

# Level Crossing Rate of System with Macrodiversity and Three Branches Microdiversity Reception in Gamma Shadowed Rician Fading Channels

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**Abstract**—In wireless communication systems, a received signal may suffer both short-term fading and long-term fading (shadowing). The simultaneous use of both microdiversity and macrodiversity is essential to mitigate channel degradations in composite fading environments. In this paper, a wireless communication system with microdiversity and macrodiversity reception in gamma-shadowed Rician fading channels is considered. An exact and rapidly converging infinite-series expression for the average level crossing rate (LCR) at the output of the system is provided. Numerical results are presented graphically to illustrate the proposed analytical presentation and to point out the effects of system's parameters on the observed characteristics.

**Keywords**—Gamma shadowing; level crossing rate; macrodiversity; microdiversity; Rician fading.

## I. INTRODUCTION

In wireless communication systems, short-term fading, or fast fading, and long-term fading, or shadowing, occur concurrently [1][2]. This leads to composite fading models. Fast fading is the result of multipath propagation due to the effects of reflection, refraction, diffraction and scattering. Shadowing is the result of large obstacles and large deviations in terrain form between transmitter and receiver. It is possible to use various distributions to describe the fading envelope of the received signal. In technical literature, the most often used are Rayleigh, Rician, Nakagami-m, Nakagami-q, Weibull,  $\alpha$ - $\mu$  [1]. The average power of the received signal is also a random variable because the shadowing is present at the same time. The distribution most often used for describing the average power is log-normal distribution. Sadly, the use of log-normal distribution does not lead to a closed-form expression for the probability density function (PDF) of the signal-to-noise ratio (SNR) [3]-[5]. With this approach, the system analysis is very unwieldy. More recent papers have shown by both theoretical results and measured data, that gamma and

lognormal distribution match sufficiently [6][7]. Such approach, with gamma distribution used for modeling the average signal power, leads to a closed-form expression for PDF of the SNR, what facilitate further analysis.

The diversity technique is one of the most used methods for reduction of fading effects and upgrading the communication system reliability without enlarging transmitting power and channel's bandwidth. Diversity techniques combine in different ways the multiple signals received. In single base station, the microdiversity technique helps mitigate short term fading effects. The diversity technique that reduces the shadow effects is called macrodiversity. Macrodiversity is implemented by combining signals received by several base stations or access points [8].

The maximal-ratio combining (MRC) is an optimal combining method [1, p. 262]. It gives the best system performance. The selection combining (SC) is a fast response hand-off mechanism that instantaneously or with minimal delay chooses the best base station [9]-[11]. The microdiversity and macrodiversity reception can be done by using the MRC and SC algorithms, respectively. If the distance between antennas in the base station is of the order of one half of the wavelength, the fading channels at micro level will be independent [8]. On the other hand, shadowing has a larger correlation distance and it is difficult to ensure that base stations operate independently, especially in microcellular systems.

The first order system characteristics (the outage probability and average bit error probability (ABEP)) of a wireless communication system with microdiversity and dual macrodiversity reception in composite Rician-gamma channel are analyzed in [7][12]. Because in some applications, such as in adaptive transmission [13], the outage probability and ABEP do not provide enough information for the overall system design, the system's second-order statistics must be determined to pointed out the correlation properties of the

fading channels and to provide a dynamic representation of the system's performance. Such characteristics are the level crossing rate (LCR) and the average fade duration (AFD). They are used for choosing the adaptive symbol rates, interleaver depth, packet length and time slot duration. Bandjur et al. [14] analyze the second-order statistics as continuation of a previous study of the first-order characteristics [7][10]. In this paper, it will be shown that a system with three microdiversity branches gives better performance than a wireless communication system with two L-branch MRC receivers at the micro level and a dual branch SC receiver at the macro level presented in [14]. The closed form expressions will be derived and shown graphically for some parameters.

This paper is organized as follows: in Section 2, we present the system and channel model. The expression for average Level Crossing Rate is derived in Section 3. In Section 4, numerical results are shown graphically and parameters influence is analyzed. We conclude in Section 5, which highlights the main contributions of this paper.

## II. SYSTEM AND CHANNEL MODEL

The model of the wireless communication system with three  $L$ -branches MRC receivers at microlevel and triple-branch SC receiver at macrolevel, analyzed in this paper, is shown in Fig. 1.

The MRC combiner's output signals from  $i$ -th ( $i=1,2,3$ ) base station are:

$$R_i = \sum_{j=1}^L r_{ij}^2$$

where  $r_{ij}$  is the envelope of the faded signal at the  $j$ -th diversity branch of the  $i$ -th base station.

If there exists a dominant line-of-sight (LoS) component in the propagation domain and the envelopes are statistically independent,  $R_i$  has the Rician distribution, which is given by [14, eq. (1)]:

$$f_{R_i}(R_i | \Omega_i) = \frac{K+1}{\Omega_i} \exp\left(-\frac{(K+1)R_i}{\Omega_i} - KL\right) \cdot \left(\frac{(K+1)R_i}{KL\Omega_i}\right)^{\frac{L-1}{2}} I_{L-1}\left(2\sqrt{\frac{KL(K+1)R_i}{\Omega_i}}\right), i=1,2,3 \quad (1)$$

where  $K$  is the Rician factor, which is defined as the ratio of the powers of the dominant signal component and the scattered signal components;  $\Omega_i$  represents the average power of the signal per base station branch;  $I_n(\cdot)$  is the modified Bessel function of the first kind and  $n$ -th order [15, eq. (8.445)];  $L$  is the number of branches at microlevel.

The influence of the number of diversity branches on the system performance was examined and it is established how much the value of the bit error rate (BER) is reduced with the change in the number of branches [16]-[18].

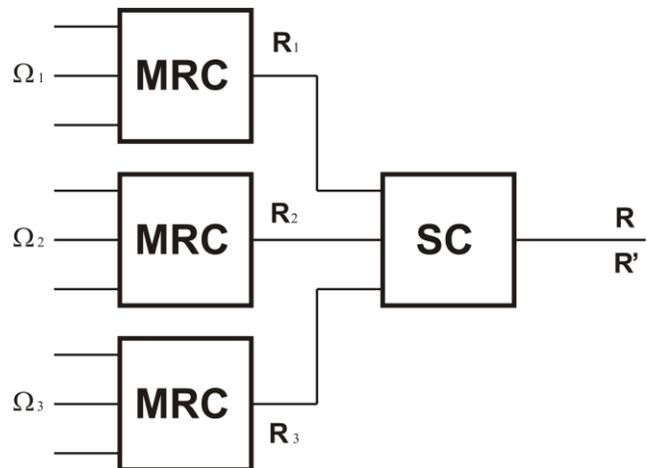


Figure 1. System model.

With the increase of the diversity order, the performance of the receiver is improved. However, a larger number of diversity branches reduces the additional gain and increases the complexity of the system.

Therefore, it is necessary to find a compromise between the performance of the system and its complexity. The increase in gain is getting smaller with increasing the number  $L$ . The power gain is the highest when the system diversity order increases from  $L=1$  to  $L=2$ ; the gain is getting less with increasing of  $L$  from  $L=2$  to  $L=3$ , and with further growth of  $L$  the gain is getting smaller. It can be noticed that the gain declines exponentially with the increase in the system diversity order [16, Fig. 2, Table I]. Also, the practical realization of the system becomes more complex and, at the same time, more expensive [16][17].

Hence, there is no need to significantly increase the number of diversity branches because the system performance improvement achieved with a few diversity branches would not increase further very much with the introduction of more branches.

So, the limitation for the number of diversity branches in the microdiversity system is the trade-off between the complexity of practical realization and requested performance improvement [18]. The obtained results in previous papers enable us to find a compromise between the efficiency (which is measured by the value of BER) and the complexity of the receiver (measured by the number of receiving antennas). Thereafter, we have chosen  $L$  to be equal to 3.

The PDF from (1) is conditional with respect to shadowing with  $\Omega_i$  being a random variable. In this paper,  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$  are correlated with each other and identically gamma distributed with the joint PDF given by [19]

$$f_{\Omega_1\Omega_2\Omega_3}(\Omega_1, \Omega_2, \Omega_3) = \frac{(\Omega_1\Omega_2\Omega_3)^{\frac{c-1}{2}}}{\rho^{c-1}(1-r)^2\Omega_0^{c+2}\Gamma(c)}$$

$$\cdot \exp\left(-\frac{\Omega_1 + \Omega_3 + (1+\rho)\Omega_2}{\Omega_0(1-\rho)}\right) \cdot I_{c-1}\left(\frac{2\sqrt{\rho}}{\Omega_0(1-\rho)}\sqrt{\Omega_1\Omega_2}\right) I_{c-1}\left(\frac{2\sqrt{\rho}}{\Omega_0(1-\rho)}\sqrt{\Omega_2\Omega_3}\right), \quad (2)$$

where  $\Omega_0$  is related to the average power of  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$ ; the correlation between  $\Omega_1$ ,  $\Omega_2$  and  $\Omega_3$  is defined by using the exponential correlation model, which is represented by the correlation matrix with dimensions of  $r \times s$ ,  $r=3$ ,  $s=3$ , with the elements of the matrix  $\rho_{rs} = \rho^{|r-s|}$ ,  $0 < \rho < 1$ , [20, eq. (13)];  $c$  is the order of Gamma distribution and  $\Gamma(\cdot)$  is the Gamma function [21, eq. (8.310/1)].

The variable  $c$  is a measure of shadowing severity; when the value of parameter  $c$  decreases, the shadowing increases. The relation between the parameters  $c$  and  $\sigma$  (standard deviation of shadowing in the log-normal shadowing) in dB is:

$$\sigma(\text{dB}) = 4.3429\sqrt{\Psi'(c)}$$

where  $\Psi'(c)$  is the trigamma function [21]. The typical values of  $\sigma$  are between 2 and 12 dB.

The joint PDF of the  $i$ -th base station output signal and its time derivative is [22]:

$$f_{R_i, \dot{R}_i}(R_i, \dot{R}_i | \Omega_i) = f_{R_i}(R_i | \Omega_i) f_{\dot{R}_i}(\dot{R}_i). \quad (3)$$

The derivative of  $R_i$  with respect to time is  $\dot{R}_i = \sum_{j=1}^L r_{ij} \dot{r}_{ij}$ , where  $\dot{r}_{ij}$  is the time derivative of  $r_{ij}$ . For isotropic scattering,  $\dot{r}_{ij}$  is a Gaussian distributed random variable with zero mean and variance:

$$\sigma_{\dot{r}_{ij}}^2 = \pi^2 f_m^2 \Omega_i / (K+1),$$

where  $f_m$  is defined as maximum Doppler frequency [23]. In that case,  $\dot{R}_i$  is also Gaussian distributed random variable with zero mean and variance:

$$\sigma_{\dot{R}_i}^2 = 4R_i \pi^2 f_m^2 \Omega_i / (K+1).$$

Then, the selection diversity is applied at macro level. Therefore, the base station with the largest total input average power is selected to provide service to the user. Then, the joint PDF of the overall output signal and its derivative, after diversity combining at both micro and macro levels, is:

$$\begin{aligned} f_{R, \dot{R}}(R, \dot{R}) &= \int_0^\infty d\Omega_1 \int_0^{\Omega_1} \int_0^{\Omega_1} f_{R_i, \dot{R}_i}(R, \dot{R} / \Omega_1) f_{\Omega_1, \Omega_2, \Omega_3}(\Omega_1, \Omega_2, \Omega_3) d\Omega_2 d\Omega_3 + \\ &+ \int_0^\infty d\Omega_2 \int_0^{\Omega_2} \int_0^{\Omega_2} f_{R_i, \dot{R}_i}(R, \dot{R} / \Omega_2) f_{\Omega_1, \Omega_2, \Omega_3}(\Omega_1, \Omega_2, \Omega_3) d\Omega_1 d\Omega_3 + \\ &+ \int_0^\infty d\Omega_3 \int_0^{\Omega_3} \int_0^{\Omega_3} f_{R_i, \dot{R}_i}(R, \dot{R} / \Omega_3) f_{\Omega_1, \Omega_2, \Omega_3}(\Omega_1, \Omega_2, \Omega_3) d\Omega_1 d\Omega_2 \quad (4) \end{aligned}$$

with  $R$  being macrodiversity SC receiver output signal and  $\dot{R}$  is the first derivative of  $R$ .

The analytical expression for the joint PDF of the signal and its derivative can be obtained if adequate joint PDFs are substituted and the integrations are done. It is given by:

$$\begin{aligned} f_{R, \dot{R}}(R, \dot{R}) &= \sum_{i=0}^\infty \sum_{j=0}^\infty \sum_{k=0}^\infty \sum_{l=0}^\infty \sum_{p=0}^\infty \frac{2\rho^{i+j}(1+\rho)^l \exp(-3K)}{f_m^i l! j! \Gamma(c) \Gamma(c) \Gamma(j+c)} \cdot \\ &\cdot \frac{3^{p+1} K^p (K+1)^{p+\frac{7}{2}} R^{p+\frac{3}{2}} (\Omega_0(1-\rho))^{-2i-2j-2c-k-l}}{\Gamma(p+3) \prod_{m=0}^k (i+c+m) \prod_{n=0}^l (i+j+c+n) \Omega_0^c (2\pi)^{3/2}} \cdot \\ &\cdot \left( \frac{\Omega_0(1-\rho)(K+1)(8\pi^2 f_m^2 R^2 + \dot{R}^2)}{8\pi^2 f_m^2 (3+\rho)} \right)^{\frac{2i+2j+3c+k+l-p-\frac{7}{2}}{2}} \cdot \\ &\cdot K^{2i+2j+3c+k+l-p-\frac{7}{2}} \left( 2\sqrt{\frac{(3+\rho)(K+1)(8\pi^2 f_m^2 R^2 + \dot{R}^2)}{\Omega_0(1-\rho)8\pi^2 f_m^2}} \right) \quad (5) \end{aligned}$$

### III. AVERAGE LEVEL CROSSING RATE

The average LCR at a specified signal level is defined as the rate (in crossings per second) at which the signal crosses the given level going towards the positive (or negative) direction. This value, for a given threshold, is given by [24]:

$$N_R(R) = \int_0^\infty \dot{R} f_{R, \dot{R}}(R, \dot{R}) d\dot{R}. \quad (6)$$

After introduction of the joint PDF of the signal and its derivative from (5) into (6) and the integration, the final infinite-series expression for the average LCR becomes:

$$\begin{aligned}
 N_R(R) = & \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \sum_{p=0}^{\infty} \frac{\sqrt{8\pi} f_m \rho^{i+j} (1+\rho)^l \exp(-3K)}{i! j! \Gamma(c) \Gamma(i+c) \Gamma(j+c)} \\
 & \cdot \frac{3^{p+1} K^p (K+1)^{\frac{2i+2j+3c+k+l+p+\frac{5}{2}}{2}} (3+\rho)^{\frac{2i+2j+3c+k+l-p-\frac{5}{2}}{2}}}{\Gamma(p+3) \prod_{m=0}^k (i+c+m) \prod_{n=0}^l (i+j+c+n) \Omega_0^c} \\
 & \cdot R^{\frac{2i+2j+3c+k+l+p+\frac{5}{2}}{2}} (\Omega_0(1-\rho))^{\frac{2i+2j+3c+k+l+p+\frac{5}{2}}{2}} \\
 & \cdot K_{2i+2j+3c+k+l-p-\frac{5}{2}} \left( 2 \sqrt{\frac{(3+\rho)(K+1)R}{\Omega_0(1-\rho)}} \right) \quad (7)
 \end{aligned}$$

where  $K_n(\cdot)$  is the  $n$ -th order modified Bessel function of the second kind.

The most important problem with the infinite-series of expressions is their convergence. The formulas for the average LCR converge rapidly, and they can be efficiently used in performance analysis. Table 1 shows the number of terms required to be summed to reach a four-significant-figure precision in the expression (7) for the normalized LCR.

TABLE I. THE NUMBER OF TERMS REQUIRED TO BE SUMMED TO REACH FOUR-SIGNIFICANT-FIGURE PRECISION

R	-20dB	0dB	20dB
$\rho=0.2$	11	12	28

The parameters used for calculation are:  $c=1.16$   $\sigma=5$  dB,  $K=2.1$  dB,  $\Omega_0=0$ dB,  $f_m=1$  Hz.

#### IV. NUMERICAL RESULTS

The numerical results are graphically presented based on the analytical process given in Section III. They show the second-order statistical characteristics for various systems parameters.

Fig. 2 and Fig. 3 depict the normalized average LCR ( $N_R/f_m$ ) versus normalized signal level ( $R_{th}=R/\Omega_0$ ). It is obvious that LCR increases as the value of normalized signal increases, until it reaches the maximum, for  $R_{th}=R_{th0}$ . Then, it starts to decrease.

The influence of fading severity on normalized level crossing rate, versus normalized signal level, is presented in Fig. 2. One can see from Fig. 2 that the normalized level crossing rate decreases with increasing of Rician factor  $k$ . The system performance is better for greater values of Rician factor. This effect is more pronounced for lower values of the signal.

The influence of shadowing severity and correlation coefficient to the normalized level crossing rate is presented in Fig. 3.

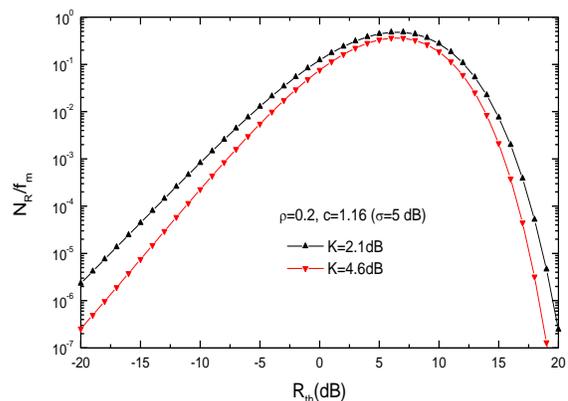


Figure 2. Normalized average LCR versus normalized signal level  $R_{th}$  for different values of Rician factor  $K$ .

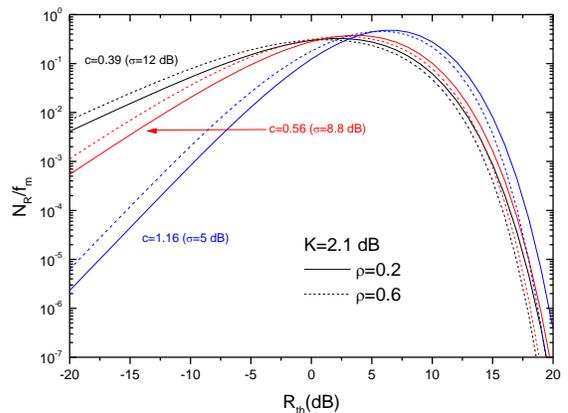


Figure 3. Normalized average LCR versus normalized signal level  $R_{th}$ , for different values of shadowing severity and correlation coefficient.

It can be seen from Fig. 3 that the normalized LCR is higher for smaller values of signal under higher shadowing, while for bigger values of the signal, the normalized LCR decreases as shadowing severity increases.

It is also visible from Fig. 3 that the influence of the correlation coefficient on the normalized LCR is more significant for small signal values. The signal changes are faster for smaller distances between the base stations, i.e., when the correlation is bigger ( $\rho=0.6$ ).

#### V. CONCLUSION

The received signal in wireless systems with macrodiversity and three branches microdiversity reception in Gamma shadowed Rician fading channel is analyzed in this paper. The numerical results for the average level crossing rate are presented graphically to illustrate the effects of the severity of fading and shadowing and correlation between base stations on the system performance. It is shown that a system with three microdiversity branches

has better performance than the dual diversity system described earlier, under the same fading conditions.

The expressions obtained here can be used for designing of wireless communication systems and optimization of system parameters in different propagation environments.

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#### REFERENCES

- [1] M. K. Simon and M. S. Alouini, *Digital Communication over Fading Channels*, A Wiley-Interscience Publication, John Wiley & Sons, Inc. New York – Chichester – Weinheim - Brisbane - Singapore – Toronto; Print ISBN 0-471-31779-9, Electronic ISBN 0-471-20069-7
- [2] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2005.
- [3] J. Zhang and V. Aalo, "Effect of Macrodiversity on Average-Error Probabilities in a Rician Fading Channel with Correlated Lognormal Shadowing", *IEEE Trans. Commun.*, vol. 49, no.1, 2001, pp. 14-18.
- [4] E. K. Al-Hussaini, A. M. Al-Bassiouni, H. M. Mourad, and H. Al-Shennawy, "Composite Macroscopic and Microscopic Diversity of Sectorized Macrocellular and Microcellular Mobile Radio Systems Employing RAKE Receiver over Nakagami Fading Plus Lognormal Shadowing Channel", *Wireless Personal Commun.*, vol. 21, no.3, 2002, pp 309-328.
- [5] A.S. Panajotovic, M.C. Stefanovic, and D.Lj. Draca, "Effect of microdiversity and macrodiversity on average bit error probability in shadowed fading channels in the presence of interference", *ETRI Journal*, vol. 31, Oct. 2009, pp. 500-505.
- [6] I. Kostic, "Analytical approach to performance analysis for channel subject to shadowing and fading", *IEEE Proc. Comm.*, vol. 152, 2005, pp. 821-827.
- [7] N. Sekulovic and M. Stefanovic, "Performance analysis of system with micro- and macrodiversity reception in correlated gamma shadowed Rician fading channels", *Wireless Personal Communications*, (publisher: Springer), vol. 65, no. 1, 2012, pp. 143-156, published online 12.Feb.2011, doi: 10.1007/s11277-011-0232-8
- [8] A. Goldsmith, *Wireless Communications*, Cambridge University Press: New York, 2005.
- [9] P. M. Shankar, "Analysis of microdiversity and dual channel macrodiversity in shadowed fading channels using a compound fading model", *Int. J. Electron Comm. AEÜ*, vol. 62, 2008, pp 445-449.
- [10] N. M. Sekulovic, M. C. Stefanovic, D. M. Milovic, and S. Z. Stanojic, "Second-order statistics of system with  $N$ -branch microdiversity and  $L$ -branch macrodiversity operating over gamma shadowed Nakagami- $m$  fading channels", *Int. J. Communication Systems* 27(2), 2014, pp. 390-400.
- [11] V. Milenković, N. Sekulović, M. Stefanović, and M. Petrović, "Effect of microdiversity and macrodiversity on average bit error probability in gamma shadowed Rician fading channels", *ETRI Journal*, vol. 32, no.3, June 2010, pp. 464-467.
- [12] N. M. Sekulovic and M. C. Stefanovic, "Performance Analysis of System with Micro- and Macrodiversity Reception in Correlated Gamma Shadowed Rician Fading Channels", *Wireless Personal Communications* 65(1), 2012, pp. 143-156.
- [13] L. Yang and M. S. Alouini, "Average outage duration of wireless communication systems", *Wireless communications systems and networks*, Springer US, 2004.
- [14] M. Bandjur, N. Sekulovic, M. Stefanovic, A. Golubovic, P. Spalevic, and D. Milic, "Second-Order Statistics of System with Microdiversity and Macrodiversity Reception in Gamma-Shadowed Rician Fading Channels", *ETRI Journal*, Vol. 35, Nr 4, Aug. 2013, pp. 722-725.
- [15] P. M. Shankar, "Performance Analysis of Diversity Combining Algorithms in Shadowed Fading Channels", *Wireless Personal Commun.*, vol. 37, 2006, pp. 61-72.
- [16] Z. J. Mitrović, B. Z. Nikolić, G. T. Đorđević, and M. Č. Stefanović, "Influence of Imperfect Carrier Signal Recovery on Performance of SC Receiver of BPSK Signals Transmitted over  $\alpha$ - $\mu$  Fading Channel", *Electronics*, vol. 13, No. 1, June 2009, pp. 58-62.
- [17] Z. Boudida, M. Abdallah, K. A. Qaraqe, and M.-S. Alouini, "Performance Analysis of Multi-Branch Switched Diversity Schemes for Spectrum Sharing Systems", *IEEE Trans. on Commun.*, TCOMM.2012.081512.110520, online Dec. 2012.
- [18] M. Stefanovic, B. Miric, P. Spalevic, and S. Panic "Second Order Statistic Analysis of Selection Macro-Diversity Combining over Gama Shadowed Rayleigh Fading Channels", *Scientific Publications of the State University of Novi Pazar, Ser. A: Appl. Math. Inform. and Mech.*, vol. 1, 1, 2009, pp. 1-9.
- [19] E. Xekalaki, J. Panaretos, and S. Psarakis, *A Predictive Model Evaluation and Selection Approach – The Correlated Gamma Ratio Distribution*, In J. Panaretos (Ed.) *Stochastic Musings: Perspectives from the Pioneers of the Late 20<sup>th</sup> Century*, Psychology Press, USA, 2003.
- [20] V. A. Aalo, "Performance of Maximal-Ratio Diversity Systems in a Correlated Nakagami-Fading Environment", *IEEE Trans. on Commun.*, vol. 43, No. 8, Aug. 1995, pp. 2360-2369.
- [21] I. S. Gradshteyn and I. M. Ryzhnik, *Table of Integrals, Series and Products*, Academic Press, 7<sup>th</sup> ed. 2007.
- [22] J. D. Parsons, *The Mobile Radio Propagation Channel*, 2nd ed., England: John Wiley & Sons, Inc., 2000.
- [23] X. Dong and N.C. Beaulieu, "Average Level Crossing Rate and Average Fade Duration of Selection Diversity", *IEEE Commun. Lett.*, vol. 5, no. 10, Oct. 2001, pp. 396-398.
- [24] S. O. Rice, "Statistical Properties of a Sine Wave Plus Noise", *Bell Syst. Tech. J.*, vol. 27, Jan. 1948, pp. 109-157.