# Achievement of Collision Avoidance and Formation for Nonlinear Multi-Ship Systems Using an Interval Type-2 Fuzzy Tracking Approach

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Abstract—This paper addresses the challenges of formation and collision avoidance for Nonlinear Multi-Ship Systems (NM-ASs) using an Interval Type-2 (IT-2) fuzzy tracking controller. Since its significant value in military applications, the control of Multi-Agent Systems (M-ASs) has garnered considerable attention. To allocate tasks more properly within M-ASs, the leader-follower control scheme has been developed. However, the nonlinearities and uncertainties in ship dynamics continue to hinder task execution effectiveness. Compared to Type-1 Takagi-Sugeno Fuzzy Models (T-SFMs), IT-2 fuzzy control offers superior uncertainty handling and provides more precise control for NM-ASs. Previous research has introduced an IT-2 Formation-and-Containment (F-and-C) fuzzy control approach for multiple ships and combined some performance constraints to enhance the control efficiency of the leaders. Nonetheless, the safety of the leader ships, who are the most critical components of whole system, remains a concern until they can avoid obstacles and other ships. In this research, the Artificial Potential Fields (APFs) based-collision avoidance control is integrated with the IT-2 fuzzy control theory. Based on the IT-2 T-SFM, a fuzzy tracking control approach is developed to simultaneously achieve both collision avoidance and formation tasks. Finally, simulation results for four leader ships are presented to verify the efficiency and applicability of the proposed IT-2 fuzzy tracking controller.

Keywords-nonlinear multi-agent system; interval Type-2 fuzzy control; formation; tracking control; collision avoidance.

### I. INTRODUCTION

Ships have long been essential in meeting the demands of both civilian and military sectors [1]. However, as the number of ships on the ocean continues to increase, safety concerns have become increasingly pressing. To address the issue, it is crucial not only to improve course management but also to ensure precise control over ship dynamics [2]. In practice, nonlinearities in ship dynamics arise from the complex marine environment [3], which makes designing controllers to ensure precise performance even more challenging. Moreover, the situation is compounded by uncertainties caused by equipment aging, rust, corrosion, biofouling, and other factors [4]. These issues further degrade the control performance of ships. In the past, the Takagi-Sugeno Fuzzy Model (T-SFM) has proven to be a powerful tool for the control and analysis of nonlinear systems. To overcome the limitations of the Type-1 T-SFM,

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the Interval Type-2 (IT-2) T-SFM has been introduced to better handle uncertainties [5]. In a complementary way, the Imperfect Premise Matching (IPM) fuzzy controller design has been systematically proposed in [5] to enable a more relaxed analysis process.

Unlike traditional applications, an increasing body of research has focused on deploying multiple unmanned vehicles to accomplish complex tasks [6]. With an appropriate cooperative topology among all units, the entire system can achieve common objectives more efficiently and effectively [7]. Consequently, control issues of Multi-Agent Systems (M-ASs) have garnered substantial attention from both academia and industry [8]. In the various objectives of M-ASs, Formation-and-Containment (F-and-C) controls remain two key topics that continue to be actively researched [9-10]. Formation control has seen significant development due to its broad applicability across diverse fields. However, nonlinearities and uncertainties in M-ASs often have a more pronounced impact than in single-agent systems. Building upon the T-SFM, many researchers have applied fuzzy control methods to complete formation or containment objectives [11-12]. Additionally, some researchers have tackled the F-and-C problems simultaneously using the T-SFM [13]. Nevertheless, the limitations of Type-1 T-SFM in managing uncertain factors can undermine the effectiveness of Nonlinear M-ASs (NM-ASs). These uncertainties can lead to imprecise system dynamics, with errors propagating sequentially one after another, ultimately causing collisions and system failure.

However, the F-and-C control based on the IT-2 T-SFM remains an open issue. Some researchers have successfully developed IT-2 fuzzy containment control approaches [14]. Nevertheless, these studies considered the leaders as openloop systems. From a practical application perspective, it is essential to ensure the stability of the leaders as well. If the leaders are unstable, the followers will also become unstable, even if they are contained within the region formed by the leaders [13-14]. Recently, our research proposed an IT-2 fuzzy control approach to achieve F-and-C [15]. In this control problem, leader crashes can significantly impact the entire NM-AS and may even result in its collapse. To further ensure the reliability of leaders, anti-disturbance capabilities were also incorporated. However, the safety of the leaders remains uncertain unless they can actively avoid other ships or obstacles. In recent decades, collision avoidance control

based on the Artificial Potential Field (APF) method has been widely adopted for its intuitive nature and ease of implementation [16]. The APF approach has proven to be an effective solution for collision avoidance due to the reduction of computational time. This advantage has also demonstrated the suitability of the APF method for application in scenarios involving multiple-ship encounters [17]. Therefore, the APF is combined with the IT-2 fuzzy tracking controller design in this research to achieve both collision avoidance and formation for leader ships.

The organization of this research is provided as follows. In Section II, the nonlinear system and the IT-2 T-SFM are presented for the leaders in NM-AS. In Section III, an IT-2 fuzzy controller design approach in terms of the IPM concept is proposed for the achievement of collision avoidance and formation. In Section IV, the simulation results are given for four leader ships. In Section V, some conclusions and future works are given for this research.

# II. NM-AS AND IT-2 T-SFM

A nonlinear system is presented to describe the dynamic behaviors of NM-AS by extending the mathematical model in [18] in this section. Taking into account the effects of uncertain factors, the IT-2 T-SFM is also built. First, the NM-AS is considered as follows.

$$\dot{x}_{1}^{\varepsilon}(t) = \mathcal{G}^{\varepsilon}(t) x_{4}^{\varepsilon}(t) - \phi^{\varepsilon}(t) x_{5}^{\varepsilon}(t)$$

$$\tag{1}$$

$$\dot{x}_{2}^{\varepsilon}(t) = \phi^{\varepsilon}(t)x_{4}^{\varepsilon}(t) + \mathcal{G}^{\varepsilon}(t)x_{5}^{\varepsilon}(t)$$
(2)

$$\dot{x}_{3}^{\varepsilon}(t) = \varphi^{\varepsilon}(t) x_{6}^{\varepsilon}(t)$$
(3)

$$\dot{x}_{4}^{\varepsilon}(t) = -0.0318x_{4}^{\varepsilon}(t) + 0.8870u_{1}^{\varepsilon}(t)$$
(4)

$$x_{5}(t) = -0.0628x_{5}(t) - 0.0030x_{6}(t) + 0.5415u_{5}^{2}(t) + 0.3152u_{5}^{2}(t)$$
(5)

$$\dot{x}_{6}^{\varepsilon}(t) = -0.0045x_{5}^{\varepsilon}(t) - 0.2427x_{6}^{\varepsilon}(t)$$

$$+0.3152u_2^{\varepsilon}(t)+8.0082u_3^{\varepsilon}(t)$$
 (6)

where  $\varphi^{\varepsilon}(t) = 1 + \Delta^{\varepsilon}(t)$ ,  $\mathscr{P}^{\varepsilon}(t) = \cos(x_{3}^{\varepsilon}(t)) + \Delta^{\varepsilon}(t)$ ,  $\varphi^{\varepsilon}(t) = \sin(x_{3}^{\varepsilon}(t)) + \Delta^{\varepsilon}(t)$ ,  $x_{1}^{\varepsilon}(t)$ ,  $x_{2}^{\varepsilon}(t)$  and  $x_{3}^{\varepsilon}(t)$  are the (x, y) position and yaw angle on earth-fixed coordinate,  $x_{4}^{\varepsilon}(t)$ ,  $x_{5}^{\varepsilon}(t)$  and  $x_{6}^{\varepsilon}(t)$  are the velocities of surge motion, sway motion and the yaw angle variation,  $u_{1}^{\varepsilon}(t)$ ,  $u_{2}^{\varepsilon}(t)$  and  $u_{3}^{\varepsilon}(t)$  are the forces and moments generated by thrusters,  $\Delta^{\varepsilon}(t)$  are the uncertain factors, each leader is indexed with the number  $\varepsilon = 1, 2, 3, 4$ . In this research, the different situations of uncertainties,  $\Delta^{1,2}(t) = 0.1\cos(t)$  and  $\Delta^{3,4}(t) = 0.1\sin(t)$ , are considered for four leaders.

To make the control problem clearer, Fig. 1 is presented for four leaders with the four target trajectories and two obstacles.



Figure 1. Formation and collision avoidance problems.

According to Fig. 1, the formation objective of the four leader ships is to track the trajectory and maintain the rectangular formation until reaching the destination on the left-hand side. It is assumed that there is an uncrossable obstacle between the initial positions of the four leaders and the destination. Within the target trajectories, two obstacles need to be avoided.

Remark 1

Based on the findings in [15], the individual tracking controller for each leader can efficiently accomplish the formation task and define the dynamics of the entire system. Furthermore, communication between the leaders that are farthest apart from each other is not required. In this research, only the formation problem of the leaders in [15] is considered, so the communication topology is not necessary.

Covering the uncertain factors in the representation, the IT-2 T-SFM is constructed for the NM-AS (1)-(6) as follows.

$$\dot{x}^{\varepsilon}(t) = \sum_{\alpha=1}^{3} \tilde{\Omega}_{\alpha} \left( x_{3}^{\varepsilon}(t) \right) \left\{ \mathbf{A}_{\alpha} x^{\varepsilon}(t) + \mathbf{B}_{\alpha} u^{\varepsilon}(t) \right\}$$
(7)

where  $u^{\varepsilon}(t) = \begin{bmatrix} u_1^{\varepsilon}(t) & u_2^{\varepsilon}(t) & u_3^{\varepsilon}(t) \end{bmatrix}^{\mathrm{T}}$ ,

 $x^{\varepsilon}(t) = \begin{bmatrix} x_1^{\varepsilon}(t) & x_2^{\varepsilon}(t) & x_3^{\varepsilon}(t) & x_4^{\varepsilon}(t) & x_5^{\varepsilon}(t) & x_6^{\varepsilon}(t) \end{bmatrix}^{T}$ . To save space, the model matrices in (7) are referred to [15] and will not be presented. The upper and lower bound membership functions are designed as follows.



Figure 2. IT-2 membership function of NM-AS.

#### Remark 2

It is worth noting that nearly all T-SFMs in the existing literature are established for an operating range between  $-90^{\circ}$  and  $90^{\circ}$ . However, the desired yaw angle, obtained through the APF for collision avoidance, may fall outside this range. This could lead to instability of the ships, as the desired yaw angle must be tracked. To address this issue, the membership functions originally defined for the ranges from  $0^{\circ}$  to  $90^{\circ}$  and  $-90^{\circ}$  to  $0^{\circ}$  are extended to cover the ranges from  $-180^{\circ}$  to  $-90^{\circ}$  and  $90^{\circ}$  to  $180^{\circ}$ , as shown in Fig. 2.

According to [5] and the IT-2 membership function of Fig. 2, the firing strength can be obtained as follows.

$$\mathbf{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) = \left[\underline{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right), \overline{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\right]$$
(8)

where upper bound and lower bound membership functions of (8) are denoted as Fig. 2. Then, the following calculation can be obtained for the IT-2 T-SFM.

$$\tilde{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) = \overline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\overline{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) + \underline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\underline{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$$

$$\tag{9}$$

where  $\overline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$  and  $\underline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$  are the functions associated with uncertainties and not necessary to be known. As the membership functions, these functions satisfy the conditions  $\overline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) + \underline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) = 1$  and  $1 \ge \overline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) \ge \underline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) \ge 0$ .

Referring to (7), the following IT-2 T-SFM is also constructed for IT-2 fuzzy tracking approach to achieve the formation and collision avoidance purposes.

$${}^{m}\dot{x}_{d}^{\varepsilon}(t) = \sum_{\alpha=1}^{3} \tilde{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}(t)\right) \left\{\mathbf{A}_{\alpha}{}^{m}x_{d}^{\varepsilon}(t)\right\}$$
(10)

where  ${}^{m}x_{d}^{\varepsilon}(t)$  is the desired system states to be tracked, m = c, f denotes the collision avoidance mode and formation mode of leader ships.

Subtracting the T-SFM (10) from (7), the error dynamic system can be obtained as follows.

$${}^{m}\dot{e}^{\varepsilon}\left(t\right) = \sum_{\alpha=1}^{3} \tilde{\Omega}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) \left\{\mathbf{A}_{\alpha}{}^{m}e^{\varepsilon}\left(t\right) + \mathbf{B}_{\alpha}{}^{m}u^{\varepsilon}\left(t\right)\right\} \quad (11)$$

where  ${}^{m}e^{\varepsilon}(t) = {}^{m}x^{\varepsilon}(t) - {}^{m}x_{d}^{\varepsilon}(t)$ .

According to the IT-2 T-SFM (11), a fuzzy tracking controller design approach is proposed to simultaneously achieve the collision avoidance and formation for leaders.

# III. IT-2 FUZZY COLLISION AVOIDANCE AND FORMATION CONTROLLER DESIGN

In this section, the IT-2 fuzzy controller design and stability analysis are proposed based on the IPM concept for leader ships in NM-AS (1)-(6). Moreover, the information of IT-2 membership function, which is more flexible than Type-1 membership function, is combined into the stability condition to reduce the conservativeness. According to the IPM concept in [5] and T-SFM (11), the IT-2 fuzzy tracking controller is proposed as follows.

$${}^{m}u^{\varepsilon}(t) = \sum_{\beta=1}^{2} \Gamma_{\beta}\left(x_{3}^{\varepsilon}(t)\right) \left\{ \mathbf{F}_{\beta} {}^{m}e^{\varepsilon}(t) \right\}$$
(12)

where  $\mathbf{F}_{\beta}$  are the feedback gains to be designed for the tracking purpose. The IT-2 membership function for the fuzzy controller (12) is designed as follows.



Figure 3. IT-2 membership function of fuzzy controller.

Note that one of the advantages of the IPM approach is that the form of the membership function and the number of rules can be designed differently from those in the T-S fuzzy model. Similar to the process (8)-(9), the firing strength of fuzzy controller (12) can be defined as follows.

$$\Gamma_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) = \left[\underline{\Gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right), \overline{\Gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\right]$$
(13)

where upper and lower bound membership functions are presented in Fig. 3. For the fuzzy controller (12), the IT-2 membership function is given as follows.

$$\widetilde{\Gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) = \overline{\gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\overline{\Gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right) + \underline{\gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)\underline{\Gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$$
(14)

where  $\overline{\gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$  and  $\underline{\gamma}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$  are the functions satisfy same conditions as  $\overline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$  and  $\underline{\tau}_{\alpha}\left(x_{3}^{\varepsilon}\left(t\right)\right)$ . The functions are predefined since the fuzzy controller is designed by users.

Substituting the IT-2 fuzzy tracking controller (12) into the T-SFM (11), the following closed-loop model is derived.

$${}^{m}\dot{e}(t) = \sum_{\alpha=1}^{3} \sum_{\beta=1}^{2} \tilde{\Xi}_{\alpha\beta} \left( x_{3}(t) \right) \left\{ \left( \mathbf{I} \otimes \left( \mathbf{A}_{\alpha} + \mathbf{B}_{\alpha} \mathbf{F}_{\beta} \right) \right) {}^{m} e(t) \right\}$$
(15)

where  $\tilde{\Xi}_{\alpha\beta}(x_3(t)) = \tilde{\Omega}_{\alpha}(x_3(t))\tilde{\Gamma}_{\beta}(x_3(t))$  and  $\otimes$  is the Kronecker product.

Note that according to [15], the stability analysis process needs to be developed only for one leader ship in (15). Referring to [5], the stability criterion for the tracking purpose is obtained as follows.

# Theorem 1

Given the scalars  $\overline{\sigma}_{\alpha\beta i_3}$  and  $\underline{\sigma}_{\alpha\beta i_3}$ , if there exist the positive definite matrices **Q**,  $\mathbf{N}_{\alpha\beta}$  and the symmetric matrix **M** such that the following sufficient conditions are all satisfied, the tracking purpose for the collision avoidance and formation can be achieved for leader ships.

$$\sum_{\alpha=1}^{3} \sum_{\beta=1}^{2} \left( \underline{\sigma}_{\alpha\beta i_{3}} \Phi_{\alpha\beta} - \left( \underline{\sigma}_{\alpha\beta i_{3}} - \overline{\sigma}_{\alpha\beta i_{3}} \right) \mathbf{N}_{\alpha\beta} + \underline{\sigma}_{\alpha\beta i_{3}} \mathbf{M} \right) - \mathbf{M} < 0$$
  
for all  $i_{3} = 1, 2$  (16)  
 $\Phi_{\alpha\beta} - \mathbf{N}_{\alpha\beta} + \mathbf{M} < 0$  for all  $\alpha, \beta$  (17)

where  $\Phi_{\alpha\beta} = \mathbf{A}_{\alpha}\mathbf{Q} + \mathbf{B}_{\alpha}\mathbf{G}_{\beta} + \mathbf{Q}\mathbf{A}_{\alpha}^{\mathrm{T}} + \mathbf{G}_{\beta}^{\mathrm{T}}\mathbf{B}_{\alpha}^{\mathrm{T}}$ ,  $\mathbf{G}_{\beta} = \mathbf{F}_{\beta}\mathbf{Q}$ ,  $\mathbf{Q} = \mathbf{P}^{-1}$ . Note that  $\overline{\sigma}_{\alpha\beta i_{3}}$ ,  $\underline{\sigma}_{\alpha\beta i_{3}}$  and  $i_{3}$  are the parameters related to the IT-2 membership function in Figs. 2-3. To save the place, the derivation process will not be provided and it can refer to [5] and [15].

After the IT-2 fuzzy tracking controller is designed by Theorem 1, the purpose of formation and collision avoidance can be determined by the following process.

First, the desired system state to be tracked in the formation mode is designed as follows.

$${}^{f}e^{\varepsilon}(t) = \begin{bmatrix} {}^{f}e_{1}^{\varepsilon}(t) & {}^{f}e_{2}^{\varepsilon}(t) & {}^{f}e_{3}^{\varepsilon}(t) & x_{4}^{\varepsilon}(t) & x_{5}^{\varepsilon}(t) & x_{6}^{\varepsilon}(t) \end{bmatrix}^{\mathrm{T}}$$
(18)

In this research, the tracking purpose is focused on the first three states. This is because the first two states directly specify the ships' positions, and the third state ensures the correction of the ships' course. As extended from Remark 2, most of the existing literature deals primarily with the stability of ships, which means that the yaw angle must track solely the zero value. However, the T-SFM-based fuzzy controller design methods cannot be applied to this research due to the operating range limitations.

Note that the desired x and y positions are obtained using a first-order hold between two positions. Moreover, the desired yaw angle is also derived from these two positions. Then, the purpose of collision avoidance mode is ensured by the desired system states as follows.

$${}^{c}e^{\varepsilon}(t) = \begin{bmatrix} {}^{c}e_{1}^{\varepsilon}(t) & {}^{c}e_{2}^{\varepsilon}(t) & {}^{c}e_{3}^{\varepsilon}(t) & x_{4}^{\varepsilon}(t) & x_{5}^{\varepsilon}(t) & x_{6}^{\varepsilon}(t) \end{bmatrix}^{1}$$
(19)

Building on the results in [17] and [19], the desired yaw angle can be derived from the combination of the source and vortex fields of the APF approach for collision avoidance. First, the gradient of vortex velocity field is given as follows.

$$\nabla\Theta_{v} = \left(v_{xv}, v_{yv}\right) = \left(-\frac{y_{s}\left(t\right) - y_{a}\left(t\right)}{r^{2}}, \frac{x_{s}\left(t\right) - x_{a}\left(t\right)}{r^{2}}\right) \quad (20)$$

where  $x_a$ ,  $y_a$  are the (x, y) positions of ships to be avoided and  $x_o$ ,  $y_o$  are the (x, y) positions of ship itself. In addition, the gradient of source velocity field is given as follows.

$$\nabla\Theta_{s} = \left(v_{xs}, v_{ys}\right) = \left(v_{r}\cos\theta + v_{\theta}\sin\theta, v_{r}\sin\theta + v_{\theta}\cos\theta\right) \quad (21)$$

where  $v_r = \frac{1}{r}$  and  $v_{\theta} = 0$ . Note that for both vortex and source potential fields, the radius and angle are derived by the following process.

$${}_{p}r = \sqrt{\left(x_{s}(t) - {}_{p}x_{a}(t)\right)^{2} + \left(y_{s}(t) - {}_{p}y_{a}(t)\right)^{2}}$$
(22)

$$_{p}\theta = tan^{-1}\left(\left(y_{s}(t) - _{p}y_{a}(t)\right) / \left(x_{s}(t) - _{p}x_{a}(t)\right)\right)$$
 (23)

where p is the number of ships to be avoided in multiple ship encounter scenario. Then, the desired yaw angle for ship to avoid the collision is derived as follows for both vortex and source fields.

$$\psi_{d} = tan^{-1}(v_{yy}/v_{xy})$$
 or  $tan^{-1}(v_{ys}/v_{xs})$  (24)

However, the x and y positions are also required to be tracked in this research. Therefore, the desired positions are also derived from desired yaw angle, which is obtained by APF approach (20)-(24), as follows.

$$x_{1d}(t) = x_s(t) + L\cos(\psi_d)$$
(25)

$$x_{2d}(t) = y_s(t) + Lsin(\psi_d)$$
(26)

where L is the distance between the desired point and the ships' own point. In the combination of the two APFs, the vortex field is used to change the ship's course for collision avoidance, while the source field generates a repulsive course to prevent the ship from getting too close to obstacles. To determine when to use which APF, the following condition is provided.

Condition 1

If 
$$R_s < r \le R_v$$
, then  $\psi_d = tan^{-1} \left( \frac{v_{yv}}{v_{xv}} \right)$ .  
If  $0 < r \le R_s$ , then  $\psi_d = tan^{-1} \left( \frac{v_{ys}}{v_{xs}} \right)$ .

where  $R_v$  and  $R_s$  are the triggered distance of vortex and source velocity field. Moreover, the following condition is also given for the determination of tracking and avoidance mode by referring to [17].

# Condition 2

If 
$$DCPA \le R_f$$
 and TCPA > 0, then  $\psi_d = tan^{-1} (v_{yy} / v_{xy});$ 

otherwise,  $\psi_d = x_{3d}^{\varepsilon}(t)$ , where  $R_f$  is the safe collision range of leader ships.

In the next section, the simulation results with four leader ships are presented to demonstrate the effectiveness and applicability of the designed IT-2 fuzzy controller.

# IV. SIMULATION OF NM-AS

With the vortex and source velocity APFs, the collision avoidance approach for multiple ships is proposed with an IT-2 fuzzy tracking controller design. First, the control gains are obtained as follows by solving the control problem in Theorem 1 using MATLAB.

$$\mathbf{F}_{1} = \begin{bmatrix} -0.6013 & -0.4007 & -0.0101 & -2.4117 & -0.2879 & -0.0029\\ 0.7777 & -0.6088 & 0.0346 & 0.5683 & -2.6147 & 0.0690\\ -0.0311 & 0.0050 & -0.4411 & -0.0231 & 0.0359 & -0.8179 \end{bmatrix}$$
(27)  
$$\mathbf{F}_{2} = \begin{bmatrix} -0.6269 & 0.3987 & 0.0101 & -2.5153 & 0.2857 & 0.0029\\ -0.7723 & -0.6624 & 0.0381 & -0.5650 & -2.8564 & 0.0770\\ 0.0309 & 0.0077 & -0.4563 & 0.0229 & 0.0493 & -0.8580 \end{bmatrix}$$
(28)

According to Remark 2, the following control gains can be derived for the case of  $-180^{\circ}$  to  $-90^{\circ}$  and  $90^{\circ}$  to  $180^{\circ}$ .

$$\mathbf{F}_{1} = \begin{bmatrix} 0.6013 & 0.4007 & -0.0101 & -2.4117 & -0.2879 & -0.0029 \\ -0.7777 & 0.6088 & 0.0346 & 0.5683 & -2.6147 & 0.0690 \\ 0.0311 & -0.0050 & -0.4411 & -0.0231 & 0.0359 & -0.8179 \end{bmatrix}$$

$$\mathbf{F}_{2} = \begin{bmatrix} 0.6269 & -0.3987 & 0.0101 & -2.5153 & 0.2857 & 0.0029 \\ 0.7723 & 0.6624 & 0.0381 & -0.5650 & -2.8564 & 0.0770 \\ -0.0309 & -0.0077 & -0.4563 & 0.0229 & 0.0493 & -0.8580 \end{bmatrix}$$
(30)

To begin with the simulation, the ranges for collision avoidance mode are designed as follows.

$$R_f = 4$$
,  $R_v = 4$ ,  $R_s = 2$  and  $L = 2$  (31)

Then, by applying the IT-2 fuzzy controller (12) with the gains (27)-(30) for different ranges, the simulation results are presented as follows based on the consideration of (31).



Figure 4. Trajectories of four leader ships.



Figure 5. Trajectories of four leader ships in tracking situation.



Figure 6. Trajectories of four leader ships in avoidance situation.

From Fig. 4, one can see that four leader ships successfully track the individual desired trajectory and thus form the rectangular region. Moreover, the region can be sustained until the destination is reached. In Fig. 5, the trajectories of the four ships before the first formation are presented. It is observed that the ships can efficiently avoid each other before tracking their desired trajectories. The vortex velocity APF turns the ships in the right direction,

complying with the COLREGs as [17], and all ships move to the right when encountering others. More importantly, the four leader ships can simultaneously avoid two obstacles and each other when the obstacles are on their trajectories in Fig. 6. The outer and inner black dot circles denote the ranges of the triggering vortex and source APFs, respectively.

# V. CONCLUSIONS

An IT-2 fuzzy tracking controller design approach has been developed in this research to simultaneously meet the requirements of collision avoidance and formation control for multiple ships. Based on the IT-2 T-SFM, the dynamic behaviors of NM-ASs can be more completely described by accounting for the uncertain factors in the IT-2 membership function. According to the IPM concept, a more flexible fuzzy tracking controller design process can be developed. By combining vortex and source velocity APFs, the IT-2 fuzzy controller can achieve the avoidance objective. As stated in Conditions 1-2, all ships can properly switch between formation and collision avoidance modes. Based on the simulation results, the IT-2 fuzzy controller can efficiently accomplish both formation and collision avoidance tasks, even under multiple encounter scenarios. In the future, the fuzzy control theory can be considered to obtain the smoother adjustment between different modes. The containment purpose of follower ships can also be integrated into the IT-2 fuzzy controller design.

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