Spectral Resource Sharing for Uncoordinated Networks

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***Abstract*—Spectrum resource sharing strategies for uncoordinated networks are investigated. The average capacity of 2-pair device-to-device (D2D) communication systems is derived, assuming a flat response of the frequency spectrum. Three different frequency allocation strategies are considered: (i) separate spectral allocation; (ii) full spectral allocation; and (iii) overlapped spectral allocation, considering deterministic and random locations of the communication devices. The analytical results, supported by simulations, show that at low to moderate received SNR the throughput assigned to each communication device can be enhanced by overlapping the frequency spectrum allocations.**

***Index Terms*—Uncoordinated networks, frequency allocation strategies, D2D communication.**

I. INTRODUCTION

Recently, device-to-device (D2D) communication has attracted more attention for its promising results in enhancing the system capacity [1]. With the increasing demand in establishing home networks, which connect a variety of communication devices including mobile phones, laptops, or other electronic appliances, D2D communication becomes a common paradigm for establishing these connections [2]. D2D communication schemes require low power consumption, cost, and human intervention [3]. Many studies have been performed to investigate the capacity of specific uncoordinated networks, i.e., cellular networks [4]. However, the results of these investigations cannot be generalized to other types of communication systems. Therefore, in this paper, we study the capacity of uncoordinated networks using physical parameters independent of the system specifications, such as the locations of communication devices, the number of devices which can co-exist in a certain region, and the environment.

We focus our study on 2-pair D2D communication systems considering three different spectral allocation strategies: separated, whole and overlapped spectral allocations. For tractability, we consider channels with a flat frequency response.

First, in Section II, we present the sum capacity of 2-pair D2D systems with deterministic locations. Then, in Section III, we derive the average sum capacity considering random locations of the communication devices. The results are introduced in Section IV. Finally, Section V concludes the paper.

II. SUM CAPACITY OF 2-PAIR D2D SYSTEMS WITH DETERMINISTIC LOCATIONS

In this section, we present the sum capacity of a circular cell in which two pairs of transmitters, (i.e., Tx_1 and Tx_2) and receivers (i.e., Rx_1 and Rx_2) are located in fixed positions inside the cell as shown in Fig. 1. In this figure, the solid lines represent the desired signals from the transmitters to the desired destinations (i.e., Tx_1-Rx_1 and Tx_2-Rx_2), while interfering signals (i.e., Tx_1-Rx_2 and Tx_2-Rx_1) are denoted by dotted lines.

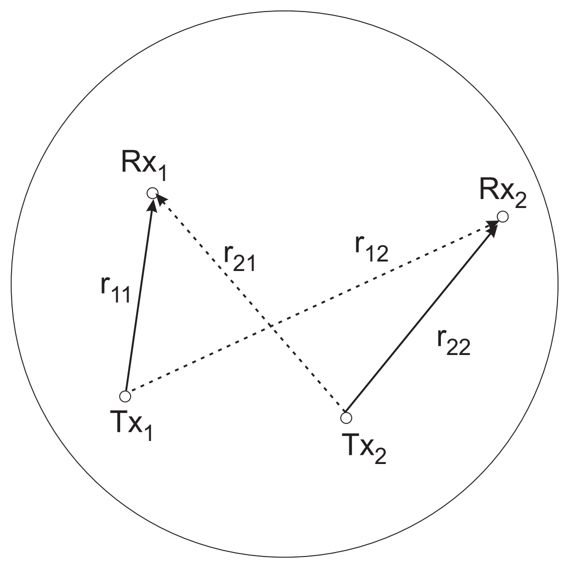

Fig. 1. Positions of communication devices inside a cell.

For simplicity, the channels of the desired and interfering communication links are modeled using the single-slope path loss, where the received power at a distance r from the transmitter is given by

$$P_r(r) \propto P_t r^{-\alpha} \tag{1}$$

where P_t is the transmit power spectral density, and α is the path loss exponent. We assume that P_t is the same for all links, and $\alpha = 4$. To simplify our analysis, we neglect the coefficient of proportionality and consider the effective transmit power density P_t such that (1) contains an equality.

Assuming that the frequency spectrum under investigation has a flat response over different frequencies, we use an

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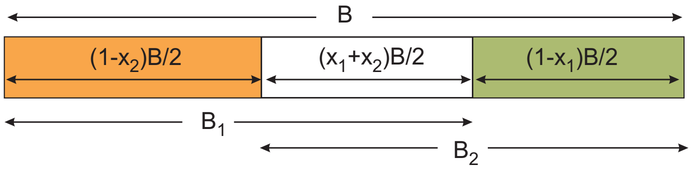

Fig. 2. Overlapped frequency spectrum allocations for 2-pair D2D communication systems.

overlapped spectral allocation strategy, where the bandwidth assigned to each transmitter is extended with a specific ratio over the neighboring frequency bands as shown in Fig. 2. The parameters x_1 and x_2 represent the spectral overlapping ratios of Tx_1 on Tx_2 and Tx_2 on Tx_1, respectively.

In this case, the average capacity of the 2-pair D2D communication systems can be defined by

$$C_T = C_{F1} + C_{F2} + C_{O1} + C_{O2} \tag{2}$$

where C_{F1} and C_{F2} are the capacity of Tx_1 and Tx_2 in the region of the spectrum which is free of interference, respectively. The components C_{O1} and C_{O2} are the capacity of Tx_1 and Tx_2 in the overlapped spectral region between the two transmitters, respectively. The expressions corresponding to these capacities are

$$C_{F1} = (1 - x_2)\frac{B}{2}\log_2\left(1 + \frac{\frac{P_t}{r_{11}^{\alpha}}}{N}\right) \tag{3}$$

$$C_{F2} = (1 - x_1)\frac{B}{2}\log_2\left(1 + \frac{\frac{P_t}{r_{22}^{\alpha}}}{N}\right) \tag{4}$$

$$C_{O1} = (x_2 + x_1)\frac{B}{2}\log_2\left(1 + \frac{\frac{P_t}{r_{11}^{\alpha}}}{\frac{P_t}{r_{21}^{\alpha}} + N}\right) \tag{5}$$

$$C_{O2} = (x_2 + x_1)\frac{B}{2}\log_2\left(1 + \frac{\frac{P_t}{r_{22}^{\alpha}}}{\frac{P_t}{r_{12}^{\alpha}} + N}\right) \tag{6}$$

where B is the total frequency bandwidth, and N is the noise power spectral density. The distance $\{r_{ij}\}, i, j \in \{1, 2\}$, represents the length of the communication link between $\mathrm{Tx_i}$ and $\mathrm{Rx_j}$. Also, $\{r_{ij}\}$ takes values between r_m and $2R$, where r_m is the minimum distance between two communication devices inside the cell, and R is the cell radius.

III. Sum Capacity of 2-Pair D2D Systems with Random Locations

Here, we assume that Tx_1, Tx_2, Rx_1 and Rx_2 are located in random positions inside the cell. In this case, r_{11}, r_{21}, r_{12}, and r_{22} become variable parameters, which have independent identical distributions defined as follows. Let r represent the Euclidian distance between two points randomly located in a circle of radius R. Then, the probability distribution function of r is given by [5]

$$f(r) = \frac{2r}{R^2}\left(\frac{2}{\pi}\cos^{-1}\left(\frac{r}{2R}\right) - \frac{r}{\pi R}\sqrt{1 - \frac{r^2}{4R^2}}\right) \tag{7}$$

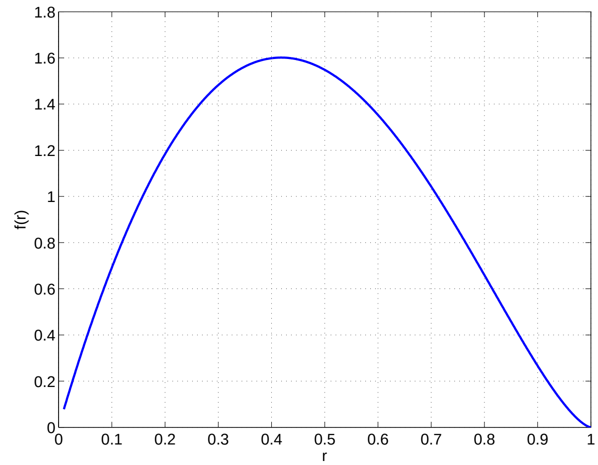

Fig. 3. Distribution of r inside a unit circle ($R = 0.5$).

which is plotted in Fig. 3. The validity of this probability distribution is investigated by Moltchanov in [5].

Using the closed-form expression of $f(r)$ in (7), we can estimate the average sum capacity of 2-pair D2D communications over a flat frequency response as follows

$$\bar{C}_T = \bar{C}_{F1} + \bar{C}_{O1} + \bar{C}_{O2} + \bar{C}_{F2} \tag{8}$$

where

$$\bar{C}_{F1} = \int_{r_m}^{2R} C_{F1}(r_{11})f(r_{11})dr_{11} \tag{9}$$

$$\bar{C}_{F2} = \int_{r_m}^{2R} C_{F2}(r_{22})f(r_{22})dr_{22} \tag{10}$$

$$\bar{C}_{O1} = \int_{r_m}^{2R}\int_{r_m}^{2R} C_{O1}(r_{11}, r_{21})f(r_{11}, r_{21})dr_{11}dr_{21} \tag{11}$$

$$\bar{C}_{O2} = \int_{r_m}^{2R}\int_{r_m}^{2R} C_{O2}(r_{12}r_{22})f(r_{12}, r_{22})dr_{12}dr_{22}. \tag{12}$$

In (9) and (10), the closed-form expression of $\bar{C}_{F1}$ and $\bar{C}_{F2}$ can be obtained by deriving the average capacity of 1-pair D2D communication systems over bandwidths $(1 - x_2)\frac{B}{2}$ and $(1 - x_1)\frac{B}{2}$, respectively. The closed-form expressions of $\bar{C}_{O1}$ and $\bar{C}_{O2}$ can be obtained by deriving the average capacity of 1-pair D2D communication systems with one interferer over a bandwidth $(x_1 + x_2)\frac{B}{2}$. In the following subsections, we will show how the average capacity of the aforementioned cases can be estimated.

A. Average capacity of 1-pair D2D communications with no interference

This case is represented by (9) or (10), where C_{F1} is a function only of r_{11}, or C_{F2} is a function only of r_{22}. First, consider the derivation of $\bar{C}_{F1}$. Using (7), the average capacity of 1-pair D2D communication systems over a frequency bandwidth, $\tilde{B} = (1 - x_2)\frac{B}{2}$, is given by

$$\bar{C}_{F1} = \tilde{B}\left(\frac{4}{\pi R^2}\bar{c}_1 - \frac{2}{\pi R^3}\bar{c}_2\right) \tag{13}$$

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where

$$\bar{c}_1 = \int_{r_m}^{2R} \log_2\left(1+\gamma_r\right) r_{11} \cos^{-1}\left(\frac{r_{11}}{2R}\right) dr_{11} \tag{14}$$

$$\bar{c}_2 = \int_{r_m}^{2R} \log_2\left(1+\gamma_r\right) r_{11}^2 \sqrt{1-\frac{r_{11}^2}{4R^2}} dr \tag{15}$$

where γ_r is the received signal-to-noise ratio (SNR) and given by $\gamma_r = \frac{\gamma_t}{r_{11}^{\alpha}}$, where $\gamma_t = \frac{P_t}{N}$. The mathematical derivations of the integrations in (14) and (15) are not tractable; therefore, we use the series expansions of the following mathematical functions to reduce the complexity of these integrations [6].

For $|\gamma_r| \le 1$, the logarithmic function $\log_2(1+\gamma_r)$ can be expressed as

$$\log_2\left(1+\gamma_r\right) = \frac{1}{\ln(2)}\left(\gamma_r - \frac{\gamma_r^2}{2} + \frac{\gamma_r^3}{3} + \dots\right) \tag{16a}$$

whereas for $|\gamma_r| > 1$,

$$\log_2\left(1+\gamma_r\right) = \frac{1}{\ln(2)}\left(\log(\gamma_r) + \frac{1}{\gamma_r} - \frac{2}{2\gamma_r^2} + \frac{1}{3\gamma_r^3} - \dots\right) \tag{16b}$$

where ln is the natural logarithm. Also, for $\left|\frac{r}{2R} < 1\right|$,

$$\sqrt{1-\frac{r_{11}^2}{4R^2}} = 1 - \frac{r_{11}^2}{8R^2} - \frac{r_{11}^4}{128R^4} - \dots \tag{17}$$

For $|\gamma_r| \le 1$, we substitute for $\log_2(1+\gamma_r)$ using the first three terms of the series expansion in (16a) since the values of the remaining terms are small and can be neglected. Accordingly, $\bar{c}_1$ can be approximated using

$$\bar{c}_1 \approx \frac{1}{\ln(2)}\left(\bar{c}_{a1} - \frac{\bar{c}_{a2}}{2} + \frac{\bar{c}_{a3}}{3}\right) \quad \forall |\gamma_r| \le 1 \tag{18}$$

where

$$\bar{c}_{a1} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{\gamma_t}{r_{11}^{\alpha-1}} dr_{11} \tag{19}$$

$$\bar{c}_{a2} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{\gamma_t^2}{r_{11}^{2\alpha-1}} dr_{11} \tag{20}$$

$$\bar{c}_{a3} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{\gamma_t^3}{r_{11}^{3\alpha-1}} dr_{11}. \tag{21}$$

When $|\gamma_r| > 1$, we also include up to the third-order term, i.e., the first four terms of the series expansion in (16b), and consequently $\bar{c}_1$ can be given by

$$\bar{c}_1 \approx \frac{1}{\ln(2)}\left(\bar{c}_{b1} + \bar{c}_{b2} - \frac{\bar{c}_{b3}}{2} + \frac{\bar{c}_{b4}}{3}\right) \tag{22}$$

where

$$\bar{c}_{b1} = \int_{r_m}^{2R} r_{11} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \log\left(\frac{\gamma_t}{r_{11}^{\alpha}}\right) dr_{11} \tag{23}$$

$$\bar{c}_{b2} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{r_{11}^{\alpha+1}}{\gamma_t} dr_{11} \tag{24}$$

$$\bar{c}_{b3} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{r_{11}^{2\alpha+1}}{2\gamma_t^2} dr_{11} \tag{25}$$

$$\bar{c}_{b4} = \int_{r_m}^{2R} \cos^{-1}\left(\frac{r_{11}}{2R}\right) \frac{r_{11}^{3\alpha+1}}{3\gamma_t^3} dr_{11}. \tag{26}$$

Finally, we estimate $\bar{c}_2$ using the series expansion in (17) as follows

$$\bar{c}_2 \approx \bar{c}_{e1} - \frac{1}{8R^2}\bar{c}_{e2} - \frac{1}{128R^4}\bar{c}_{e3} \tag{27}$$

where

$$\bar{c}_{e1} = \int_{r_m}^{2R} \log_2\left(1+\frac{\gamma_t}{r_{11}^{\alpha}}\right) r_{11}^2 dr_{11} \tag{28}$$

$$\bar{c}_{e2} = \int_{r_m}^{2R} \log_2\left(1+\frac{\gamma_t}{r_{11}^{\alpha}}\right) r_{11}^4 dr_{11} \tag{29}$$

$$\bar{c}_{e3} = \int_{r_m}^{2R} \log_2\left(1+\frac{\gamma_t}{r_{11}^{\alpha}}\right) r_{11}^6 dr_{11}. \tag{30}$$

The closed-form expression of the integrations in (19)-(21), (23)-(26) and (28)-(30) can be obtained from handbooks of integrations; however, due to the space limitation of the paper, we refer the reader to [6]. Similarly, we use the same mathematical derivations to get $\bar{C}_{F2}$ over bandwidth $(1-x_1)\frac{B}{2}$ from (10).

B. Average capacity of one-pair D2D communications with one interferer

Now, we assume that there is an interfering signal to the desired communication link from a randomly located communication device. The average capacity of this case is presented in (11) and (12). In the following, we first solve the double integration in (11), and then we use the same procedure to evaluate the integrations in (12). Assuming that the locations of Tx_1 and Tx_2 inside the cell are statistically independent, i.e., $f(r_{11}, r_{21}) = f(r_{11})f(r_{22})$, (11) is reduced to

$$\bar{C}_{O1} = \tilde{B}_1 \int_{r_m}^{2R} \bar{\Phi}(r_{11}) f(r_{11}) dr_{11} \tag{31}$$

where $\tilde{B}_1 = (x_1 + x_2)\frac{B}{2}$, and

$$\begin{aligned} \bar{\Phi}(r_{11}) &= \int_{r_m}^{2R} \log_2\left(1+\frac{r_{21}^{\alpha}}{r_{11}^{\alpha}(1+\gamma_t^{-1}r_{21}^{\alpha})}\right) f(r_{21}) dr_{21} \\ &= \left(\frac{4}{\pi R^2}\bar{\Phi}_1(r_{11}) - \frac{2}{\pi R^3}\bar{\Phi}_2(r_{11})\right). \end{aligned} \tag{32}$$

In (32),

$$\begin{aligned} \bar{\Phi}_1(r_{11}) = \int_{r_m}^{2R} & \log_2\left(1+\frac{r_{21}^{\alpha}}{r_{11}^{\alpha}(1+\gamma_t^{-1}r_{21}^{\alpha})}\right) r_{21} \\ & \times \cos^{-1}\left(\frac{r_{21}}{2R}\right) dr_{21} \end{aligned} \tag{33}$$

$$\begin{aligned} \bar{\Phi}_2(r_{11}) = \int_{r_m}^{2R} & \log_2\left(1+\frac{r_{21}^{\alpha}}{r_{11}^{\alpha}(1+\gamma_t^{-1}r_{21}^{\alpha})}\right) r_{21}^2 \\ & \times \sqrt{1-\frac{r_{21}^2}{4R^2}} dr_{21}. \end{aligned} \tag{34}$$

In (33), the integration can be simplified by substituting for $\cos^{-1}(\theta)$ by the first two terms of its series expansion, which is given by

$$\cos^{-1}(\theta) = \frac{\pi}{2} - \theta - \dots \tag{35}$$

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Also, we evaluate these integrations for $\alpha = 4$. Thus, the integrations in (33) can be approximated by

$$\bar{\Phi}_1(r_{11}) \approx \frac{\pi}{2}\bar{\Phi}_{11}(r_{11}) - \frac{1}{2R}\bar{\Phi}_{12}(r_{11}) \quad (36)$$

where

$$\begin{aligned}\bar{\Phi}_{11}(r_{11}) &= \int_{r_m}^{2R} r_{21}\log_2\left(1+\frac{r_{21}^4}{r_{11}^4(1+\gamma_t^{-1}r_{21}^4)}\right)dr_{21}\\
&= \left[\frac{r_{21}^2}{2}\log\left(\frac{r_{21}^4}{r_{11}^4(\gamma_t^{-1}r_{21}^4+1)}+1\right) - \frac{r_{11}^2}{\sqrt{\gamma_t^{-1}r_{11}^4+1}}\right.\\
&\times\tan^{-1}\left(1-\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right) - \frac{r_{11}^2}{\sqrt{\gamma_t^{-1}r_{11}^4+1}}\\
&\times\tan^{-1}\left(1+\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right) + \frac{1}{\gamma_t^{-\frac{1}{2}}}\\
&\left.\times\left(\tan^{-1}\left(1-\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}\right) + \tan^{-1}\left(1+\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}\right)\right)\right]_{r_m}^{2R}\end{aligned} \quad (37)$$

and

$$\begin{aligned}\bar{\Phi}_{12}(r_{11}) &= \int_{r_m}^{2R} r_{21}^2\log_2\left(1+\frac{r_{21}^4}{r_{11}^4(1+\gamma_t^{-1}r_{21}^4)}\right)dr_{21}\\
&= \frac{1}{6\sqrt{2}}\left[-\frac{\sqrt{2}}{\gamma_t^{-\frac{3}{4}}}\log\left(\gamma_t^{-\frac{1}{2}}r_{21}^2-\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}+1\right)+\frac{\sqrt{2}}{\gamma_t^{-\frac{3}{4}}}\right.\\
&\times\log\left(\gamma_t^{-\frac{1}{2}}r_{21}^2+\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}+1\right)+\frac{2\sqrt{2}}{\gamma_t^{-\frac{3}{4}}}\\
&\times\tan^{-1}\left(1-\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}\right)-\frac{2\sqrt{2}}{\gamma_t^{-\frac{3}{4}}}\tan^{-1}\left(1+\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}\right)\\
&+2r_{21}^3\log\left(\frac{r_{21}^4}{(r_{11}^4(\gamma_t^{-1}r_{21}^4+1))}+1\right)-\frac{2\sqrt{2}r_{11}^3}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{3}{4}}}\\
&\times\tan^{-1}\left(1-\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right)+\frac{2\sqrt{2}r_{11}^3}{(\gamma_t^{-1}r_{11}^4+1)^{3/4}}\\
&\times\tan^{-1}\left(1+\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right)+\frac{\sqrt{2}r_{11}^3}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{3}{4}}}\\
&\log\left(r_{21}^2\sqrt{\gamma_t^{-1}r_{11}^4+1}-\sqrt{2}r_{11}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}+r_{11}^2\right)\\
&-\frac{\sqrt{2}r_{11}^3}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{3}{4}}}\log\left(r_{21}^2\sqrt{\gamma_t^{-1}r_{11}^4+1}+\sqrt{2}r_{11}r_{21}\right.\\
&\left.\left.\times(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}+r_{11}^2\right)\right]_{r_m}^{2R}.\end{aligned} \quad (38)$$

The integrations of (37) and (38) can be evaluated with the aid of a handbook of integrations, e.g., [6]. Using the first two terms of the expansion in (17) and considering $\alpha = 4$, the integration in (34) can be approximated as

$$\bar{\Phi}_2(r_{11}) \approx \bar{\Phi}_{21}(r_{11}) - \frac{1}{8R^2}\bar{\Phi}_{22}(r_{11}) \quad (39)$$

where $\bar{\Phi}_{21}(r_{11}) = \bar{\Phi}_{12}(r_{11})$, given by (38) and

$$\begin{aligned}\Phi_{22}(r_{11}) &= \int_{r_m}^{2R} r_{21}^4\log_2\left(1+\frac{r_{21}^4}{r_{11}^4(1+\gamma_t^{-1}r_{22}^4)}\right)dr_{21}\\
&= \frac{1}{10\log(2)}\left[-\frac{\sqrt{2}}{\gamma_t^{-\frac{5}{4}}}\log\left(\gamma_t^{-\frac{1}{2}}r_{21}^2-\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}+1\right)+\frac{\sqrt{2}}{\gamma_t^{-\frac{5}{4}}}\right.\\
&\times\log\left(\gamma_t^{-\frac{1}{2}}r_{21}^2+\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}+1\right)-\frac{2\sqrt{2}}{\gamma_t^{-\frac{5}{4}}}\tan^{-1}\left(1-\sqrt{2}\gamma_t^{-1/4}\right.\\
&\left.\times r_{21}\right)+\frac{2\sqrt{2}}{\gamma_t^{-\frac{5}{4}}}\tan^{-1}\left(1+\sqrt{2}\gamma_t^{-\frac{1}{4}}r_{21}\right)-\frac{8r_{21}}{\gamma_t^{-2}r_{11}^4+\gamma_t^{-1}}\\
&+2r_{21}^5\log\left(\frac{r_{21}^4}{r_{11}^4(\gamma_t^{-1}r_{21}^4+1)}+1\right)+\frac{2^{\frac{3}{2}}r_{11}^5}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{5}{4}}}\\
&\times\tan^{-1}\left(1-\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right)-\frac{2^{\frac{3}{2}}r_{11}^5}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{5}{4}}}\\
&\times\tan^{-1}\left(1+\frac{\sqrt{2}r_{21}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}}{r_{11}}\right)+\frac{\sqrt{2}r_{11}^5}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{5}{4}}}\\
&\times\log\left(r_{21}^2\sqrt{\gamma_t^{-1}r_{11}^4+1}-\sqrt{2}r_{21}r_{11}(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}+r_{11}^2\right)\\
&-\frac{\sqrt{2}r_{11}^5}{(\gamma_t^{-1}r_{11}^4+1)^{\frac{5}{4}}}\log\left(r_{21}^2\sqrt{\gamma_t^{-1}r_{11}^4+1}+\sqrt{2}r_{21}r_{11}\right.\\
&\left.\left.\times(\gamma_t^{-1}r_{11}^4+1)^{\frac{1}{4}}+r_{11}^2\right)\right]_{r_m}^{2R}.\end{aligned} \quad (40)$$

The integration in (40) is obtained using [6]. Substituting (36) and (39) in (32), we derive the closed-form expression of $\bar{\Phi}(r_{11})$, and subsequently, we can perform the second integration in (31) with respect to r_{11}. However, due to the high complexity of the resultant integrations, we obtain the final estimate of $\bar{C}_{O1}$ using numerical integration techniques, specifically, we used the adaptive Gauss-Kronrod quadrature method [7].

IV. SIMULATION RESULTS

In this section, we examine the accuracy of the derived expression of the average sum capacity in (8) of 2-pair D2D systems considering different spectral allocation strategies. For illustrative purposes, we perform our simulations using design parameters similar to those of GSM systems: carrier frequency f_c = 2GHz, bandwidth B = 200kHz and cell with radius R = 5km. The value of r_m, which is the minimum distance between two devices inside the cell to initiate transmission, is taken as 1% of the cell diameter, i.e., $r_m = 0.02R$. In other words, the two devises are not allowed to communicate with each other when $r < r_m$. As noted previously, the channel between different communication devices is modeled using the single-slope path loss with $\alpha = 4$. In our results, we present the average capacity versus the received SNR at the median distance r_e between $\mathrm{Tx_i}$ and $\mathrm{Rx_i}$, which is denoted as γ_m. The value of r_e can be estimated from (7) or can be detected from Fig. 3. With this value of γ_m, the transmit power density,

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P_t, can be determined from

$$P_t = N r_e^{\alpha} \gamma_m. \tag{41}$$

In the following results, the average sum capacity $\bar{C}_T$ is estimated by averaging (2) over 10^5 realizations of $\{r_{ij}\},\ i,j \in \{1,2\}$.

The average capacity of 1-pair D2D systems is shown in Fig. 4 for two cases: (i) no interferer (0-Interferer); and (ii) 1-Interferer. The analytical results, supported by the simulations, show that the average capacity of 1-pair systems can be enhanced by increasing P_t; however, this enhancement becomes limited when an interferer is present. The figure also shows the accuracy of the analytical derivation of the scenario of 0-Interferer. Even for the 1-Interferer case, the error in the derivations at high values of γ_m is less than 23%, which can be reduced by adding more terms from the series expansions in (17) and (35), but this requires additional mathematical computations.

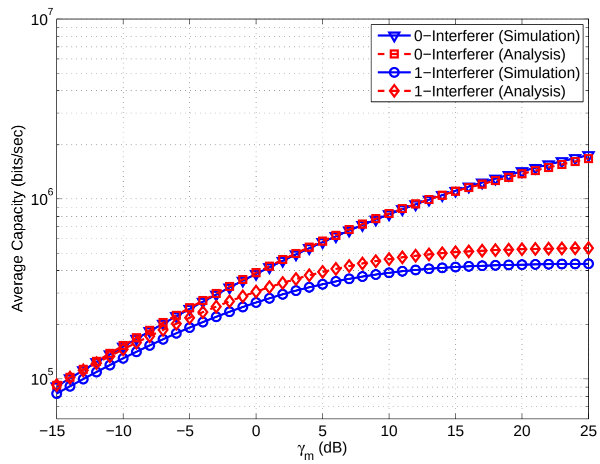

Fig. 4. Average capacity of 1-pair D2D systems with (i) 0-Interferer, (ii) 1-Interferer (R=5km and B=200kHz).

The average sum capacity, i.e., $\bar{C}_T$, of different frequency allocation strategies are shown in Fig. 5. The values of the overlapping ratios, i.e., x_1 and x_2, vary from 0 to 1. The case of $x_1 = x_2 = 0$ represents the separated spectral allocation, while the values $x_1 = x_2 = 1$ refer to the fully overlapped spectral allocation. The results show that at low to moderate values of γ_m, $\bar{C}_T$ can be enhanced by overlapping the frequency bands assigned to each pair of communication devices, which means that a fully-overlapped spectral allocation can be the preferred strategy at low received SNR values. At high values of γ_m, the interference power increases and thus limits the capacity enhancement provided by the overlapped spectral allocation.

V. Conclusion

We derived the average capacity of 1-pair and 2-pair D2D communication systems assuming deterministic and random locations of the transmitting and receiving devices. We found

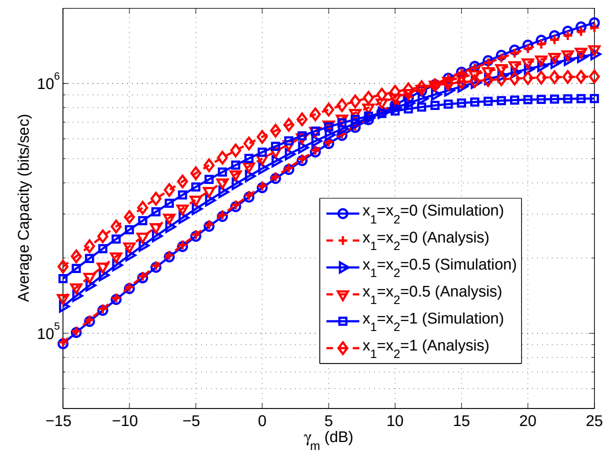

Fig. 5. Average capacity of 2-pair D2D systems (R=5km and B=200kHz).

that at low and moderate received SNR values, the average system capacity can be enhanced by allowing an overlapping between the frequency spectrum allocations. As future work, it would be interesting to find the optimal overlapping ratios, x_1^{opt} and x_2^{opt}, which maximize the average throughput assigned to each user considering the limits on the transmit power and the number of users which can co-exist in the area under investigation.

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