Outage Probability of Full-duplex Systems in Multi-spectrum Environments

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***Abstract*—The goal of this paper is to investigate outage performance in a multi-spectral bi-directional full-duplex system (M-BFD). The system considered is two-way communication between two nodes equipped with a single shared antenna. Because full-duplex transmission is being employed, the required SNR associated with the target rate at each node for BFD is smaller than that for a bi-directional half-duplex system (BHD). In a single spectrum environment, therefore, BFD outperforms BHD in terms of outage probability. From a practical perspective, we investigate whether BFD remains advantageous over BHD in a multi-spectrum environment. We investigate the optimal spectrum selection strategy in terms of outage probability for M-BFD. The outage probability is derived as a closed-form expression under this selection strategy.**

The results show that the diversity orders for M-BFD and multi-spectral BHD (M-BHD) are identical. In the high signal-to-noise ratio (SNR) range, furthermore, the SNR difference between the outage curves for M-BFD and for M-BHD is shown to be inversely proportional to the number of available spectra, but in proportional to the target data rate.

***Index Terms*—bi-directional full duplex; multi-spectrum environment; spectrum selection.**

I. Introduction

Bi-directional full-duplex systems (BFD) have the potential to increase the system capacity of two-way networks with multiple antennas. In BFD, two source nodes simultaneously exchange signals by utilizing the same frequency resource [1]. If we assume that self-interference cancellation is perfect, BFD can achieve up to double the system capacity of bi-directional half-duplex systems (BHD). Numerous papers have shown the superiority of BFD over BHD in terms of system capacity [1]-[4], but system reliability is also a key metric in measuring system performance.

In BFD, it is inevitable that both nodes divide the spatial resources relatively into two subsets for simultaneous transmission and reception [1][2]. As a result, the achievable diversity order decreases, since the division reduces the number of transmit and receive antennas at each node.[1] In order to improve the achievable diversity order in BFD, a transmit antenna-switched receive diversity for the bi-directional beamforming scheme was proposed [6]. It was assumed that the antennas at each node are divided into two subsets which specifically perform only transmission or reception in [6]. Shared antennas can be exploited to avoid any division of spatial resources which causes diversity order reduction [7]. Throughout this paper, we will consider a two-way communication system equipped with a single shared antenna.

Recently, there has been growing interest in exploiting the multi-spectrum environment [8]-[10]. In [8], a closed-form expression of the capacity gain achieved from spectrum selection diversity was calculated in cognitive radio environments. In [9], a fair scheduling scheme was proposed in multi-spectrum environments. Those works argue that spectrum selection diversity can improve system reliability. In particular, the outage probability in a multi-spectrum system is investigated [10]. One way to enhance system performance, therefore, would be to bring BFD into the multi-spectrum network. Based on the above, BFD should also be considered in multi-spectrum environments. To the best of the authors' knowledge, however, there has been no investigation of the outage probability for M-BFD to date.

Multi-spectral BFD (M-BFD) requires an efficient spectrum selection strategy. The outage probability is defined as the probability that one of the SNRs at both nodes will be unable to support the required SNR [4]. For two-way communication in M-BFD, both nodes simultaneously utilize the same spectrum so that just one selection is needed. In a multi-spectral bi-directional half-duplex system (M-BHD), in contrast, each node independently selects the spectrum during its own transmission period so that twice as many spectrum selections are needed. In order to avoid outage events in M-BFD, both links should be considered simultaneously.

The remainder of this paper is organized as follows: Section II describes the system model. Section III investigates a spectrum selection strategy for M-BFD which minimizes the outage probability. Under this spectrum selection strategy, in Section IV, we will then derive the outage probabilities for M-BFD and M-BHD as a closed-form expression. We will also investigate the diversity orders and the asymptotic SNR difference between the outage curves for M-BFD and M-BHD. Finally, our conclusions are presented in Section V.

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[1]Multiple input multiple output (MIMO) system with M transmit and N receive antennas; the maximum achievable diversity order is MN in a slow Rayleigh fading environment [5]. In BFD, at least one antenna at each node is used for opposite directional communication so that the maximum diversity order is $(M-1)(N-1)$ [6].

II. SYSTEM MODEL

Let us first consider our two-way communication system. It consists of two transceivers, node a and node b, equipped with a single antenna [7]. Let $link_{ab}$ and $link_{ba}$ denote the data transmission links from a to b and b to a, respectively. We assume that each transmission link uses time T sec and one spectrum B Hz. We also assume the number of available spectra to be K.

Fig. 1 describes the M-BFD system [11]. Node a and node b simultaneously transmit and receive signals during time period T with the same selected spectrum from among K available spectra. We presume that the antennas at each node are able to transmit and receive signals simultaneously using a circulator in the BFD system with self-interference cancellation [7]. We also assume that the self-interference is perfectly eliminated as described in [1][2] and [6].[2]

Fig. 2 shows the M-BHD system. In order to perform two-way communication, M-BHD requires time $2T$ [1]. Each single time period of T sec is utilized for a single transmission link using the independently selected spectrum. In other words, during time $[0, T]$, node a transmits signals to node b through $link_{ab}$ using the selected k_{ab}^H-th spectrum. Then, during time $[T, 2T]$, node b transmits signals to node a through $link_{ba}$ using the selected k_{ba}^H-th spectrum.

$(\cdot)^F$ and $(\cdot)^H$ stand for $(\cdot)$ for M-BFD and M-BHD, respectively. The received signal at node i which is transmitted by node j, y_i $(i, j \in \{a, b\})$, can then be expressed respectively as

$$y_{k,i}^m = h_k^m x_j^m + n_i, \tag{1}$$

where $m \in \{F, H\}$. x_j is the transmitted signal from node j.[3] n_i is the additive noise at receiver node i and distributed as $\mathcal{CN}(0, \sigma^2)$. The channel coefficient for the k-th selected spectrum, h_k $(k \in \{1, ..., K\})$, is distributed as $\mathcal{CN}(0, 1^2)$. The channel state is constant in a single period, since it is assumed that the coherence time and the coherence bandwidth of each channel are T and B, respectively. Therefore, the h_k's are independent and identically distributed (i.i.d) Rayleigh fading. In M-BFD, the channel coefficients for both links are identical, since we assume channel reciprocity between both links [15]. Note that each node in M-BFD simultaneously receives signals during time $[0, T]$ while each node in M-BHD receives signals during different periods, $[0, T]$ and $[T, 2T]$, respectively.

[2]Up to 70dB of self-interference can be eliminated by isolating the antennas[7]. Since each node knows its own transmitted data, self-interference can be subtracted from the desired signal [12]. A number of papers have been published on practical full-duplex transmission [13][14].

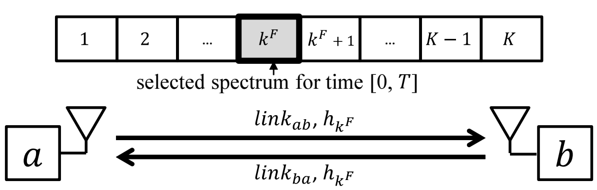

Fig. 1. M-BFD system model.

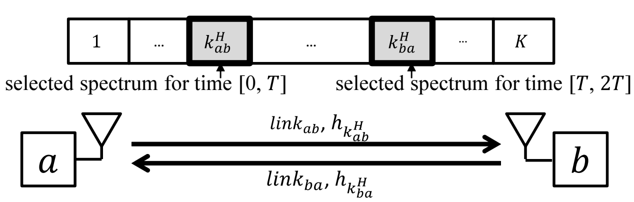

Fig. 2. M-BHD system model.

III. SPECTRUM SELECTION CRITERION AND OUTAGE PROBABILITY

In this section, we investigate the spectrum selection criteria for M-BFD and M-BHD in order to minimize the outage probability. The outage probabilities are derived as closed-form expressions under the spectrum selection criteria, after which the diversity orders are then also derived.

A. Optimal Spectrum Selection Criterion in Terms of Outage Probability

In two-way communication, an outage occurs when one of the two links cannot support the required SNR, which is denoted by γ_{th} [4]. We assume that perfect CSI is available [16]. In this case, the outage probability, P_O, is expressed as

$$P_O = 1 - \Pr\left[\gamma_{k,a} > \gamma_{th}, \gamma_{k,b} > \gamma_{th}\right], \tag{2}$$

where $\gamma_{k,i} = \frac{|h_k x_j|^2}{\sigma^2}$ is the SNR at receiver node i utilizing the k-th selected channel.

Let us first consider the spectrum selection criterion for M-BFD. In M-BFD, the received SNRs at nodes a and b utilizing the selected spectrum both need to simultaneously support the required SNR, since each node utilizes the same time and spectrum resources. The outage probability can then be expressed as [10]

$$P_O^F = 1 - \Pr\left[\min_{i=a,b} \gamma_{k,i}^F > \gamma_{th}^F\right]. \tag{3}$$

In order to minimize the outage probability for M-BFD, we need to select the spectrum that maximizes the SNR for the weaker link. Therefore, the spectrum selection criterion for M-BFD can be expressed as

$$k^F = \arg \max_{k=1,\ldots,K} \min_{i=a,b} \gamma_{k,i}^F. \tag{4}$$

In M-BFD, the SNRs at both nodes are identical, since we assume channel reciprocity between both links [15]. Therefore, without loss of generality, (4) can be expressed as

$$k^F = \arg \max_{k=1,\ldots,K} \gamma_{k,a}^F. \tag{5}$$

[3]For the sake of fair comparison, the M-BFD has half the transmission power of the M-BHD, i.e., $\mathcal{E}\left[\left|x^F\right|^2\right] = \frac{P}{2}$ and $\mathcal{E}\left[\left|x^H\right|^2\right] = P$, where $\mathcal{E}\left[\cdot\right]$ stands for the expectation operator.

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On the other hand, in M-BHD, note that the received SNRs at node a and b are independent, since the spectra can be independently selected at each node during different time periods. Therefore, the outage probability for M-BHD can be expressed as

$$P_O^H = 1 - \Pr[\gamma_{k,a}^H > \gamma_{th}^H] \Pr[\gamma_{k,b}^H > \gamma_{th}^H]. \tag{6}$$

In order to minimize the outage probability for M-BHD, each node should independently select the spectrum that maximizes the received SNR at the corresponding transmission time. The spectrum selection criterion for each link in M-BHD can then be expressed as

$$\begin{aligned} k_{ab}^H &= \arg \max_{k=1,\ldots,K} \gamma_{k,b}^H, \\ k_{ba}^H &= \arg \max_{k=1,\ldots,K} \gamma_{k,a}^H. \end{aligned} \tag{7}$$

From (5) and (7), we can observe the effects on the outage probability by the number of the available spectra. Both (5) and (7) can achieve full spectrum selection gain since the spectra which maximize the received SNRs are independently selected at each node during the corresponding transmission time periods.

B. Outage Probability

In this subsection, we derive the outage probabilities for M-BFD and M-BHD as closed-form expressions. We first consider the outage probability for M-BFD. As mentioned above, an outage occurs when one of the two nodes cannot support the required SNR. We define the target rate as R bps/Hz, so that the required SNR for M-BFD, γ_{th}^F, is $2^R - 1$. The received SNRs at node i in M-BFD are

$$\gamma_{k,i}^F = \frac{|h_k|^2 \frac{P}{2}}{\sigma^2} = \frac{|h_k|^2 \eta}{2}, \tag{8}$$

where η is the common SNR, which is expressed as P/σ^2. The $|h_k|^2$'s are modeled as i.i.d exponential random variables whose mean value is 1 because it is assumed that the h_k's are i.i.d Rayleigh fading. We define $f_{|h_k|^2}(x)$ as the PDF of $|h_k|^2$, which are expressed as e^{-x} and independent with respect to k. Then, using (3), (5), and (8), the outage probability for M-BFD can be expressed as

$$\begin{aligned} P_O^F &= 1 - \Pr\left[\gamma_{k^F,a}^F > \gamma_{th}^F\right] \\ &= 1 - \Pr\left[\max_{k=1,\ldots,K} |h_k|^2 > \frac{(2^R - 1)}{\eta/2}\right]. \end{aligned} \tag{9}$$

Using (9) and order statistics [17], the outage probability for M-BFD can be expressed as

$$\begin{aligned} P_O^F &= 1 - \left(1 - \left(\int_0^{\frac{(2^R-1)}{\eta/2}} f_{|h_k|^2}(x) dx\right)^K\right) \\ &= \left(1 - e^{-\frac{(2^R-1)}{\eta/2}}\right)^K. \end{aligned} \tag{10}$$

In M-BHD, an outage occurs when one of the SNRs received at the two nodes cannot satisfy the required SNR at the corresponding transmission time. Since M-BHD utilizes double the time for two way communication compared to M-BFD, M-BHD transmission for each link needs to be performed twice as fast as that for M-BFD in order to achieve the same target rate. Therefore, the required SNR for M-BHD, γ_{th}^H, can be expressed by $2^{2R} - 1$. The received SNRs at node i for M-BHD can then be expressed as

$$\gamma_{k,i}^H = \frac{|h_k|^2 P}{\sigma^2} = |h_k|^2 \eta, \tag{11}$$

Using (6), (7), and (11), the outage probability for M-BHD can be expressed as

$$\begin{aligned} P_O^H &= 1 - \Pr\left[\gamma_{k_{ba}^H,a}^H > \gamma_{th}^H\right] \Pr\left[\gamma_{k_{ba}^H,b}^H > \gamma_{th}^H\right] \\ &= 1 - \left(\Pr\left[\max_{k=1,\ldots,K} |h_k|^2 > \frac{2^{2R} - 1}{\eta}\right]\right)^2. \end{aligned} \tag{12}$$

Using (12) and order statistics [17], the outage probability for M-BHD can be expressed as

$$\begin{aligned} P_O^H(\eta, R, K) &= 1 - \left(1 - \left(\int_0^{\frac{2^{2R}-1}{\eta}} f_{|h_k|^2}(x) dx\right)^K\right)^2 \\ &= 1 - \left(1 - \left(1 - e^{-\frac{2^{2R}-1}{\eta}}\right)^K\right)^2. \end{aligned} \tag{13}$$

In the following subsections we will move on to investigating the outage probabilities with respect to diversity order and SNR difference the in high SNR regime.

C. Diversity Order

In this subsection, we derive the diversity orders for M-BFD and M-BHD in order to investigate the effect of the multi-spectrum diversity gain on the outage probability. We define the diversity order as the magnitude of the slope in the high SNR regime where the outage probability versus SNR is represented on a log scale [18]. We can define the diversity order, d, as

$$d = \lim_{\eta \to \infty} \left(-\eta \frac{\partial \log P_O}{\partial \eta}\right). \tag{14}$$

Substituting (10) and (13) into (14), the diversity orders for M-BFD and M-BHD can be derived respectively as

$$d^F = \lim_{\eta \to \infty} \left(-\eta \frac{\partial \log P_O^F}{\partial \eta}\right) = K \tag{15}$$

and

$$d^H = \lim_{\eta \to \infty} \left(-\eta \frac{\partial \log P_O^H}{\partial \eta}\right) = K. \tag{16}$$

Note that each system achieves the same diversity, K. This is because both M-BFD and M-BHD select one spectrum from among K available spectra in a single time period. It can be inferred that M-BFD and M-BHD are identically affected by the number of available spectra in terms of reliability. On the other hand, M-BFD has half the transmission time compared to M-BHD with the given common SNR. Considering both reliability and transmission time, it can be inferred that M-BFD offers an advantage in two-way communications.

D. Asymptotic Difference Between Outage Probability

In this subsection, we investigate the asymptotic difference between the outage probability for M-BFD and for M-BHD. From (15) and (16), it can be inferred that the log-scaled outage probability versus the dB-scaled SNR curves for M-BFD is a shifted version of that for the corresponding M-BHD in the high SNR regime. We define the SNR difference between the outage probability curve for M-BFD and M-BHD as the *SNR gap*, $\Delta(R, K)$. We assume ζ as a desired outage probability for a given target rate R. Then,

$$P_O^F\left(\eta^F, R, K\right) = P_O^H\left(\eta^H, R, K\right) = \zeta, \tag{17}$$

where η^F and η^H denote the SNR values which achieve the desired outage probability for M-BFD and M-BHD, respectively. From Appendix A, the overall SNR gap can be obtained as

$$\Delta(R, K) = \underbrace{\frac{10\log_{10}2}{K}}_{\substack{spectrum\,selection\\ diversity\,gain\\ difference}} + \underbrace{10\log_{10}\left(2^R+1\right)}_{required\,SNR\,gain} - \underbrace{10\log_{10}2}_{power\,gain}. \tag{18}$$

Note that the SNR gap is determined by the number of available spectra and the target data rate. The SNR gap is in inverse proportion to K, but increases almost linearly along with R.

The first term of the SNR gap originates from the difference in the spectrum selection diversity gain between M-BFD and M-BHD. For two-way communication, both nodes in M-BFD simultaneously utilize the same spectrum so that just one selection is needed. In M-BHD, in contrast, each node independently selects the spectrum in its transmission time so that twice as many spectrum selections are needed. From this perspective, M-BFD is more advantageous with respect to avoiding outages than M-BHD. As K increases, the probability difference between the existence of a satisfactory spectrum in one time period and that in two consecutive time periods decreases. Therefore, as shown in (18), the gain decreases along with K.

The second term and the third term originate from the difference in the required SNR and power usage between M-BFD and M-BHD, respectively. The required SNR gain of M-BFD increases logarithmically with the ratio of the required SNR for M-BHD to that for M-BFD which can be expressed as 2^R+1. The negative constant power gain of M-BFD originates from the fact that both nodes in M-BFD transmit with half the power of those in M-BHD, as mentioned in (8) and (11).

We also find that the SNR gap has a non-negative value. This is because $\frac{10\log_{10}2}{K} \geq 0$ and $10\log_{10}\left(2^R+1\right) \geq \log_{10}2$, since K and R are non-negative. Hence, it can be inferred that M-BFD outperforms M-BHD in terms of outage probability.

IV. Numerical Results

In this section, we present the outage probabilities for M-BFD and M-BHD to verify our analysis. In the figures, the lines and symbols represent the theoretical results and simulation results, respectively.

Fig. 3 compares the outage probabilities for M-BFD and M-BHD according to the SNR with various numbers of available spectra ($K = 1, 5, 10$). We set the target data rate, R, to be 1 bps/Hz. Each system has the same diversity order under fixed R and K. In addition, we can see that the outage probabilities for both systems decrease with the power of K. This means that the effect of the number of available spectra on the diversity orders in each system is equivalent, as shown

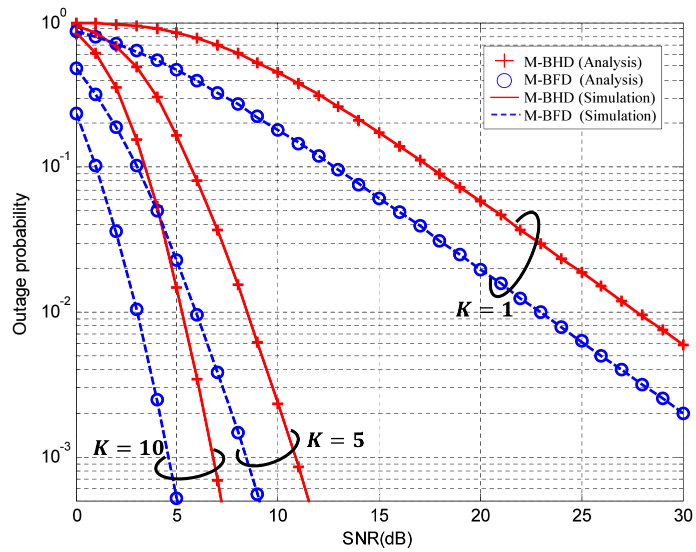

Fig. 3. Outage probabilities for M-BFD and M-BHD. (R = 1bps/Hz, K = 1,5,10)

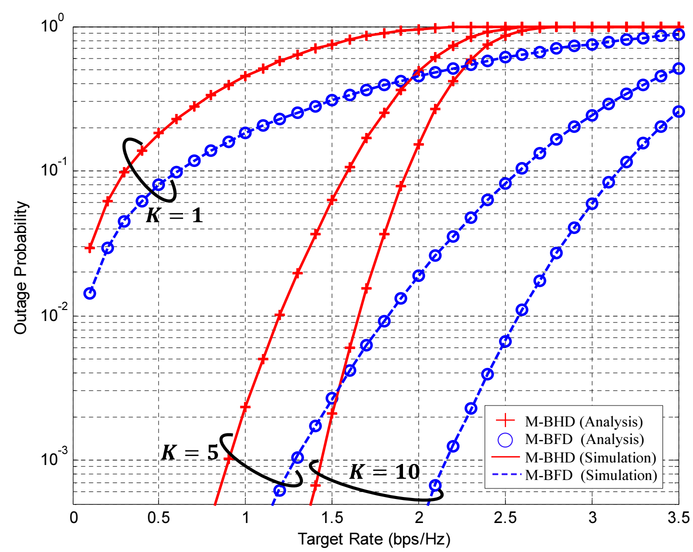

Fig. 4. Outage probabilities for M-BFD and M-BHD. (SNR = 10dB, K = 1,5,10)

in (15) and (16). The SNR gap decreases as K increases under fixed R and with the desired outage probability, but becomes saturated at $\log_{10}\left(\frac{2^R+1}{2}\right)$ dB as shown in (18). From these results, we can confirm that M-BFD outperforms M-BHD.

Fig. 4 compares the outage probabilities according to the target rate with various numbers of available spectra ($K = 1, 5, 10$). We set the SNR to be 10 dB. The difference in the achievable data rate between M-BFD and M-BHD increases along with K under fixed SNR and with the desired outage probability. This originates from the fact that the ratio of the required SNR for M-BHD on M-BFD for the desired outage probability increases along with K as shown in (10) and (13). From this result, it can be inferred that M-BFD can achieve a higher data rate compared to M-BHD, and that the difference increases along with K.

V. CONCLUSIONS

We investigated spectrum selection strategies for M-BFD and M-BHD in order to minimize outage. After applying the spectrum selection strategies, M-BFD was shown to be superior to M-BHD in terms of outage probability. Due to the smaller spectrum selection, M-BFD achieves a greater spectrum selection diversity gain than M-BHD despite the fact that the difference decreases along with the number of available spectra. A reduction in time usage due to simultaneous transmission at both source nodes leads M-BFD to achieve the required SNR gain at a better rate than M-BHD. The advantage of M-BFD expands at high target data rates with a large number of available spectra.

APPENDIX

A. Proof of the SNR Gap

Let ω be the ratio of η^H to η^F in high SNR regime which can be expressed as

$$\omega = \lim_{\eta^H\to\infty} \frac{\eta^H}{\eta^F}. \tag{19}$$

Substituting (10), (13) and (19) to (18), we can obtain

$$\begin{aligned}&\lim_{\eta^H\to\infty} \frac{1-\left(1-\left(1-e^{-\frac{2^{2R}-1}{\eta^H}}\right)^K\right)^2}{\left(1-e^{-\frac{(2^R-1)\omega}{\eta^H/2}}\right)^K}\\ &=\lim_{\eta^H\to\infty} \frac{\left(2-\left(1-e^{-\frac{2^{2R}-1}{\eta^H}}\right)^K\right)\left(1-e^{-\frac{2^{2R}-1}{\eta^H}}\right)^K}{\left(1-e^{-\frac{(2^R-1)\omega}{\eta^H/2}}\right)^K}\\ &=1.\end{aligned} \tag{20}$$

Applying L'Hopital's rule, we have

$$2\left(\frac{2^R+1}{2\omega}\right)^K = 1. \tag{21}$$

From (21), ω can be obtained as

$$\omega = \left(\frac{2^R+1}{2}\right)2^{\frac{1}{K}}. \tag{22}$$

The SNR gap in dB-scale, Δ, can then be expressed as

$$\Delta = 10\log_{10}\omega. \tag{23}$$

Substituting (22) to (23), the SNR gap is obtained as

$$\Delta(R,K) = \frac{10\log_{10}2}{K} + 10\log_{10}\left(2^R+1\right) - 10\log_{10}2. \tag{24}$$

From (24), we can investigate the SNR with respect to the spectrum selection diversity gain difference, required SNR gain, and power gain.

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