Solving Stationary Gas Transport Problems with Compressors of Piston and Generic Type

Anton Baldin
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Anton.Baldin@scai.fraunhofer.de

Kläre Cassirer
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Klaere.Cassirer@scai.fraunhofer.de

Tanja Clees
University of Applied Sciences Bonn-Rhein-Sieg and Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Tanja.Clees@scai.fraunhofer.de

Bernhard Klaassen
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Bernhard.Klaassen@scai.fraunhofer.de

Igor Nikitin
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Igor.Nikitin@scai.fraunhofer.de

Lialia Nikitina
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Lialia.Nikitina@scai.fraunhofer.de

Sabine Pott
Fraunhofer Institute for Algorithms and Scientific Computing
Sankt Augustin, Germany
email: Sabine.Pott@scai.fraunhofer.de

***Abstract*—In this paper, modeling of piston and generic type gas compressors for a globally convergent algorithm for solving stationary gas transport problems is carried out. A theoretical analysis of the simulation stability, its practical implementation and verification of convergence on a realistic gas network have been carried out. The relevance of the paper for the topics of the conference is defined by a significance of gas transport networks as an advanced application of simulation and modeling, including the development of novel mathematical and numerical algorithms and methods.**

***Index Terms*—simulation and modeling; mathematical and numerical algorithms and methods; advanced applications; gas transport networks**

I. Introduction

In this paper, we will continue the study of globally converging methods for solving stationary network problems on the example of gas transport networks. Differently from our previous works, where the gas compressors of the most common turbine type were considered, in this paper, we investigate compressors of piston and generic type. In our work [1], we introduced the concept of generalized resistivity of network elements and formulated stability conditions for the algorithm solving the corresponding network problems. In the works [2] [3] [4] we have considered in detail the modeling of gas compressors of the turbine type. For these compressors, individually calibrated characteristics and data resampling on a regular grid were used. Now we consider compressors of piston and generic type, which are characterized by the existence of analytical solutions and a simpler representation of control equations. This simulation extends our system MYNTS (Multi-phYsics NeTwork Simulation) [5].

Globally convergent methods in applications to electric networks were formulated in [6], as well as in a more general form for piecewise linear systems in [7] [8] and for general smooth systems in [9]. Modeling of gas networks is described in detail in [10] [11]. This modeling is based on the nonlinear friction law in pipes [12] [13] and empirical approximations for the equation of state of a real gas [14] [15] [16].

In this paper, in Section II, we recall the general concepts of element resistivity and describe their physical meaning in more detail. In Section III, we will look at compressors of piston type and in Section IV – of generic type. In Section V, we will carry out a numerical solution of a realistic network problem with compressors of the described types.

II. Transport variables in stationary network problems

Network problems of a stationary type are described by a system of equations that includes linear Kirchhoff equations of the form $\sum Q_i = 0$, which describe the conservation of flows in network nodes, and equations of elements of the form $f(P_{in}, P_{out}, Q) = 0$, in the general case, nonlinear, introduced on each edge of the network graph. Here the transport variables $P_{in/out}$ are used – nodal variables for the input and output of the element, for gas networks – pressure values, Q – the flow through the element. In gas problems, flows are considered in different normalizations, which is indicated by the index: Q_m – mass flow, Q_ν –

molar flow, Q_N – volumetric flow under normal conditions, $Q_{vol,in/out}$ – volumetric flow in input or output conditions (by default, input conditions are taken), etc. An element is called generalized resistive if its equation has derivatives of the following signature:

$$\partial f/\partial P_{in} > 0,\ \partial f/\partial P_{out} < 0,\ \partial f/\partial Q < 0. \quad (1)$$

The work [1] shows that stationary network problems in which all elements have a given signature have a unique solution that can be found by the standard stabilized Newton algorithm with an arbitrary choice of starting point. Technically, it also requires a supply with a set pressure P_{set} in each disconnected component of the graph, as well as a proper condition for the behavior of functions at infinity, which can be satisfied if there are linear continuations of the equations of elements outside the working region that have the signature (1). Also, the completely inverse signature is formally admissible, since the sign change of $f \to -f$ is admissible for stationary problems. To eliminate this trivial ambiguity, one can choose the sign of f, postulating the fulfillment of one of the conditions (1), for example, the first one.

The physical meaning of these conditions is illustrated in Figure 1. It shows the serial connection of the tested element (in this case the compressor, a circle) and a linear resistor (a rectangle). Pressure $P_{set1,2}$ is set at the free ends. The intermediate node must satisfy the equation

$$P_{out}(P_{set1}, Q) = P_{set2} + RQ, \quad (2)$$

graphically depicted in the central and lower parts of the figure. Here $R > 0$ is the resistance value, the corresponding line on the figure increases monotonically. If the tested element has the signature (1), then the function $P_{out}(P_{set1}, Q)$ decreases monotonically in Q, which corresponds to the central part of the figure. In this case, the intersection of lines exists and is unique. It can also occur outside this graph, when the above condition is met at infinity (continuation of the element's characteristic by a linearly strictly decreasing function outside the working region). In the case, if the signature (1) would be violated and the function $P_{out}(P_{set1}, Q)$ would increase in Q, then by choosing the parameters P_{set2} and R it is possible to achieve that the lines will have several intersections or no intersection. Even if the function $P_{out}(P_{set1}, Q)$ increases in Q only locally, a linear resistor can be fitted to it, which will give several solutions to the problem under consideration. It is also clear that a non-linear resistor can also be used for this purpose, as long as its characteristic increases and has enough parameters for tuning.

Similarly, by connecting elements in reverse order, as well as considering their parallel connection, it can be shown that any violation of the condition (1) leads to a violation of the uniqueness of solution. If the signature is violated, then the tested element can be connected to an elementary resistive element in such a way that the equation will have several solutions or none. The case when the signature is satisfied for all elements and the system has a unique solution is, of course, more preferable in practical applications.

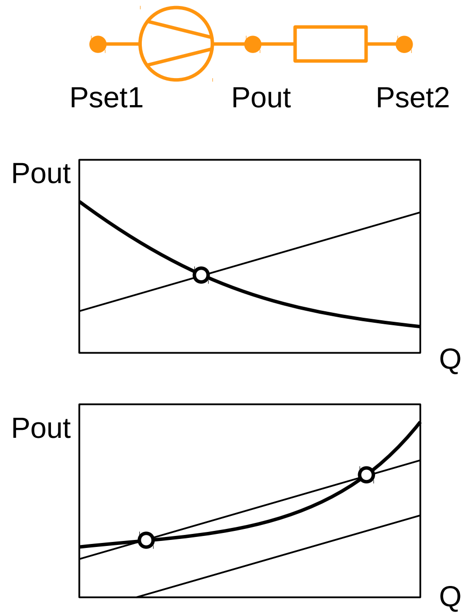

Fig. 1. On the top: a serial connection of compressor (circle) and resistor (rectangle); in the center: decreasing compressor $P_{out}(Q)$ characteristics (thick line) and increasing resistor $P_{out}(Q)$ characteristics (thin line) have a single intersection (stable case); at the bottom: increasing compressor $P_{out}(Q)$ characteristics (thick line) and increasing resistor $P_{out}(Q)$ characteristics (thin lines) can have multiple intersections or no intersection (unstable case).

Compressors are the most complex elements in gas problems; several levels of modeling are used to represent them. The main purpose of introduction of these levels is the gradual sophistication of modeling, where the solution of a simple model is used as a starting point for the more complex one. Also, it allows to separate effects dependent on individual calibration of compressors from their basic representation.

Free model: is the simplest, formulated only in terms of transport variables, and is described by a piecewise linear formula of the form

$$\max(\min(\ P_{in} - P_L, -P_{out} + P_H, -Q + Q_H), \quad (3)$$

$$P_{in} - P_{out}, -Q) + \epsilon(P_{in} - P_{out} - Q) = 0, \quad (4)$$

where parameters P_L, P_H, Q_H define target values, for example, $P_H = SPO$ for specified output pressure, or upper and lower limits for other controlled values. This formula defines a polyhedral surface in the space of transport variables in the so-called *maxmin* representation [8]. Particular attention should be paid to the last term in the equation, which is controlled by a small positive parameter ϵ. The reason for its introduction is that the exact equation satisfies the signature condition (1) only marginally, some derivatives vanish. The geometric interpretation of this is that the normals to the faces of the polyhedron described by the equation are directed strictly along the axes, although they should be directed inside the octant described by the condition (1). Such marginality leads to degeneracy of the Jacobi matrix, ambiguity of solutions, bad condition numbers, and other troubles for the numerical solution procedure. The introduction of a regularizing ϵ term

 ISBN: 978-1-61208-961-4

formally eliminates this problem by making the condition (1) strictly satisfied. At the same time, adjusting this parameter represents a compromise between the physical accuracy and the numerical stability of the solution procedure. In practice, the values $\epsilon = 10^{-6}...10^{-3}$ are tolerable, meaning the relative violation of, e.g., SPO-condition, up to 0.1%, simultaneously keeping the convergence rate near 100%.

Advanced model: introduces additional internal variables for compressors: revolution number rev, adiabatic enthalpy increase H_{ad}, performance $Perf$, efficiency η, torque M_t, and additional equations:

$$P = \rho RTz/\mu,\ Q_m = Q_{vol}\rho_{in}, \tag{5}$$

$$H_{ad} = P_{in}/(\rho_{in}\alpha)\cdot((P_{out}/P_{in})^{\alpha} - 1), \tag{6}$$

$$Perf = Q_m H_{ad}/\eta,\ M_t = Perf/(2\pi\cdot rev), \tag{7}$$

$$\alpha = (\kappa - 1)/\kappa,\ 0 < \alpha < 1,\ 0 < \eta < 1, \tag{8}$$

where the equation of state is written first with its parameters: density ρ, universal gas constant R, absolute temperature T, compressibility factor z, molar mass μ; the second is the relationship between the mass flow and the volumetric flow in the input conditions; the following are definitions of internal variables in terms of transport variables; $\kappa > 1$ is the adiabatic exponent.

For the turbocompressors considered in [2] [3] [4], additional relationships between internal variables are introduced based on the calibration procedure. We will now consider piston and generic type compressors, for which there is a simpler model that allows an analytical solution. The general strategy is to resolve all internal variables from the corresponding equations, obtain a formula in terms of transport variables, check its signature, and use it in the standard solution algorithm.

III. Piston compressors

Compressors of piston types are modeled by direct proportionality

$$Q_{vol} = V\cdot rev \tag{9}$$

with given constants η and V – compressor chamber volume. The control equation has the following patches:

$$f_1 = rev_{max} - rev \geq 0, \tag{10}$$

$$f_2 = M_{t,max} - M_t \geq 0, \tag{11}$$

$$f_3 = Perf_{max} - Perf \geq 0, \tag{12}$$

$$f_4 = rel_{max} - P_{out}/P_{in} \geq 0, \tag{13}$$

$$f_5 = \Delta P_{max} - (P_{out} - P_{in}) \geq 0, \tag{14}$$

with given constants rev_{max}, $M_{t,max}$, rel_{max}, ΔP_{max} and the function $Perf_{max}(rev)$ determined by the characteristics of the compressor drive.

Stability analysis: calculating the derivatives of f_i with respect to (P_{in}, P_{out}, Q_m) in the working region $0 < P_{in} \leq P_{out}$, $Q_m > 0$, $rev > 0$, we get the signatures given in Table I. In this case, the above formulas are used, as well as the stability of the equation of state: $\rho > 0$, $\partial\rho/\partial P > 0$.

TABLE I
PATCH SIGNATURES OF PISTON COMPRESSOR

patch	sgn	condition
f_1	$(+\ 0\ -)$	
f_2	$(+-0)$	$P_{out}/P_{in} < \beta$
f_3	$(+--)$	$P_{out}/P_{in} < \beta$, $\partial M_{t,drv}/\partial\, rev < 0$
f_4	$(+-0)$	
f_5	$(+-0)$	

TABLE II
PATCH SIGNATURES OF GENERIC COMPRESSOR

patch	sgn	condition
f_1	$(+\ 0\ -)$	
f_2	$(+-0)$	$\partial z_{in}/\partial P_{in} < 0$ or small
f_3	$(+--)$	$\partial z_{in}/\partial P_{in} < 0$ or small

In particular, $rev = Q_m/(\rho_{in}V)$ has signature $(-0+)$, which implies the signature of f_1 in the table. $M_t = H_{ad}\rho_{in}V/(2\pi\eta)$ has signature $(*+0)$, where $* = \partial(H_{ad}\rho_{in})/\partial P_{in} < 0$ for $P_{out}/P_{in} < (1-\alpha)^{(-1/\alpha)} = \beta$. Thus, the signature f_2 is correct only if the compressor raises the pressure by no more than the factor β, with the value $\kappa = 1.29$ typical for natural gas, we get $\beta = 3.10408$. To eliminate the fold in the equation, f_2 should be replaced with $H_{ad}\rho_{in}|P_{in} \to \max(P_{in}, P_{out}/\beta)$. It is convenient to divide the expression f_3 by $(2\pi\, rev)$ and consider the signature $\tilde{f}_3 = M_{t,drv}(rev) - M_t$. As noted in [4], for drive equations to be stable it is necessary that $M_{t,drv}$ decrease with rev. Therefore, the first term in $\tilde{f}_3$ has the signature $(+0-)$, and the second already calculated $(+-0)$ in the region $P_{out}/P_{in} < \beta$, which gives the complete signature $(+--)$. Calculation of other derivatives is trivial. We also note that the presence of zeros in the signatures means that the rule (1) is satisfied marginally, which is corrected by adding a regularizing ϵ-term to the element equation. Also, for the practical implementation of these formulas, it is necessary to introduce clamps, which force all variables to the working region: $Q_m \to \max(Q_m, 0)$, $P_{out}/P_{in} \to \max(P_{out}/P_{in}, 1)$, etc.

IV. Generic compressors

Compressors of generic type can also be considered as an intermediate level of modeling (generic model). In this model, the variable rev is not introduced, and restrictions are introduced on other variables

$$f_1 = Q_{vol,max} - Q_{vol} \geq 0, \tag{15}$$

$$f_2 = H_{ad,max} - H_{ad} \geq 0, \tag{16}$$

$$f_3 = Perf_{max} - Perf \geq 0, \tag{17}$$

with constant $Q_{vol,max}$, $H_{ad,max}$ and $Perf_{max}$.

Stability analysis: Calculating derivatives similarly, for $Q_{vol} = Q_m/\rho_{in}$ we have signature $(-0+)$, hence $(+0-)$ for f_1. For $H_{ad} = RT_{in}z_{in}/(\mu_{in}\alpha)((P_{out}/P_{in})^{\alpha} - 1)$ we get $(*+0)$, where $* = \partial(z_{in}((P_{out}/P_{in})^{\alpha} - 1))/\partial P_{in} < 0$. For an ideal gas $z = 1$, hence, obviously, $* = -$. For natural gas z is a decreasing function of P, in this case also $* = -$. For

 ISBN: 978-1-61208-961-4

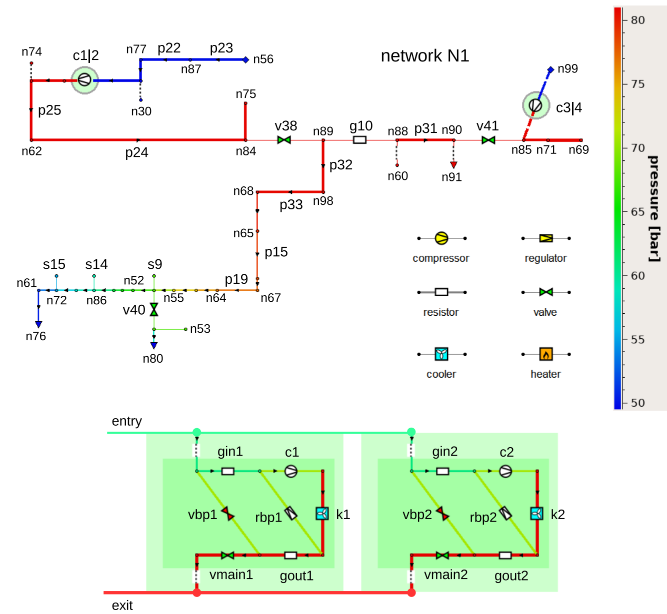

Fig. 2. On the top: test network N1; at the bottom: the structure of parallel compressor station. Images from [1].

some gases, such as hydrogen, z may increase with P, but it remains close to 1 and changes so slowly that the remaining decreasing dependence of H_{ad} on P_{in} dominates. Under these conditions, f_2 has signature $(+-0)$. For $Perf = Q_m H_{ad}/\eta$ the signature $(-++)$ under the same conditions on z_{in}, thus f_3 has the signature $(+--)$.

V. Numerical tests

The described patches are inserted into the free formula as follows:

$$\max(\min(\ P_{in} - P_L, -P_{out} + P_H, -Q + Q_H, \tag{18}$$
$$\underline{f_1, ..., f_n}\ ), \tag{19}$$
$$P_{in} - P_{out}, -Q) + \epsilon(P_{in} - P_{out} - Q) = 0, \tag{20}$$

after that the stabilized Newton algorithm described in [1] can be used to solve the system. The tests were carried out on the network N1 shown in Figure 2 on the top. This network has 100 nodes and 111 edges, of which 4 compressors are organized into two compressor stations c1|2 and c3|4 with individual compressors connected in parallel, as shown in Figure 2 at the bottom. Compressors in station c1|2 are configured as piston ones, in station c3|4 as generic ones. Values P_H, Q_H are set to unreachable high values, thereby activating the f_i patches described above. Note that the stations also include other elements, but they have trivial equations and are eliminated by the topological cleaning filter used in the solution procedure. The procedure consists of several phases with a gradual increase in the modeling level. First (init) the compressors are set to fulfill the main target values, e.g., $P = P_H$, then (free) the modeling level (3)-(4) is used, taking into account additional conditions, then (adv) the modeling level (18)-(20) is taken. The solution procedure described in

TABLE III
Timing for different phases of the solution procedure*

phase	translate	solve
init	15	8
free	15	7
adv	17	20
total	47	35

* in milliseconds, for 2.6 GHz Intel i7 CPU 16 GB RAM computer.

[5] consists of the translation phase of the system from the network description language to the language understood by the numerical solver, and the actual numerical solution phase. The corresponding timing is given in Table III; approximately the same results are obtained if turbocompressors are used instead of piston/generic ones. The performed numerical experiment shows that the inclusion of piston and generic compressors in the system does not lead to any divergences or slowdown of the solution procedure, which is a direct consequence of the implementation of the stability criteria described above.

We also performed numerical experiments with test networks from work [4]. The test set contains 85 networks with complexity up to four thousand nodes and up to 42 compressors. Among them are multiple piston and generic compressors, in parallel and series connections. We have found that the presence and placement of such compressors does not affect performance in any way, and this is consistent with the convergence conditions we developed. The extension of the convergence theory to the dynamic case is the subject of our further work.

VI. Conclusion

In this work, modeling of piston and generic type gas compressors was carried out. The signatures of the derivatives of the control equation are analyzed, the ranges of parameter values are identified, under which the conditions for the stable operation of the algorithm for solving stationary network problems are satisfied. After the practical implementation of the modeling, in a numerical experiment on a realistic gas network, the convergence of the solution algorithm is shown.

Our future plans include extending the described methods to dynamic problems.

Acknowledgment

The work has been supported by the project TransHyDE-Sys, grant 03HY201M.

References

[1] T. Clees, I. Nikitin, and L. Nikitina, “Making Network Solvers Globally Convergent”, Advances in Intelligent Systems and Computing, vol. 676, 2018, pp. 140-153.
[2] T. Clees, I. Nikitin, L. Nikitina, and L. Segiet, “Modeling of Gas Compressors and Hierarchical Reduction for Globally Convergent Stationary Network Solvers”, International Journal On Advances in Systems and Measurements, vol. 11, 2018, pp. 61-71.
[3] A. Baldin, T. Clees, B. Klaassen, I. Nikitin, and L. Nikitina, “Topological Reduction of Stationary Network Problems: Example of Gas Transport”, International Journal On Advances in Systems and Measurements, vol. 13, 2020, pp. 83-93.

 ISBN: 978-1-61208-961-4

[4] A. Baldin et al., “AdvWarp: A Transformation Algorithm for Advanced Modeling of Gas Compressors and Drives”, in Proc. of SIMULTECH 2021, International Conference on Simulation and Modeling Methodologies, Technologies and Applications, pp. 231-238, SciTePress, 2021.
[5] T. Clees et al., “MYNTS: Multi-phYsics NeTwork Simulator”, in Proc. of SIMULTECH 2016, International Conference on Simulation and Modeling Methodologies, Technologies and Applications, pp. 179-186, SciTePress, 2016.
[6] J. Katzenelson, “An algorithm for solving nonlinear resistor networks”, Bell System Technical Journal, vol. 44, 1965, pp. 1605-1620.
[7] M. J. Chien and E. S. Kuh, “Solving piecewise-linear equations for resistive networks”, International Journal of Circuit Theory and Applications, vol. 4, 1976, pp. 1-24.
[8] A. Griewank, J.-U. Bernt, M. Radons, and T. Streubel, “Solving piecewise linear systems in abs-normal form”, Linear Algebra and its Applications, vol. 471, 2015, pp. 500-530.
[9] C. T. Kelley, Iterative Methods for Linear and Nonlinear Equations, SIAM, Philadelphia, 1995.
[10] J. Mischner, H. G. Fasold, and K. Kadner, System-planning basics of gas supply, Oldenbourg Industrieverlag GmbH, 2011 (in German).
[11] M. Schmidt, M. C. Steinbach, and B. M. Willert, “High detail stationary optimization models for gas networks”, Optimization and Engineering, vol. 16, 2015, pp. 131-164.
[12] J. Nikuradse, “Laws of flow in rough pipes”, NACA Technical Memorandum 1292, Washington, 1950.
[13] C. F. Colebrook and C. M. White, “Experiments with Fluid Friction in Roughened Pipes”, in Proc. of the Royal Society of London, Series A, Mathematical and Physical Sciences, vol. 161, num. 906, 1937, pp. 367-381.
[14] J. Saleh, Fluid Flow Handbook, McGraw-Hill 2002.
[15] DIN EN ISO 12213-2: Natural gas - Calculation of compression factor, European Committee for Standardization, 2010.
[16] O. Kunz and W. Wagner, “The GERG-2008 wide-range equation of state for natural gases and other mixtures: An expansion of GERG-2004”, J. Chem. Eng. Data, vol. 57, 2012, pp. 3032-3091.