontal axis with the value on the vertical axis often degrades the importance of the value on horizontal axis in ranking fuzzy numbers." Instead, Wang and Lee proposed a technique to overcome the shortcomings. The revised method is given by the following criteria:

- If $\bar{z}(W_{a_i}) > \bar{z}\left(W_{a_j}\right)$, then $W_{a_i} > W_{a_j}$.
- If $\bar{z}(W_{a_i}) < \bar{z}\left(W_{a_j}\right)$, then $W_{a_i} < W_{a_j}$.
- If $\bar{z}(W_{a_i}) = \bar{z}\left(W_{a_j}\right)$, then $W_{a_i} = W_{a_j}$, thereby we check the following conditions:
- If $\bar{\mu}(W_{a_i}) > \bar{\mu}\left(W_{a_j}\right)$, then $W_{a_i} > W_{a_j}$.
- If $\bar{\mu}(W_{a_i}) < \bar{\mu}\left(W_{a_j}\right)$, then $W_{a_i} < W_{a_j}$.
- If $\bar{\mu}(W_{a_i}) = \bar{\mu}\left(W_{a_j}\right)$, then $W_{a_i} = W_{a_j}$,

in which

$$\bar{z}(W_{a_i}) = \frac{\int_{b^-_{W_{a_i}}}^{m_{W_{a_i}}} zLeft\left(\mu_{W_{a_i}}(z)\right)dz + \int_{m_{W_{a_i}}}^{b^+_{W_{a_i}}} zLeft\left(\mu_{W_{a_i}}(z)\right)dz}{\int_{b^-_{W_{a_i}}}^{m_{W_{a_i}}} Left\left(\mu_{W_{a_i}}(z)\right)dz + \int_{m_{W_{a_i}}}^{b^+_{W_{a_i}}} Left\left(\mu_{W_{a_i}}(z)\right)dz} \tag{14}$$

and

$$\bar{\mu}_{W_{a_i}}(z) = \frac{\int_0^1 \mu Left\left(\mu_{W_{a_i}}(z)\right)^{-1} d\mu + \int_0^1 \mu Right\left(\mu_{W_{a_i}}(z)\right)^{-1} d\mu}{\int_0^1 Left\left(\mu_{W_{a_i}}(z)\right)^{-1} d\mu + \int_0^1 Right\left(\mu_{W_{a_i}}(z)\right)^{-1} d\mu} \tag{15}$$

Here, $\left(\bar{z}(W_{a_i}), \bar{\mu}_{W_{a_i}}(z)\right)$ is the centroid point of the fuzzy number W_{a_i}, $Left\left(\mu_{W_{a_i}}(z)\right)$ and $Right\left(\mu_{W_{a_i}}(z)\right)$ are called the left and the right membership functions of W_{a_i}, $Left\left(\mu_{W_{a_i}}(z)\right)^{-1}$ and $Right\left(\mu_{W_{a_i}}(z)\right)^{-1}$ are known as the inverse functions of $Left\left(\mu_{W_{a_i}}(z)\right)$ and $Right\left(\mu_{W_{a_i}}(z)\right)$, respectively.

III. PRACTICAL STUDIES

In this section, we want to present two practical studies in medical group decision-making task. The physicians from a MDT group (urologists and medical oncologists) are independently asked for providing the opinions on some treatment schemes for two separate prostate cancer patients. The methods of probabilistic model, the 2-tuple linguistic model are considered for the first prostate cancer patient and the approach of hesitant fuzzy linguistic term sets is applied to the second patient.

A. *The Probabilistic Model*

Let us suppose that $E = \{e_1, e_2, e_3, e_4\}$ denotes a collection consisting of four physicians. And another set $A = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ contains sex types of treatment schemes for a prostate cancer patient, where a_1= "wait and see", a_2= "active monitoring", a_3= "symptom based treatment", a_4 = "brachytherapy", a_5 = "external beam radiation therapy" and a_6= "radical prostatectomy". Also, $L = \{s_0, s_1, s_2, s_3, s_4, s_5, s_6\}$ includes seven linguistic terms, in which s_0= "contraindicated", s_1= "doubtful", s_2= "acceptable", s_3= "possible", s_4 = "suitable", s_5= "recommended" and s_6= "strongly recommended".

By inserting $z_{\min} = 0$, $h_z = 0.167$ and l = 0 in (1), we obtain the function for s_0 = "contraindicated" expanded by

$$\mu_{s_0}(z) = \begin{cases} 2\left(\frac{z+0.167}{0.167}\right)^2 & \text{for} \quad -0.167 \le z \le -0.0835, \\ 1-2\left(\frac{z}{0.167}\right)^2 & \text{for} \quad -0.0835 \le z \le 0, \\ 1-2\left(\frac{z}{0.167}\right)^2 & \text{for} \quad 0 \le z \le 0.0835, \\ 2\left(\frac{z-0.167}{0.167}\right)^2 & \text{for} \quad 0.0835 \le z \le 0.167. \end{cases} \tag{16}$$

By following the same procedure for l =1, 2, 3, 4, 5 and 6 we generate membership functions

$$\mu_{s_1}(z) = \begin{cases} 2\left(\frac{z}{0.167}\right)^2 & \text{for} \quad 0 \le z \le 0.0835, \\ 1-2\left(\frac{z-0.167}{0.167}\right)^2 & \text{for} \quad 0.0835 \le z \le 0.167, \\ 1-2\left(\frac{z-0.167}{0.167}\right)^2 & \text{for} \quad 0.167 \le z \le 0.2505, \\ 2\left(\frac{z-0.334}{0.167}\right)^2 & \text{for} \quad 0.2505 \le z \le 0.334, \end{cases} \tag{17}$$

$$\mu_{s_2}(z) = \begin{cases} 2\left(\frac{z-0.167}{0.167}\right)^2 & \text{for} \quad 0.167 \le z \le 0.2505, \\ 1-2\left(\frac{z-0.334}{0.167}\right)^2 & \text{for} \quad 0.2505 \le z \le 0.334, \\ 1-2\left(\frac{z-0.334}{0.167}\right)^2 & \text{for} \quad 0.334 \le z \le 0.4175, \\ 2\left(\frac{z-0.501}{0.167}\right)^2 & \text{for} \quad 0.4175 \le z \le 0.501, \end{cases} \tag{18}$$

$$\mu_{s_3}(z) = \begin{cases} 2\left(\frac{z-0.334}{0.167}\right)^2 & \text{for} \quad 0.334 \le z \le 0.4175, \\ 1-2\left(\frac{z-0.501}{0.167}\right)^2 & \text{for} \quad 0.4175 \le z \le 0.501, \\ 1-2\left(\frac{z-0.501}{0.167}\right)^2 & \text{for} \quad 0.501 \le z \le 0.5845, \\ 2\left(\frac{z-0.668}{0.167}\right)^2 & \text{for} \quad 0.5845 \le z \le 0.668, \end{cases} \quad (19)$$

$$\mu_{s_4}(z) = \begin{cases} 2\left(\frac{z-0.501}{0.167}\right)^2 & \text{for} \quad 0.501 \le z \le 0.5845, \\ 1-2\left(\frac{z-0.668}{0.167}\right)^2 & \text{for} \quad 0.5845 \le z \le 0.668, \\ 1-2\left(\frac{z-0.668}{0.167}\right)^2 & \text{for} \quad 0.668 \le z \le 0.7515, \\ 2\left(\frac{z-0.835}{0.167}\right)^2 & \text{for} \quad 0.7515 \le z \le 0.835, \end{cases} \quad (20)$$

$$\mu_{s_5}(z) = \begin{cases} 2\left(\frac{z-0.668}{0.167}\right)^2 & \text{for} \quad 0.668 \le z \le 0.7515, \\ 1-2\left(\frac{z-0.835}{0.167}\right)^2 & \text{for} \quad 0.7515 \le z \le 0.835, \\ 1-2\left(\frac{z-0.835}{0.167}\right)^2 & \text{for} \quad 0.835 \le z \le 0.9185, \\ 2\left(\frac{z-1.002}{0.167}\right)^2 & \text{for} \quad 0.9185 \le z \le 1.002, \end{cases} \quad (21)$$

and

$$\mu_{s_6}(z) = \begin{cases} 2\left(\frac{z-0.835}{0.167}\right)^2 & \text{for} \quad 0.835 \le z \le 0.9185, \\ 1-2\left(\frac{z-1.002}{0.167}\right)^2 & \text{for} \quad 0.9185 \le z \le 1.002, \\ 1-2\left(\frac{z-1.002}{0.167}\right)^2 & \text{for} \quad 1.002 \le z \le 1.0855, \\ 2\left(\frac{z-1.169}{0.167}\right)^2 & \text{for} \quad 1.0855 \le z \le 1.169. \end{cases} \quad (22)$$

We sample all functions (16)–(22) in a family of fuzzy numbers restrictions, which are plotted in Figure 2.

By using the probabilistic model, we collect all the experts' judgments in Table VII, whereas the random preference value of each judgment is given in Table VIII.

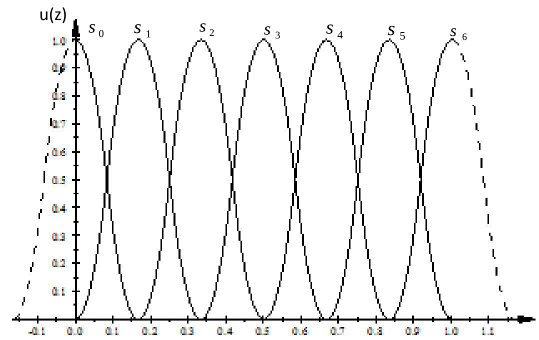

Figure 2. S-parametric membership functions for linguistic fuzzy sets $s_0 - s_6$

TABLE VII. THE COLLECTION OF THE JUDGMENTS FOR THE FIRST PATIENT

Alternatives	Experts			
	e_1	e_2	e_3	e_4
a_1	s_0	s_0	s_0	s_0
a_2	s_6	s_6	s_5	s_5
a_3	s_0	s_0	s_0	s_0
a_4	s_3	s_2	s_4	s_4
a_5	s_3	s_1	s_3	s_4
a_6	s_4	s_5	s_4	s_5

TABLE VIII. THE AGGREGATION OF RANDOM PREFERENCES FOR THE FIRST PATIENT

	Random Preferences						
	s_0	s_1	s_2	s_3	s_4	s_5	s_6
X_{a_1}	1	0	0	0	0	0	0
X_{a_2}	0	0	0	0	0	0.5	0.5
X_{a_3}	1	0	0	0	0	0	0
X_{a_4}	0	0	0.25	0.25	0.5	0	0
X_{a_5}	0	0.25	0	0.5	0.25	0	0
X_{a_6}	0	0	0	0	0.5	0.5	0

By using (3), we calculate the choice value for a_1 as the following structure:

$$\begin{aligned} V(a_1) &= \sum_{1\neq j} P\left(X_{a_1} \ge X_{a_j}\right) \\ &= \sum_{1\neq j}\sum_{s_l\in S}[P(X_{a_1} = s_l) \sum_{\substack{L_{ij}\in S \\ s_l \ge L_{ij}}} P(X_{a_j} = L_{ij})] \\ &= P\left(X_{a_1} \ge X_{a_2}\right) + \cdots + P\left(X_{a_1} \ge X_{a_6}\right) \\ &= 0 + 1 + 0 + 0 + 0 = 1. \end{aligned}$$

TABLE IX. THE COLLECTION OF CHOICE VALUES

The Collection of Choice Values for Each Alternative					
$V(a_1)$	$V(a_2)$	$V(a_3)$	$V(a_4)$	$V(a_5)$	$V(a_6)$
1	5	1	3	2.625	4.25

For other a_i, i = 2,3,4,5,6, $V(a_i)$ are calculated in the similar way as

$V(a_2) = 1 + 1 + 1 + 1 + 1 = 5,$
$V(a_3) = 1 + 0 + 0 + 0 + 0 = 1,$
$V(a_4) = 1 + 0 + 1 + 0.75 + 0.25 = 3,$
$V(a_5) = 1 + 0 + 1 + 0.5 + 0.125 = 2.625,$
and
$V(a_6) = 1 + 0.25 + 1 + 1 + 1 = 4.25.$

The collection of choice values for each $a_i, i = 1, \cdots, 6$ is aggregated in Table IX. We choose the optimal therapy alternative by means of (4) as

$$a_{\text{optimal}} = \max_{a_i \in A}\{V(a_i)\} = \max\{1, 5, 1, 3, 2.625, 4.25\} = 5 = V(a_2).$$

The value of 5 indicates the choice value of a_2 to be maximal. This means that the second therapy alternative is the most efficacious.

We want to confirm the result by applying the model of 2-tuple fuzzy linguistic representations.

B. The Model of 2-tuple Linguistic Representation

According to the algorithm for the model of 2-tuple representation, the judgment that is transformed into 2-tuples is given in Table X.

TABLE X. THE JUDGMENTS EXPRESSED IN THE 2-TUPLES REPRESENTATION MODEL FOR THE FIRST PATIENT

Experts	Alternatives					
	a_1	a_2	a_3	a_4	a_5	a_6
e_1	(C, 0)	(SR, 0)	(C, 0)	(P, 0)	(P, 0)	(S, 0)
e_2	(C, 0)	(SR, 0)	(C, 0)	(A, 0)	(H, 0)	(R, 0)
e_3	(C, 0)	(R, 0)	(C, 0)	(S, 0)	(P, 0)	(S, 0)
e_4	(C, 0)	(R, 0)	(C, 0)	(S, 0)	(S, 0)	(R, 0)

We calculate the arithmetic mean for the first alternative a_1 by means of (5).
$x_{a_1} = \{(\text{C}, 0), (\text{C}, 0), (\text{C}, 0), (\text{C}, 0)\}$ is a finite set consisting of four 2-tuple linguistic representations for the alternative a_1. By adopting (5), the arithmetic mean value for a_1 is calculated as:

$$\bar{x}^e_{a_1} = \Delta^2\left(\frac{1}{4}(0 + 0 + 0 + 0)\right) = \Delta^2(0) = (s_0, 0).$$

For the second alternative the arithmetic means value is given as follows:

$$\bar{x}^e_{a_2} = \Delta^2\left(\frac{1}{4}(6 + 6 + 5 + 5)\right) = \Delta^2(5.5) = (s_5, 0.5).$$

By the same reasoning, when setting i = 3,4,5,6 in (5), we implement

$$\bar{x}^e_{a_3} = \Delta^2\left(\frac{1}{4}(0 + 0 + 0 + 0)\right) = \Delta^2(0) = (s_0, 0),$$

$$\bar{x}^e_{a_4} = \Delta^2\left(\frac{1}{4}(3 + 2 + 4 + 1)\right) = \Delta^2(2.5) = (s_2, 0.5),$$

$$\bar{x}^e_{a_5} = \Delta^2\left(\frac{1}{4}(3 + 1 + 3 + 4)\right) = \Delta^2(2.75) = (s_3, -0.25),$$

and

$$\bar{x}^e_{a_6} = \Delta^2\left(\frac{1}{4}(4 + 5 + 4 + 5)\right) = \Delta^2(4.5) = (s_4, 0.5).$$

The collection of the arithmetic mean values for all alternatives is presented in Table XI.

TABLE XI. TABLE OF THE ARITHMETIC VALUES

The Collection of the Arithmetic Mean Values					
$\bar{x}^e_{a_1}$	$\bar{x}^e_{a_2}$	$\bar{x}^e_{a_3}$	$\bar{x}^e_{a_4}$	$\bar{x}^e_{a_5}$	$\bar{x}^e_{a_6}$
$(s_0, 0)$	$(s_5, 0.5)$	$(s_0, 0)$	$(s_2, 0.5)$	$(s_3, -0.25)$	$(s_4, 0.5)$

According to the computational technique presented earlier, we compare the above 2-tuples that represent the arithmetic values for all the alternatives. We obtain the result presented as $a_2 > a_6 > a_5 > a_4 > a_1 = a_3$, which shows that alternative a_2 is the most efficacious treatment scheme for this particular patient. This result converges to the previous result from "the probabilistic model".

C. The Hesitant Fuzzy Linguistic Term Sets

In this medical application, another prostate cancer patient is considered. We have five health professionals constitute the expert group $E = \{e_1, e_2, e_3, e_4, e_5\}$. The set $A = \{a_1, a_2, a_3, a_4, a_5, a_6, a_7\}$ contains seven types of treatment schemes alternatives, in which a_1= "active expectance", a_2= "active monitoring", a_3= "symptom based treatment", a_4 = "brachytherapy", a_5 = "external beam radiation therapy", a_6 = "adjuvant hormonal therapy" and a_7= "radical prostatectomy".

Furthermore, a linguistic term set $S = \{s_0, s_1, s_2, s_3, s_4, s_5\}$ includes six linguistic terms, in which s_0= "contraindicated" = "C", s_1= "acceptable" = "A", s_2= "possible"= "P", s_3 = "suitable" = "S", s_4= "recommended" = "R", and s_5= "strongly recommended" = "SR". Each linguistic term is associated with a fuzzy number restricted by general *s*-parametric function [16]-[18] given by (1). By choosing l = 0, …, 5, we obtain a family of six membership functions that map the effectiveness of the treatment therapies. Functions s_l are presented in Figure 3.

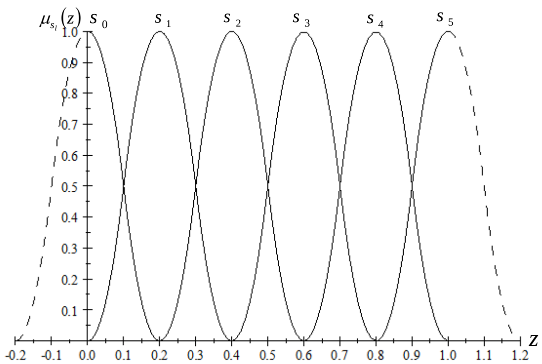

Figure 3. The family of hesitant membership functions $s_0 - s_5$

Here, s_0= "contraindicated" = "C" = $(0, 0.2, 0.2)_{LR}$, s_1= "acceptable" = "A" = $(0.2, 0.2, 0.2)_{LR}$, , s_2= "possible"= "P" = $(0.4, 0.2, 0.2)_{LR}$, s_3 = "suitable" = "S" = $(0.6, 0.2, 0.2)_{LR}$, s_4= "recommended" = "R" = $(0.8, 0.2, 0.2)_{LR}$ and s_5= "strongly recommended" = "SR" = $(1.0, 0.2, 0.2)_{LR}$. By combining comparative terms with single words, the experts express the preferences of the treatment therapies in a broader spectrum. These assessments are aggregated in Table XII.

TABLE XII. THE HESITANT JUDGMENT TABLE DESIGNED BY EXPERTS FOR THE SECOND PATIENT

Alternatives	Experts				
	e_1	e_2	e_3	e_4	e_5
a_1	[A, P]	≥ S	C	[A, P]	≤ P
a_2	≤ A	C	[A, P]	≤ A	R
a_3	[A, S]	C	C	C	≤ A
a_4	[A, S]	[P, S]	C	[A, S]	≤ S
a_5	[S, R]	[P, S]	≥ S	≥ P	≤ A
a_6	[A, S]	[P, S]	C	[P, R]	C
a_7	[S, R]	[C, P]	[P, S]	[P, R]	[P, R]

The assessment "[A, P]" denotes a comparative term, which indicates the terms $s \in S$ between "acceptable" and "possible". It is also a hesitant fuzzy linguistic term set H_S^{11}= $\{s_1, s_2\}$ in which $\{s_1, s_2\} \subset S$. "$\geq S$ " can be interpreted as "greater than suitable", which symbolizes another hesitant fuzzy linguistic term set H_S^{12}= $\{s_0, s_1, s_2, s_3\}$. Furthermore, "$\leq$ A " means "less than acceptable", which assigns H_S^{21}= $\{s_0, s_1\}$.

In order to obtain the assessments as comprehensive as possible and prevent the information loss, for individual alternative, we perform the operation of union on fuzzy sets to aggregate all possible preferences in one set. Therefore,

$$U_{a_1} = \{A, P\} \cup \{S, R, SR\} \cup \{C\} \cup \{A, P\} \cup \{C, A\ P\} = \{C, A, P, S, R, SR\} = \{s_0, s_1, s_2, s_3, s_4, s_5\},$$

$$U_{a_2} = \{C, A\} \cup \{C\} \cup \{A, P\} \cup \{C, A\} \cup \{R\} = \{C, A, P, R\} = \{s_0, s_1, s_2, s_4\},$$

$$U_{a_3} = \{A, P, S\} \cup \{C\} \cup \{C\} \cup \{C\} \cup \{C, A\} = \{C, A, P, S\} = \{s_0, s_1, s_2, s_3\},$$

$$U_{a_4} = \{A, P, S\} \cup \{P, S\} \cup \{C\} \cup \{A, P, S\} \cup \{C, A, P, S\} = \{C, A, P, S\} = \{s_0, s_1, s_2, s_3\},$$

$$U_{a_5} = \{S, R\} \cup \{P, S\} \cup \{S, R, SR\} \cup \{P, S, R, SR\} \cup \{C, A\} = \{C, A, P, S, R, SR\} = \{s_0, s_1, s_2, s_3, s_4, s_5\}$$

$$U_{a_6} = \{A, P, S\} \cup \{C\} \cup \{P, S\} \cup \{C\} \cup \{P, S, R\} = \{C, A, P, S, R\} = \{s_0, s_1, s_2, s_3, s_4\}$$

and

$$U_{a_7} = \{S, R\} \cup \{C, A, P\} \cup \{P, S\} \cup \{P, S, R\} = \{C, A, P, S, R\} = \{s_0, s_1, s_2, s_3, s_4\}.$$

We recall the union of two fuzzy numbers which can be performed by (9). Thereby, the effectiveness of a_1 and a_5 can be calculated as

$$Eff(a_1) = W_{a_1} = s_0 + s_1 + s_2 + s_3 + s_4 + s_5 = (0, 0.2, 0.2)_{LR} + \cdots + (1.0, 0.2, 0.2)_{LR} = (3.0, 1.2, 1.2)_{LR} \equiv [1.8, 3.0, 4.2]_{int}$$

and

$$Eff(a_5) = W_{a_1} = s_0 + s_1 + s_2 + s_3 + s_4 + s_5 = (0, 0.2, 0.2)_{LR} + \cdots + (1.0, 0.2, 0.2)_{LR} = (3.0, 1.2, 1.2)_{LR} \equiv [1.8, 3.0, 4.2]_{int}.$$

The membership functions for a_1 and a_5 are given by

$$y = \mu_{W_{a_1}}(z) = \mu_{W_{a_5}}(z) = \begin{cases} Left\left(\mu_{W_{a_1}}(z)\right) & \text{for} \quad z \leq 3.0, \\ Right\left(\mu_{W_{a_1}}(z)\right) & \text{for} \quad z \geq 3.0, \end{cases} \quad (23)$$

in which

$$Left\left(\mu_{W_{a_1}}(z)\right) = Left\left(\mu_{W_{a_5}}(z)\right) = \begin{cases} 2\left(\dfrac{z-1.8}{3.0-1.8}\right)^2 & \text{for} \quad 1.8 \leq z \leq 2.4, \\ 1-2\left(\dfrac{z-3.0}{3.0-1.8}\right)^2 & \text{for} \quad 2.4 \leq z \leq 3.0, \end{cases} \quad (24)$$

and

$$Right\left(\mu_{W_{a_1}}(z)\right) = Right\left(\mu_{W_{a_5}}(z)\right) = \begin{cases} 1 - 2\left(\frac{z-3.0}{4.2-3.0}\right)^2 & \text{for} \quad 3.0 \le z \le 3.6, \\ 2\left(\frac{z-4.2}{4.2-3.0}\right)^2 & \text{for} \quad 3.6 \le z \le 4.2. \end{cases} \quad (25)$$

The membership function of W_{a_1} and W_{a_5} is depicted in Figure 4.

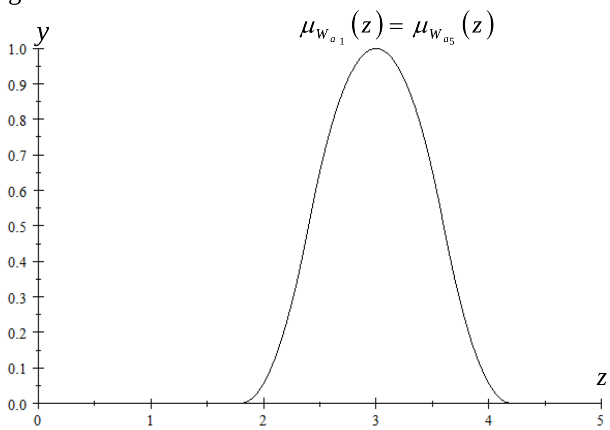

Figure 4. The membership function of W_{a_1} and W_{a_5}

The effectiveness of W_{a_2} is given by $Eff(a_1) = W_{a_2} = s_0 + s_1 + s_2 + s_3 + s_4 = (0, 0.2, 0.2)_{LR} + \cdots + (0.8, 0.2, 0.2)_{LR} = (1.4, 0.8, 0.8)_{LR} \equiv [0.6, 1.4, 2.2]_{int}$. The membership function of W_{a_2} is given by

$$Left\left(\mu_{W_{a_2}}(z)\right) = \begin{cases} 2\left(\frac{z-0.6}{1.4-0.6}\right)^2 & \text{for} \quad 0.6 \le z \le 1.0, \\ 1 - 2\left(\frac{z-1.4}{1.4-0.6}\right)^2 & \text{for} \quad 1.0 \le z \le 1.4, \end{cases} \quad (26)$$

and

$$Right\left(\mu_{W_{a_2}}(z)\right) = \begin{cases} 1 - 2\left(\frac{z-1.4}{2.2-1.4}\right)^2 & \text{for} \quad 1.4 \le z \le 1.8, \\ 2\left(\frac{z-2.2}{2.2-1.4}\right)^2 & \text{for} \quad 1.8 \le z \le 2.2. \end{cases} \quad (27)$$

The membership function of W_{a_2} is plotted in Figure 5.

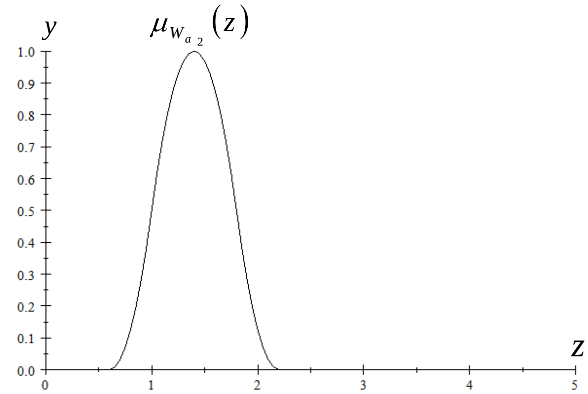

Figure 5. The membership function of W_{a_2}

For a_3 and a_4, the result is shown by $Eff(a_3) = W_{a_3} = s_0 + s_1 + s_2 + s_3 = (0, 0.2, 0.2)_{LR} + \cdots + (0.6, 0.2, 0.2)_{LR} = (1.2, 0.8, 0.8)_{LR} \equiv [0.4, 1.2, 2.0]_{int}$

and

$Eff(a_4) = W_{a_4} = s_0 + s_1 + s_2 + s_3 = (0, 0.2, 0.2)_{LR} + \cdots + (0.6, 0.2, 0.2)_{LR} = (1.2, 0.8, 0.8)_{LR} \equiv [0.4, 1.2, 2.0]_{int}$

The membership functions for W_{a_3} and W_{a_4} are given by

$$Left\left(\mu_{W_{a_3}}(z)\right) = Left\left(\mu_{W_{a_4}}(z)\right) = \begin{cases} 2\left(\frac{z-0.4}{1.2-0.4}\right)^2 & \text{for} \quad 0.4 \le z \le 0.8, \\ 1 - 2\left(\frac{z-0.4}{1.2-0.4}\right)^2 & \text{for} \quad 0.8 \le z \le 1.2, \end{cases} \quad (28)$$

and

$$Right\left(\mu_{W_{a_3}}(z)\right) = Right\left(\mu_{W_{a_4}}(z)\right) = \begin{cases} 1 - 2\left(\frac{z-1.2}{2.0-1.2}\right)^2 & \text{for} \quad 1.2 \le z \le 1.6, \\ 2\left(\frac{z-2.0}{2.0-1.2}\right)^2 & \text{for} \quad 1.6 \le z \le 2.0. \end{cases} \quad (29)$$

Figure 6 represents the membership function of W_{a_3} and W_{a_4}.

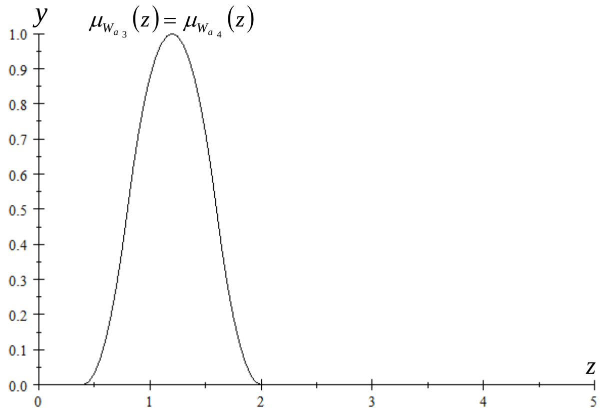

Figure 6. The membership function of $W_{a_3}$and W_{a_4}

And finally, the effectiveness of a_6 and a_7 is yielded by $Eff(a_6) = W_{a_6} = s_0 + s_1 + s_2 + s_3 + s_4 = (0, 0.2, 0.2)_{LR}$ $+ \ldots + (0.8, 0.2, 0.2)_{LR} = (2.0, 1.0, 1.0)_{LR} \equiv [1.0, 2.0, 3.0]_{int}$

and

$Eff(a_7) = W_{a_7} = s_0 + s_1 + s_2 + s_3 + s_4 = (0, 0.2, 0.2)_{LR}$ $+ \ldots + (0.8, 0.2, 0.2)_{LR} = (2.0, 1.0, 1.0)_{LR} \equiv [1.0, 2.0, 3.0]_{int}$.

The membership functions for W_{a_6} and W_{a_7} are given by

$$Left\left(\mu_{W_{a_6}}(z)\right) = Left\left(\mu_{W_{a_7}}(z)\right) = \begin{cases} 2\left(\dfrac{z-1.0}{2.0-1.0}\right)^2 & \text{for} \quad 1.0 \le z \le 1.5, \\ 1-2\left(\dfrac{z-2.0}{2.0-1.0}\right)^2 & \text{for} \quad 1.5 \le z \le 2.0, \end{cases} \tag{30}$$

and

$$Right\left(\mu_{W_{a_6}}(z)\right) = Right\left(\mu_{W_{a_7}}(z)\right) = \begin{cases} 1-2\left(\dfrac{z-2.0}{3.0-2.0}\right)^2 & \text{for} \quad 2.0 \le z \le 2.5, \\ 2\left(\dfrac{z-3.0}{3.0-2.0}\right)^2 & \text{for} \quad 2.5 \le z \le 3.0. \end{cases} \tag{31}$$

Figure 7 represents the membership function of W_{a_6} and W_{a_7}.

We first use (14) to calculate the horizontal coordinate of the centroid point of each fuzzy number W_{a_i}. If there exists identical horizontal coordinates, then (15) will be used to compute the vertical coordinate.

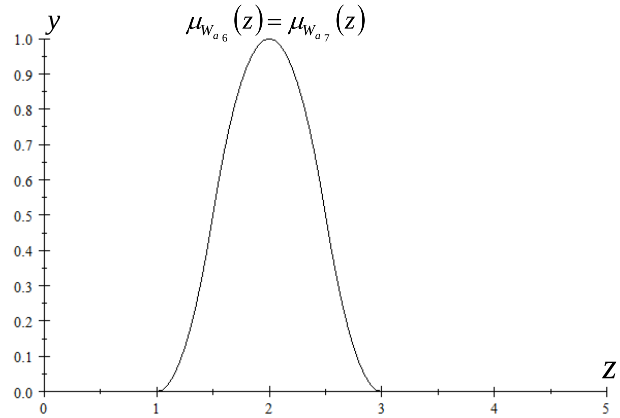

Figure 7. The membership function of W_{a_6} and W_{a_7}

By the insertion of the left respective the right membership functions and the borders of W_{a_1} and W_{a_5} in (14), we obtain the horizontal coordinate of $W_{a_1}$and W_{a_5} presented as follows:

$$\bar{z}(W_{a_1}) = \bar{z}(W_{a_5}) = = \frac{\int_{1.8}^{3.0} zLeft\left(\mu_{W_{a_1}}(z)\right)dz + \int_{3.0}^{4.2} zLeft\left(\mu_{W_{a_1}}(z)\right)dz}{\int_{1.8}^{3.0} Left\left(\mu_{W_{a_1}}(z)\right)dz + \int_{3.0}^{4.2} Left\left(\mu_{W_{a_1}}(z)\right)dz} = = \frac{3.6}{1.2} = 3.0.$$

By the same procedure, we obtain the horizontal coordinates for the remained alternatives presented below:

$$\bar{z}(W_{a_2}) = \frac{\int_{0.6}^{1.4} zLeft\left(\mu_{W_{a_2}}(z)\right)dz + \int_{1.4}^{2.2} zLeft\left(\mu_{W_{a_2}}(z)\right)dz}{\int_{0.6}^{1.4} Left\left(\mu_{W_{a_2}}(z)\right)dz + \int_{1.4}^{2.2} Left\left(\mu_{W_{a_2}}(z)\right)dz} = = \frac{1.12}{0.8} = 1.4$$

$$\bar{z}(W_{a_3}) = \bar{z}(W_{a_4}) = \frac{\int_{0.4}^{1.2} zLeft\left(\mu_{W_{a_3}}(z)\right)dz + \int_{1.2}^{2.0} zLeft\left(\mu_{W_{a_3}}(z)\right)dz}{\int_{0.4}^{1.2} Left\left(\mu_{W_{a_3}}(z)\right)dz + \int_{1.2}^{2.0} Left\left(\mu_{W_{a_3}}(z)\right)dz} = = \frac{2.073}{0.8} \approx 2.5913$$

and finally,

$$\bar{z}(W_{a_6}) = \bar{z}(W_{a_7}) = \frac{\int_{1.0}^{2.0} zLeft\left(\mu_{W_{a_6}}(z)\right)dz + \int_{2.0}^{3.0} zLeft\left(\mu_{W_{a_6}}(z)\right)dz}{\int_{1.0}^{2.0} Left\left(\mu_{W_{a_6}}(z)\right)dz + \int_{2.0}^{3.0} Left\left(\mu_{W_{a_6}}(z)\right)dz} = = \frac{2.0}{1.0} = 2.$$

Since no identical horizontal coordinates are found, we do not need to compute the values of the vertical coordinates. By means of the criteria introduced in [35] we obtain the following results: 3 > 2.5913 > 2 > 1.4, i.e. $\bar{z}(W_{a_1}) = \bar{z}(W_{a_5}) > \bar{z}(W_{a_3}) = \bar{z}(W_{a_4}) > \bar{z}(W_{a_6}) = \bar{z}(W_{a_7}) > \bar{z}(W_{a_2})$. The first a_1 = "active expectance" and the fifth alternative a_5 = "external beam radiation therapy" have the most optimal effectiveness found for the second patient according to our computation.

IV. Conclusion

According to the physicians' requirements, we seek the arrangements of the effectiveness of treatment alternatives from the most recommended to the contraindicated for two separate prostate cancer patients.

Three approaches, such as the probabilistic model, the model of 2-tuple linguistic representations and the hesitant fuzzy linguistic term sets have been applied to two multi-expert decision-making cases. The convergence results from the first two approaches verify the high reliability of adopting the linguistic approach in solving group decision making problems. Moreover, the independent assumed preferences of each alternative make the computation of comparing the probabilities easy to be performed. Especially, the use of the 2-tuple linguistic representation model prevents the loss of information and makes the result more precise. The use of *s*-parametric membership functions not only increases the accuracy rate of the comparative analysis, but also facilitates the transformation process from the linguistic preferences to the numerical values. In the approach of hesitant fuzzy linguistic term set, the horizontal coordinates of the centroid point of fuzzy numbers are adopted for ranking the fuzzy numbers in a decision-making environment. The calculating process has its complexity but the technique is reliable.

V. Discussion

We found all three methods very interesting in decision-making process when panelists were not unanimous. The results seem to be reasonable from the clinical point of view. The process of sampling the data by filling the questionnaires was easy and quickly accomplished, especially in the probabilistic model and the model of 2-tuple linguistic representations. However, we do encountered some issues in filling questionnaires for the hesitant fuzzy linguistic term sets, mostly because it is not as intuitive as the two aforementioned methods. The filling of hesitant fuzzy linguistic term sets questionnaire needs a 2-3 minutes preparation, just to maintain the homogenicity of the answers. The authors have a feeling that the hesitant fuzzy linguistic term sets is a reliable method, but probably better to use in other conditions than in prostate cancer decisions making, mostly because of its logistical and practical problems. We hope to soon introduce one of the models in our clinical practice to assess the method in a real life conditions. Hopefully, this approach can allow us to find better treatment strategies and to give prostate cancer patients more flexibility concerning the treatment options. This should be a great complement to the current guidelines and scientific society recommendations.

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