

A New Approach To Solve Aircraft Recovery Problem

Congcong Wu

The Scientific Research Academy
Shanghai Maritime University
Shanghai 200135, P. R. China
meilongle@hotmail.com

Meilong Le

The Scientific Research Academy
Shanghai Maritime University
Shanghai 200135, P. R. China
lemeilong@126.com

Abstract—When disruptions occur, the airlines have to recover from the disrupted schedule. The recovery usually consists of aircraft recovery, crew recovery and passengers' recovery. This paper focuses on aircraft recovery. Take the total cost of assignment, cancelation and delay as an objective; we present a more practical model, in which the maintenance and other regulations are considered. Then, we present a so-called iterative tree growing with node combination method. By aggregating nodes, the possibility of routings is greatly simplified. So, it can give out the solution in more reasonable time. Finally, we use data from a main Chinese airline to test the solution algorithm. The experimental results state that this method could be used in aircraft recovery problem.

Keywords—aircraft recovery; airlines optimal recovery ; airlines recovery; recovery algorithm

I. INTRODUCTION

When disruptions caused by severe weather conditions, air traffic control or mechanical failures occur, the airlines have to recover from the disrupted schedule. The airlines recovery usually consists of aircraft recovery, crew recovery and passengers' recovery. Since the aircraft is viewed as the most important scarce resource, the most work on operational recovery problems has been reported on the aircraft recovery.

Aircraft recovery problem (ARP) is to determine new flight departure times, cancellations and rerouting for affected aircrafts including ferrying, diverting, swapping and so on. Besides that several decision rules, such as, aircrafts balance requirements, maintenance requirements and station departure curfew restrictions should also be considered. At the end of the recovery period, aircrafts should be positioned to resume operations as planned.

Being different to the aircraft rotation problem in the planning stage, the method to solve the ARP should calculate the problem in reasonable time, which is very difficult to most optimization solvers under most reasonable disruption scenarios. How to solve the ARP in reasonable time and meet these decision rules has been one of the most important keys in airline recovery study. Teodorovic and Gubernic (1984) [1] are one of the first to study the aircraft recovery problem, using a branch and bound (B&B) algorithm [2] to solve the aircraft recovery model (ARM) , but the research does not satisfy the constraints of station curfews, maintenance requirements and aircrafts balance at the recovery period in the modeling. Arguello et al. (1997) [3] creates a greedy randomized adaptive search procedure (GRASP) to reconstruct aircraft routings, which is a fast heuristic based on randomized neighborhood search, but they don't consider the maintenance requirements and crew

requirements after the aircraft routings altered. Afterwards, Bard et al. (2001) [4] develops a time-band optimization model to reconstruct cost-effective aircraft routings. The disadvantage is that the research excludes the maintenance requirements and crew requirements. Thengvall (2003) [5] presents a bundle algorithm to solve a multi-commodity network model. As in Petersen et al. (2010) [6], they integrate all kinds of recovery simultaneously, and employ the Bender's decomposition to decompose the model into a master problem (airline schedule recovery) and three sub-problems (aircraft recovery, crew recovery and passenger recovery), using an optimization-based approach to solve the situation of hub closure.

In our paper, modeling is based on flight strings instead of flights as well as defining recovery scope, in order to solve the model in reasonable time. We assign specific aircraft to flight strings while meeting maintenance requirements, station departure curfew restrictions and other aircraft requirements. As to the solution methodology, firstly, we transform our model into time-space network. Then, we create a new method (a so-called iterative tree growing with node combination method) to solve the network model, which is the most important part of our paper. We test our intelligent method with data from Chinese airlines. Computational results are presented for a daily schedule recovery, showing that the proposed approach provides faster times to optimality in some cases and always obtains feasible, near-optimal solutions for medium-size airlines recovery problem much more quickly than can be found using CPLEX. In our future study, we should do much more experience to test our new method, try it on the large-size airlines recovery problem and use it much more widely, for example, in integrated recovery combining with crew recovery or passenger recovery or all of the three.

The remainder of the paper is organized as follows. We first give in Section II a literature review of the aircraft recovery problem. In Section III, we build our aircraft recovery model. The solution methodology is described in Section IV and two scenarios are presented to test the intelligent method. We give our conclusion in Section V.

II. LITERATURE REVIEW

When one or more aircrafts are out of service, the airlines have to operate the flight schedule with a reduced number of planes. Teodorovic and Gubernic (1984) [1] are the pioneers to study ARP. The paper tries to minimize total the passenger delay by swapping or delaying flights and solved exactly by branch and bound. Subsequently, Teodorovic and Stojkovic (1990) [7] formulates a heuristic algorithm to solve the same problem as Teodorovic and Gubernic (1984) [1]. But, in their paper the chief objective is to minimize the total

passenger delay with an equal total number of cancelled flights. In addition, neither of these models considers flight delay and cancellation cost.

Yan and Yang (1996) [8] are the first to allow for delays and cancellations simultaneously. Four systematic strategic models are developed by perturbing the BSPM (basic schedule perturbation model) and combining various scheduling rules. The BSPM is designed to minimize the schedule-perturbed period after an incident and to obtain the most profitable schedule given the schedule-perturbed period. These network models are formulated as pure network flow problems or network flow problems with side constraints. With real flight data from Taiwan Airlines, the former was solved by the network simplex method while the latter was solved by Lagrangian relaxation with subgradient methods. However, the constraints of aircraft maintenance and crew scheduling are overlooked.

An extension to the network model of Arguello et al. (1997) [9] is presented by Thengvall et al. (2000) [10]. The authors presents a model in which they penalize in the objective function the deviation from the original schedule and they allow human planners to specify preferences related to the recovery operations.

Rosenberger et al. (2003) [11] models ARP as a set-packing problem with a time window and slots restrictions. In this model the objective is to minimize the cost of assigning routes to aircraft and the cost of cancelling the unassigned legs. Being different from Arguello et al. (1997) [3] and Bard et al. (2001) [4], their paper assumes an aircraft selection heuristic (ASH) for ARO (an optimization model for aircraft recovery), which selects a subset of aircraft for optimization prior to generating new routes. Compared with network model, this model can check maintenance feasibility using column generation.

Eggenberg et al. (2007) [13] introduces an extension of the time-space network model to minimize delays, cancellations and plane swappings, and make span cost.

In Massoud Bazargan (2010) [14], the paper introduces the airline irregular operation in detail and uses the time-band optimization method to solve the aircraft recovery as an example.

Le et al. (2011) [15] provides an overview recent years' of disruption management of schedule, aircraft, crew, passenger and the integrated recovery.

Something is done in our aircraft recovery model, aiming to minimize the aggregate cost comprised of assigning cost and recovering cost. We transform the aircraft recovery problem as a multi-commodity network with side constraints and using a so-called iterative tree growing with node combination method to solve the disruption.

III. THE AIRCRAFT RECOVERY PROBLEM

A. Sets

F_n	set of all flight legs in recovery scope N
$F_n^{mandatory}$	set of mandatory flight legs

$F_n^{optional}$	set of optional flight legs that are candidates for deletion
E_n	set of fleet types in recovery scope N
S_n	set of flight string s in recovery scope N
$K(e)_n$	set of aircraft of fleet type in recovery scope N
$H(e)_n$	set of aircraft of fleet type requiring maintenance within T in recovery scope N
A	set of airports
$A^{maint}(e)$	set of stations that are capable of performing schedule maintenance of aircraft of fleet type e
G^k	set of ground arcs of aircraft k which cross the count time
B. Datas	
y_j^k	a ground variable used to count the number of aircraft k on the ground j
AN_e	the number of aircrafts in fleet type
$C_{e,s}^k$	cost of assigning aircraft $k \in K(e)_n$ to flight string s
td_f	actual departure time of flight f
td_f	actual departure time of flight f
ta_f	actual arrival time of flight f
ta_f	actual arrival time of flight f
$A_{e,f}^k$	ready time of aircraft k to operate flight f
CC_f	cost of canceling flight f
CD_f	cost of 1-min delay of flight f
DT_f	expected trip (block-to-block) time of flight f
N	recovery scope index
T_f	the scheduled departure time of flight f
U	minimum connection time

$$r_s^k \quad \text{the number of flight string } s \text{ is being executed by aircraft } k \text{ cross the count time} \quad \sum_{k \in K(e)_n} \sum_{s \in S_n} r_s^k x_{e,s}^k + \sum_{k \in K(e)_n} \sum_{j \in G^k} p_j^k y_j^k \leq N_e, \forall e \in E_n \quad (7)$$

C. Variables

$$I_{m,s} = \begin{cases} 1 & \text{if an eligible maintenance station } m \in A^{\text{maint}}(e) \geq A_{e,f}^k x_{e,s}^k a_{f,s}, \forall f \in \text{first flight of } S, \\ & \text{is visited by flight string } s \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} x_{e,s}^k a_{f_{i+1},s} - x_{e,s}^k a_{f_i,s} = 0, \forall k \in K(e)_n, f_i \in s \in S_n \\ k \in K(e)_n \end{matrix} \quad (8)$$

$$a_{f,s} = \begin{cases} 1 & \text{indicator variable, if flight } f \in F \text{ in flight string } s \text{ is visited} \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} a_{f,s} \geq a_{f',s'} x_{e,s}^k a_{f',s'} + U \\ f \in \text{last flight of string } s \in S_n, \end{matrix} \quad (9)$$

$$x_{e,s}^k = \begin{cases} 1 & \text{if aircraft } k \in K(e)_n \text{ covers flight string } s \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} f' \in \{\text{first flight of string } s' \in S_n \\ T_f + DT_f \leq T_{f'} + \text{Max Delayed allowed} \} \end{matrix} \quad (10)$$

$$z_f = \begin{cases} 1 & \text{if flight } f \text{ is canceled} \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} td_{f_{i+1}} \geq ta_{f_i} + U, \forall f \in \text{flight of string } s \in S_n \\ td_f \geq T_f, \forall f \in F_n \end{matrix} \quad (11)$$

$$p_j^k = \begin{cases} 1 & \text{ground arc } j \in G^k \text{ for aircraft } k \\ & \text{crosses the count time} \\ 0 & \text{otherwise} \end{cases} \quad \begin{matrix} ta_f = td_f + DT_f(1 - z_f), \forall f \in F_n \\ x_{e,s}^k \in \{0,1\} \\ z_f \in \{0,1\} \end{matrix} \quad (12)$$

D. Mathematical formulation

$$\min \sum_{k \in K(e)_n} \sum_{s \in S_n} c_{e,s}^k x_{e,s}^k + \sum_{f \in F_n} CD_f(1 - z_f)[td_f - T_f] + \sum_{f \in F_n} CC_f z_f \quad (1)$$

Subject to:

$$\sum_{k \in K(e)_n} \sum_{s \in S_n} x_{e,s}^k a_{f,s} + z_f = 1, \forall f \in F_n \quad (2)$$

$$\sum_{k \in K(e)_n} \sum_{s \in S_n} x_{e,s}^k a_{f,s} = 1, \forall f \in F_n^{\text{mandatory}} \quad (3)$$

$$\sum_{k \in K(e)_n} \sum_{s \in S_n} x_{e,s}^k a_{f,s} \leq 1, \forall f \in F_n^{\text{optional}} \quad (4)$$

$$\sum_{s \in S_n} \sum_{m \in A^{\text{maint}}(e)} I_{m,s} x_{e,s}^k = 1, \forall k \in K(e)_n \quad (5)$$

$$x_{e,s}^k a_{f,s} + \sum_{f'} x_{e,s}^k a_{f',s'} \leq 1$$

$$f \in \text{first flight of } s \in S_n,$$

$$f' \in \{\text{first flight of } s' \in S_n \mid T_{f'} > T_f, \quad (6)$$

$$T_{\text{last flight of } s} + DT_{\text{last flight of } s} \geq$$

$$T_{f'} + \text{Max Delayed allowed}, \forall k \in K(e)_n$$

The objective (1) tries to minimize the aggregate cost comprised of assigning strings (assignment cost) and recovering aircrafts (delay cost and cancellation cost) in the recovery scope. Either a flight must be contained in exactly one string or cancelled, as seen in (2). The cover constraints are split into (3) and (4) to distinguish between the mandatory and optional leg sets, ensure each aircraft is assigned to no more than one string. Maintenance cover constraints are seen in (5). This simply ensures a maintenance opportunity is built in, and the specific maintenance planning can be done post-optimization. Constraint (6) ensures that each available aircraft cannot be assigned to two different strings in the same time. The count constraint (7), make sure that the total number of aircraft in the air and on the ground does not exceed the size of fleet type e. Constraint (8) defines rotations aircraft usage. All flights in a rotation use one aircraft not different ones. By using the concept of rotation and defining rotations in the model, aircraft balance at each airport is satisfied. Constraints (9)–(12) determine the departure time of each flight. A flight cannot depart earlier than the ready time of its assigned aircraft, as stated in (9). Constraints in (10) ensure that when two flight strings are flown by the same aircraft, the second string cannot depart earlier than real arrival time of first string (because of the minimum connecting time). In a flight string, the departure time of a flight cannot be earlier than the arrival time of its previous flight, as stated in (11).

Constraints in (12) state that no flight is allowed to depart before its scheduled departure time. Constraints in (13) relate the departure and arrival times for each flight. Constraints (14)–(16) ensure that the x, z are binary variables.

IV. SOLUTION METHODOLOGY

Even by limiting the scope of the problem to get computational result, to most airlines, the problem is likely too large and complex to return a globally optimal solution with optimization solver for most reasonable disruption scenarios. Thus, we seek the hybrid method, which is optimization method with heuristic approach. The heuristic we used is so-called iterative tree growing with node-combination. The time-space graph is used to describe our heuristic method. In the graph, the cities and times are represented horizontally and vertically respectively. Each node represents an airport-departure or airport-arrival event. All the arcs denote flights. Except first node (time-earliest node) and last node (time-termination node, usually night curfew time for departure), we draw all parallel arcs (copy arcs) if the arc lies above the node and originates from the same node (airport). As the flight arcs are placed in the graph iteratively, the tree grows downward. There are three kinds of arcs in the graph. One is original flight arcs. The other is copy arcs, which is actually opportunity flight arcs due to flight cancellation. The third is overfly arcs, which is actually delay flight arcs. Most probably, each copy arc generates a new node. From this new node, copy arcs and overfly arcs

can be originated or connected again. It is an iterative procedure. By so stretching, the tree grows downward.

Obviously, there will be more and more nodes and arcs as the graph stretches downward. Every route from top to down represents a routing. In order to simplify such a combinatorial problem, we use circle to replace a dot to represent a node. In other words, a node does not represent a single airport-departure or airport-arrival event, but a cluster of airport-departure or airport-arrival events. All the airport-departure or airport-arrival dots within the certain time circle are aggregated to this node. The delay time is counted from departure circle node to arrival circle node, not the difference between real departure and arrival time. Under the extreme condition, such aggregating method may calculate delay time the whole circle diameter difference.

The test instances used as benchmark problems in this study are acquired from real flight schedule of one medium-size airlines in China. The schedule consists of 170 flights served by 5 fleets, 35 aircrafts over a network of 51 airports all over the country.

We choose test instances from the flight schedule. The relative data is listed in TABLE I. The computations also use the following assumptions:

- Each station requires a minimum of 40 minutes turnaround time;
- Execute midnight arrival/departure curfew (no arrival or departure after midnight is allowed);
- Each minute of delay on any flight costs the airline \$20.

TABLE I THE FLIGHT SCHEDULE AND CANCELANATION COST

Fleet type	Flight string	Aircraft	Flight	Pax	DStat	STD1	AStat	STA1	Duration	Cancellation cost
737-800	S1	1	9131	100	SHA	815	TSN	1005	1:50	\$17,490
			9125	72	TSN	1100	SZX	1350	2:50	\$15,780
			9126	100	SZX	1450	TSN	1755	3:05	\$21,050
			9132	100	TSN	1855	SHA	2040	1:45	\$16,980
737-800	S2	2	9380	14	SZX	845	SHA	1050	2:05	\$14,120
			9371	14	SHA	1305	SZX	1515	2:10	\$14,870
			9372	49	SZX	1610	SHA	1820	2:10	\$17,120
			9369	150	SHA	1910	SZX	2115	2:05	\$19,870
737-800	S3	3	9304	104	CAN	1130	SHA	1335	2:05	\$18,740
			9375	78	SHA	1435	SZX	1645	2:10	\$17,290
			9376	78	SZX	1750	SHA	2015	2:25	\$18,110
			9303	78	SHA	2100	CAN	2315	2:15	\$17,890

We use two scenarios to test the method.

Scenario 1—Delay

The aircraft 2 in airport SZX must be grounded at 8:00 and is available until 15:00. That is, aircraft 2 is unavailable from 8:00 to 15:00. The trivial solution 1 is to cancel flights 9380 and 9371 which are flown by aircraft 2 during 8:00 to 15:00. The total cancellation cost is \$28,990. The trivial solution 2 is to delay flight string 2(9380, 9371, 9372, 9369). The ready time of flight 9369 is 23:25. Against the curfew, so the flight 9369 should be canceled. The solution got from our method is listed in TABLE II. The total cost is \$38,270.

Scenario 2—Delay and grounded combination

In this case, we assume that aircraft 1 in airport SHA becomes grounded owing to some mechanical failure at 8:00 and is unavailable for the rest of the day. The obvious solution without permitting any rerouting of other aircraft is to cancel flights 9131, 9125, 9126 and 9132. These cancellations cost the airline a total of \$71,300 (the sum of all cancellation costs for flight string1).

TABLE II TRIVAL OPTION 2

Aircraft tail	Flight	Pax	DStat	STD1	AStat	STA1	Option	Cancellation cost	Delay cost
1	9131	100	SHA	815	TSN	1005	/	/	/
	9125	72	TSN	1100	SZX	1350	/	/	/
	9126	100	SZX	1450	TSN	1755	/	/	/
	9132	100	TSN	1855	SHA	2120	/	/	/
2	9380	14	SZX	1500	SHA	1745	Delay	/	\$7,500
	9371	14	SHA	1745	SZX	2035	Delay	/	\$5,600
	9372	49	SZX	2035	SHA	2325	Delay	/	\$5,300
	9369	150	SHA	1910	SZX	2115	Cancel	\$19,870	/
3	9304	104	CAN	1130	SHA	1335	/	/	/
	9375	78	SHA	1435	SZX	1645	/	/	/
	9376	78	SZX	1750	SHA	2015	/	/	/
	9303	78	SHA	2100	CAN	2315	/	/	/
Total							\$19,870	\$18,400	

The according graph is drawn in Figure 1. The figure on the arc is flight number. The figure besides the node is departure or arrival time. The node is marked according to vertical time coordinate and horizontal airport coordinate. In order to reflect whether two flight legs can be connected, the arrival time has been added turnaround time. For example, flight 9131 arrives in TSN at 10:05 and connects to flight 9125 which is available for departure at 10:45. We use 30 minutes as the diameter of the circle. So, flight 9131 is not ready for departure at 10:45 but 11:00.

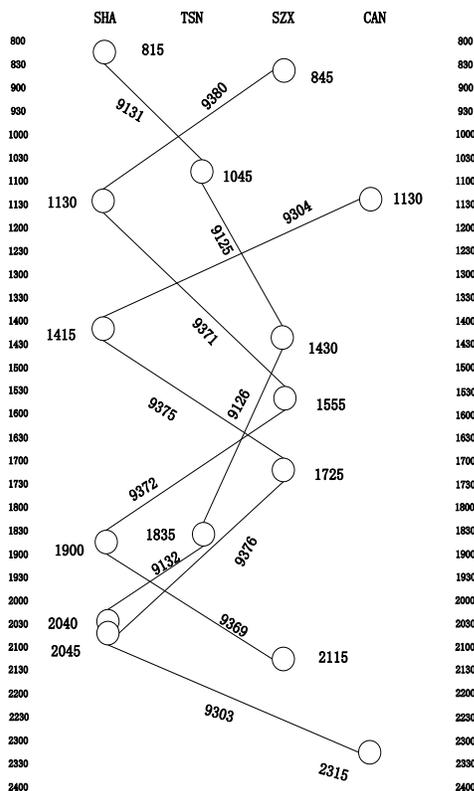


Figure 1 The flights graph

In the Figure 1, one arc is drawn from node 2 to node 16, it represents flight 9131. Actually it is a copy arc

representing flight 9131, the delay time is 210 minutes, not the actual time minus the schedule time. This is because flight 9131 was scheduled to leave SHA at 8:15. If this flight occurs in node 2, the departure time is calculated as 11:30. Considering the nodes are within 30 minutes circle, this delay spans from 8:00 to 11:30, a total of 210 minutes. Each minute of delay costs the airline \$20, so flight 9131 has a delay cost of \$4,200 if it departs from node 2.

Figure 2 is deduced from Figure 1 based on the method mentioned above.

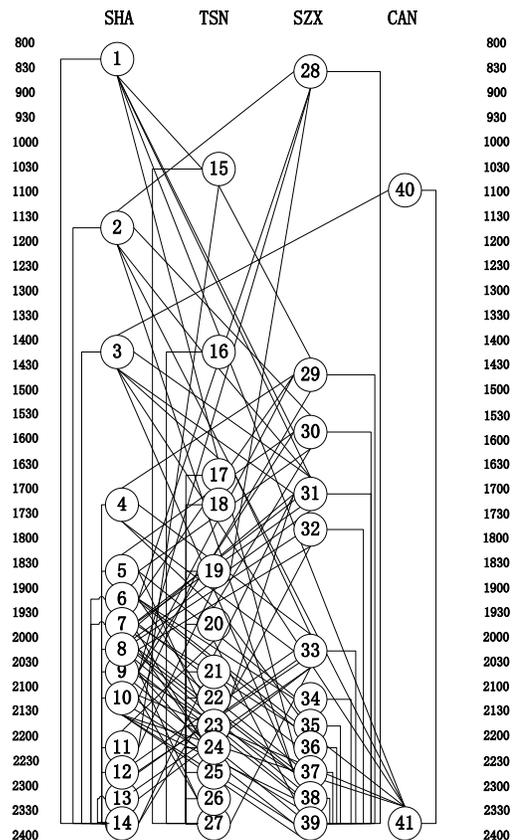


Figure 2 The stretch graph

TABLE III presents the non-zero delay costs for all flight arcs in Figure 2.

TABLE III NONE-ZERO DELAYCOSTS

Flight	Pax	Ori.node	Dest.node	Delay cost(\$)
9125	25	19	36	9000
9125	25	16	32	3600
9125	25	20	38	10200
9125	25	17	33	6600
9126	100	33	27	6600
9126	100	32	22	3600
9126	100	30	20	1200
9126	100	31	21	3000
9126	100	31	22	3000
9131	100	4	20	10800
9131	100	2	16	4200
9131	100	5	22	12600
9131	100	3	17	7200
9131	100	6	23	13200
9131	100	7	24	13800
9131	100	8	25	14400
9131	100	9	26	15000
9131	100	10	26	15600
9132	100	20	11	1200
9132	100	20	36	1200
9132	100	23	14	3600
9132	100	22	13	3000
9369	150	7	37	600
9369	150	8	37	1200
9369	150	8	38	1200
9369	150	9	39	1800

9369	150	10	38	2400
9371	14	4	33	4800
9371	14	5	35	6600
9371	14	6	35	7200
9371	14	3	31	1200
9371	14	7	37	7800
9371	14	8	38	8400
9371	14	9	39	9600
9371	14	10	39	9900
9372	49	33	12	4800
9372	49	32	8	1800
9372	49	33	13	4800
9372	49	31	7	1200
9372	49	31	8	1200
9375	78	4	33	3000
9375	78	5	35	4800
9375	78	6	35	5400
9375	78	7	37	6000
9375	78	8	38	6600
9375	78	9	39	7200
9375	78	10	39	7800
9376	78	33	13	3000
9380	14	29	4	7200
9380	14	33	12	13800
9380	14	32	8	10800
9380	14	30	5	8400
9380	14	33	13	13800
9380	14	31	7	10200
9380	14	31	8	10200

TABLE IV RECOVERY SOLUTION FOR SCENARIO 1

Aircraft tail	Flight	Pax	Dstat	Ori.node	Astat	Dest.node	Option	Cancelation cost	Delay cost
1	9131	100	SHA	1	TSN	15	/		
	9125	72	TSN	15	SZX	29	/		
	9380	14	SZX	29	SHA	4	Delay		\$7,200
	9375	78	SHA	4	SZX	33	Delay		\$3,000
	9376	78	SZX	33	SHA	13	Delay		\$3,000
2	9126	100	SZX	30	TSN	19	Delay		\$600
	9132	100	TSN	19	SHA	10	/		
	9303	78	SHA	10	CAN	39	Delay		0
3	9304	104	CAN	40	SHA	3	/		
	9371	14	SHA	3	SZX	31	Delay		\$1,200
	9372	49	SZX	31	SHA	7	Delay		\$1,200
	9369	150	SHA	7	SZX	37	Delay		\$600
Total									\$16,800

Through a series of aircraft rerouting and cancellations in an effort to minimize the total cost to the airlines, the total cost of the solution is \$16,800, less than the above two trivial options. In contrast with the method of branch and bound (B&B), using an intelligent algorithm we can quickly obtain feasible, near-optimal solutions faster times in some case study than using CPLEX (Thinkpad X201S). Through the computation results we can see our model has quite a good effect on aircraft recovery optimization. The total passenger delay is 40825 minutes. The total cost for this actual flight schedule is \$16,800, which is similar to the solution got from our method.

Using the so-called iterative tree growing with node combination method as scenario1, we can get the recovery solution for scenario2.

TABLE V RECOVERY SOLUTION FOR SCENARIO 2

Aircraft	Flight	Pax	DStat	Ori.node	AStat	Dest.node	Option	Cancelation cost	Delay cost
1	9126	100	SZX	29	TSN	19	Cancel	\$21,050	/
	9132	100	TSN	19	SHA	10	Cancel	\$16,980	/
	9371	14	SHA	2	SZX	30	Cancel	\$14,870	/
2	9380	14	SZX	28	SHA	2	/	/	/
	9131	100	SHA	2	TSN	16	Delay	/	\$4,200
	9125	72	TSN	16	SZX	32	Delay	/	\$3,600
	9372	49	SZX	32	SHA	8	Delay	/	\$1,800
	9369	150	SHA	8	SZX	38	Delay	/	\$1,200
3	9304	104	CAN	40	SHA	3	/	/	/
	9375	78	SHA	3	SZX	31	/	/	/
	9376	78	SZX	31	SHA	9	/	/	/
	9303	78	SHA	9	CAN	41	/	/	/
Total							\$52,900	\$10,800	

The total cost for scenario 2 by our method is \$63,700, smaller than the trivial solution of \$71,300 resulting from canceling all flights operated by aircraft 1. The total cost for this actual flight schedule is \$63,400, which is similar to the solution got from our method. In this scenario we can see the aircraft is one of the most important resources in airlines recovery. The shortage of aircraft resources limited the degree of airlines recovery.

V. CONCLUSION

The paper presents a more practical formulation for airline optimal recovery. In order to get the solution in a reasonable time, a new approach to solve the problem is studied. The computational results state the method could be used in airline recovery. On average, for medium-size airline recovery, the algorithm finds a feasible solution twice as fast as an exact algorithm, obtaining a high-quality feasible solution in half the time is an important improvement for our application. Often in our method, having several near-optimal solutions provide decision makers much more flexibility.

Airlines recovery is a more complex and large-scale problem. Not only should aircrafts be considered, but crew and passengers should be considered, too. In the future, a more comprehensive recovery model should be studied. Meanwhile, a more systematic evaluation of the method should be carried out.

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