# Cramer-Rao Lower Bound on RF Pattern Matching Method with Velocity in LTE System

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***Abstract*—The Radio Frequency Pattern Matching (RFPM) method is an useful solution for UE positioning, and it is not sensitive to the channel states and multipath impact compared with the time-of-arrival schemes. With the help of drive test or other estimation algorithms, the RF pattern database can be constructed efficiently. The positioning accuracy of RFPM is tightly related with the member of measurement elements for pattern matching. In this paper, the UE velocity is adopted in the RFPM positioning to improve the positioning accuracy, and the Cramer-Rao lower bound is used for accuracy evaluation and comparison. From the simulation results the positioning accuracy can be improved remarkably when the UE velocity is considered in addtion to the reference signal received power (RSRP), timing advance (TA) and reference signal time difference (RSTD) in the pattern matching.**

***Keywords - Cramer-Rao Lower Bound; RFPM***

## I. INTRODUCTION

Location estimation has received great deal of attention over the last two decades due to the requirements set forced by the US Federal Communications Commission (FCC) for Enhanced-911 (E-911) safety services [1]. RF Pattern Matching is a positioning method proposed in 3GPP RAN4 meetings. This method can locate the target UE in an area by comparing its RF measurement against a detailed model of the RF environment for that area. The RF parameters measured by the UE could be serving cell identity, reference signal strengths for serving and neighboring cells, timing advance, etc. The RF environment model could be stored in the form of a database of signal strengths vectors indexed by location coordinates of an area, and which could be constructed in advance or real time using a combination of RF propagating modeling and possibly test measurements [2].

In [5], a novel method about finger print positioning based on spatial correlation of collected signal samples and spatial diversity is proposed which can improve positioning accuracy and reduce time consumption of constructing a metropolitan-scale radio map, however only the received signal strength (RSS) have been considered in the algorithms which limit the peak performance. In [6], position estimations got from the so-called Neural Network localization are further processed through a Kalman filtering-based tracking algorithm and thereafter, the processed position is matched to the road according to the map-matching technique applied. However, it cannot work without the existing GPS. In [7], a novel positioning system is proposed, which consist of three sub-systems, and the first sub-system solves the problems related to fingerprint localization and involves neural network as key element of the positioning algorithm. It also ignores the distance and velocity information for improving positioning accuracy.

In this paper, we propose to introduce velocity into the existing RF Pattern Matching schemes for performance improvement. If the velocity estimation can be achieved at UE side with specific measurement errors, it can be adopted for RF Pattern Matching to improve the positioning accuracy using the RF database.

This paper is structured as follows. In Section 2, we will introduce RF Pattern Matching positioning method and diversified measurements with Cramer-Rao lower bound error model. In Section 3, based on the analysis, a RFPM method with velocity is presented, and mathematic expression can be provided as a new positioning error model. Section 4 shows simulation results of proposed method compared with traditional RFPM, and the analysis of simulation results show advantages of proposed method. Finally, conclusions are drawn in Section 5.

## II. ERROR MODEL ANALYSIS

The general error model is proposed as shown in the paper [2]. It assumes a model where measurements are non-linearly related to the parameter of interest (location, i.e., coordinate of the UE) and are corrupted by Gaussian Noise. More specifically, let y be a length N vector of measurements as below:

$$y = h(x_{HS}) + n \tag{2.1}$$

where $x = (x_{HS1} \quad x_{HS2})$ is the coordinate of the UE, $h(x_{HS}) = (h_1(x_{HS}) \quad h_2(x_{HS}) \quad \dots \quad h_N(x_{HS}))$ is the N vector of measurements which are the function of UE' coordinate. $n = (n_1 \quad n_2 \quad \dots \quad n_N)$ is the corresponding noise with the variance.

In estimation theory and statistics, the Cramer-Rao lower bound (CRLB) expresses a lower bound on the variance of estimators of a deterministic parameter. A location error function of Cramer-Rao lower bound can be calculated as below:

$$\sigma_{LOC} = \sqrt{trace(\sum_i I_i^{-1})} \quad (2.2)$$

$$I_i = \frac{1}{\sigma_i^2} \frac{\partial h_i(x_{HS})}{\partial x_{HS}}^T \frac{\partial h_i(x_{HS})}{\partial x_{HS}} \quad (2.3)$$

(2.3) is the ith measurement function's fisher information matrix. Error of positioning method can be calculated by accumulating all the measurement's information matrix as indicated in [2].

In mobile communication system, many measurement elements including RSRP (reference signal received power), RSTD (reference signal time difference) and TA (timing advance) can be used for RFPM.

The measurement element of RSRP is the estimated value at UE side which can be derived from transmit power and propagation loss. Hata models can be used to represent the RSRP signature models as below:

$$RSRP = RSRP_{REF} - \alpha \times 10 \times \log_{10}(d / d_{REF}) - a \quad (2.4)$$

where $\alpha$ is the pathloss exponent and $d$ is the distance between UE and serving eNodeB, $RSRP_{REF}$ is the reference RSRP calculated from reference distance $d_{REF}$ and

$$a = 0.5 \times FBR \times (1 - \cos(\theta_{HS} - \theta_{CELL}))$$

where FBR is the front-to-back ratio, $\theta_{HS}$ is the angle of UE measured positive counter-clockwise from the X-axis, $\theta_{CELL}$ is the angle of the antenna boresight measured positive counter-clockwise from the X-axis. So differential coefficient of RSRP can be get as below:

$$\frac{\partial RSRP}{\partial x_{HS}} = \begin{pmatrix} pa_1 - qa_2 & pa_2 + qa_1 \end{pmatrix} \quad (2.5)$$

Where

$$a_1 = \frac{x_{HS1} - x_{CELL1}}{\sqrt{(x_{HS1} - x_{CELL1})^2 + (x_{HS2} - x_{CELL2})^2}}$$

$$a_2 = \frac{x_{HS2} - x_{CELL2}}{\sqrt{(x_{HS1} - x_{CELL1})^2 + (x_{HS2} - x_{CELL2})^2}}$$

$$p = -\frac{10}{In10} * \frac{\alpha}{d}$$

$$q = -0.5 * FBR * \sin(\theta_{HS} - \theta_{CELL}) * \frac{1}{d}$$

$(x_{CELL1}, x_{CELL2})$ is the coordinate of the eNB of serving cell. RSRP's fisher information matrix can be calculated by using (2.3) and (2.5) as below [3]:

$$I_{RSRP} = \frac{1}{\sigma_{RSRP}^2} \begin{pmatrix} (pa_1 - qa_2)^2 & (pa_2 + qa_1)(pa_1 - qa_2) \\ (pa_2 + qa_1)(pa_1 - qa_2) & (pa_2 + qa_1)^2 \end{pmatrix} \quad (2.6)$$

$\sigma_{RSRP}$ is the RMS of RSRP measurement error.

RSTD is time difference between positioning signal arrival timing of reference cell and of neighboring cell, which means that it is also the function of coordinate of the target UE. According to RSTD definition, it can be expressed as below [3]:

$$RSTD_i = (d_i - d_1) / c \quad (2.7)$$

where $d_i$ is distance between the UE and *i*-th eNodeB and c is light speed.

$$I_{RSTD}^i = \frac{1}{(c * \sigma_{RSTD})^2} * \begin{pmatrix} (\cos\theta_1 - \cos\theta_i)^2 & (\cos\theta_1 - \cos\theta_i)(\sin\theta_1 - \sin\theta_i) \\ (\cos\theta_1 - \cos\theta_i)(\sin\theta_1 - \sin\theta_i) & (\sin\theta_1 - \sin\theta_i)^2 \end{pmatrix} \quad (2.8)$$

$\theta_i$ is the angle of the line between UE and ith cell positive counter-clockwise from the X-axis. $\sigma_{RSTD}$ is the RMS of RSTD measurement error. The detailed formula for RSTD is shown in [3].

TA can be used to estimate the distance from the UE to serving eNodeB. Information matrix of TA can be calculated by using the same principles as RSTD [3]:

$$I_{TA} = \frac{1}{(c * \sigma_{TA})^2} \begin{pmatrix} \cos^2\theta_{HS} & \cos\theta_{HS}\sin\theta_{HS} \\ \cos\theta_{HS}\sin\theta_{HS} & \sin^2\theta_{HS} \end{pmatrix} \quad (2.9)$$

$\sigma_{TA}$ is the RMS of TA measurement error. The detailed formula for TA is shown in [2].

The variance of RSRP, RSTD and TA can be assumed to as $\sigma_{RSRP}^2$, $\sigma_{RSTD}^2$, $\sigma_{TA}^2$ , respectively.

## III. PROPOSED POSITIONING METHOD

Velocity can be obtained by assuming UE move from one point to another point during the certain time and it is expressed as a function of UE's coordinates information. It is useful to distinguish UE status if the velocity can be used appropriately, so the UE can be located well and truly. Based on current Cramer-Rao lower bound in section Ⅱ, the positioning performance can be re-evaluated considering the UE velocity in formula deducing. It can be expected that positioning error lower bound of new RFPM with velocity is smaller than traditional RFPM method.

Velocity can be calculated by the UE moving distance divided the corresponding time. It is assumed that UE move from $dp_1 = \left(x_1^{dp1} \quad x_2^{dp1}\right)$ to $dp_2 = \left(x_1^{dp2} \quad x_2^{dp2}\right)$ during the time of $\Delta t$. So Velocity can be expressed as below:

$$v = \frac{\sqrt{\left(x_1^{dp1} - x_1^{dp2}\right)^2 + \left(x_2^{dp1} - x_2^{dp2}\right)^2}}{\Delta t} \quad (3.1)$$

So, differential coefficient of velocity can be obtained:

$$\frac{\partial v}{\partial dp_2} = \frac{1}{\Delta t}\left(\cos(\theta_v) \quad \sin(\theta_v)\right)$$

where

$$\cos(\theta_v) = \frac{\left(x_1^{dp2} - x_1^{dp1}\right)}{\sqrt{\left(x_1^{dp1} - x_1^{dp2}\right)^2 + \left(x_2^{dp1} - x_2^{dp2}\right)^2}}$$

$$\sin(\theta_v) = \frac{\left(x_2^{dp2} - x_2^{dp1}\right)}{\sqrt{\left(x_1^{dp1} - x_1^{dp2}\right)^2 + \left(x_2^{dp1} - x_2^{dp2}\right)^2}}$$

The information can be calculated as below:

$$I_v = \frac{1}{\sigma_v^2 * \Delta t^2}\begin{pmatrix} \cos^2(\theta_v) & \cos(\theta_v) * \sin(\theta_v) \\ \cos(\theta_v) * \sin(\theta_v) & \sin^2(\theta_v) \end{pmatrix} \quad (3.2)$$

$\theta_v$ is the angle of the line between UE last position and current position positive counter-clockwise from the X-axis.

$\sigma_v$ is the RMS of velocity measurement error. We can calculate information matrix of this new positioning method by using (2.6), (2.8),(2.9), (3.2):

$$I = I_v + I_{RSRP} + I_{RSTD} + I_{TA} \quad (3.3)$$

So, error can be calculated by using (2.2) and (3.3):

$$\sigma_{LOC} = \sqrt{\text{trace}(I^{-1})} = \sqrt{\frac{M}{N}} \quad (3.4)$$

where

$$M = \frac{p^2 + q^2}{\sigma_{RSRP}^2} + \frac{1}{(c * \sigma_{TA})^2} + \frac{r}{(c * \sigma_{RSTD})^2} + \frac{1}{\sigma_v^2 * \Delta t^2}$$

where $r = \sum_{i=[2,N_{RSTD}]} (\cos\theta_1 - \cos\theta_i)^2 + \sum_{i=[2,N_{RSTD}]} (\sin\theta_1 - \sin\theta_i)^2$

$N_{RSTD}$ is the number of cells participating the OTDOA positioning.

$$N = \frac{N_1}{\sigma_{RSRP}^2 * \sigma_{RSTD}^2 * c^2} + \frac{N_2}{\sigma_{TA}^2 * c^2 * \sigma_v^2 * \Delta t^2} + \frac{N_3}{\sigma_{TA}^2 * \sigma_{RSTD}^2 * c^4} + \frac{N_4}{\sigma_v^2 * \Delta t^2 * \sigma_{RSTD}^2 * c^2} + \frac{N_5}{\sigma_{RSRP}^2 * \sigma_{TA}^2 * c^2} + \frac{N_6}{\sigma_{RSRP}^2 * \sigma_v^2 * \Delta t^2} + \frac{N_7}{(c * \sigma_{RSTD})^4}$$

The related parameters used in above formulas can be summarized as follows:

$$N_1 = (pa_1 - qa_2)^2 \sum_{i=[2,N_{RSTD}]} (\rho_i)^2 + (pa_2 + qa_1)^2 \sum_{i=[2,N_{RSTD}]} (\lambda_i)^2 - 2 * (pa_2 + qa_1)(pa_1 - qa_2) \sum_{i=[2,N_{RSTD}]} (\rho_i \lambda_i)$$

$$N_2 = \left(\sin(\theta_{HS} - \theta_v)\right)^2$$

$$N_3 = \cos^2\theta_{HS} * \sum_{i=[2,N_{RSTD}]} (\rho_i)^2 + \sin^2\theta_{HS} * \sum_{i=[2,N_{RSTD}]} (\lambda_i)^2 - 2 * \cos\theta_{HS}\sin\theta_{HS} \sum_{i=[2,N_{RSTD}]} (\rho_i \lambda_i)$$

$$N_4 = \cos^2\theta_v * \sum_{i=[2,N_{RSTD}]} (\rho_i)^2 + \sin^2\theta_v * \sum_{i=[2,N_{RSTD}]} (\lambda_i)^2 - 2 * \cos\theta_v \sin\theta_v \sum_{i=[2,N_{RSTD}]} (\rho_i \lambda_i)$$

$$N_5 = ((pa_1 - qa_2) * \sin\theta_{HS} - (pa_2 + qa_1) * \cos\theta_{HS})^2$$

$$N_6 = ((pa_1 - qa_2) * \sin\theta_v - (pa_2 + qa_1) * \cos\theta_v)^2$$

$$N_7 = \sum_{i=[2,N_{RSTD}]} (\lambda_i)^2 * \sum_{i=[2,N_{RSTD}]} (\rho_i)^2 - (\sum_{i=[2,N_{RSTD}]} \lambda_i \rho_i)^2$$

where $\lambda_i = \cos\theta_1 - \cos\theta_i$ and $\rho_i = \sin\theta_1 - \sin\theta_i$.

## IV. Simulation Results Analysis

To verify the proposed scheme, we simulated both traditional RFPM and our strategy in Urban environment scenario. In this section, the simulator of RFPM with Velocity is described in detail.

The 19 (sites) * 3 (sectors) topology is used in the Fig. 1, and the evaluation area is covered by the 3 center green sectors where UEs are uniformly dropped in our simulator. The site indexes are illustrated, and the bound of sectors is denoted by the dash line [3].

Inter-site distance (ISD) is 500 meter. Pathloss exponent is 0.5 and FBR equals to 30dB. Reference distance can be set 100 meter so that reference power can be calculated as -80.5 dBm according to hata model. Minimum Distance between target UE and eNodeB is 35m. The RSRP measurement error is defined in the RSRP, RSTD and TA accuracy requirement of [4].

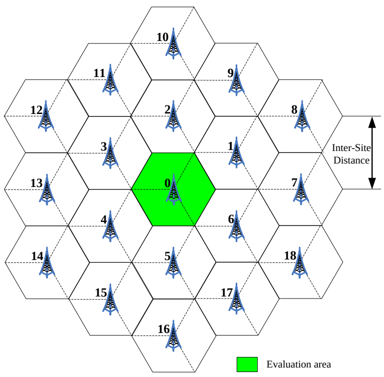


Figure 1. Network topology

For our simulation, the ±8 dB measurement error is assumed, while the RSTD and TA measurement error is assumed to be 5Ts, 10Ts respectively. Ts is the minimum time resolution of LTE systems, which equals to 1/(2048*15000)(s). Velocity variance is equipment implementation specific, and according to existing data, both1m/s and 0.5m/s are reasonable error value for this simulation. $\Delta t$ can be calculated by using the equation as below:

$$\Delta t = \frac{ISD}{v} \tag{4.1}$$

In our simulation, we make a comparison between our strategy and traditional RFPM, in which Velocity is assumed to be 3km/h and 120km/h respectively. All the detailed simulation parameters are listed in Table I.

TABLE I. SIMULATION ASSUMPTIONS

| Parameters | Value |
|---|---|
| Inter-site distance | 512 m |
| Reference distance | 100 m |
| FBR | 30 dB |
| Reference power | -80.5 dBm |
| RSRP measurement errors | 10 dB |
| TA measurement errors | 10Ts |
| RSTD measurement errors | 5Ts |
| Velocity variance | 1m/s, 0.5m/s |
| Minimum distance between target UE and eNodeB | 35 m |
| Pathloss exponent | 0.5 |
| $N_{RSTD}$ | 4 |
| UE velocity | 3/120 km/h |

RFPM is related with the number of measurement elements. It means that if velocity is used for mapping the existing RFPM performance can be enhanced from theory aspect.

Based on the assumptions and Cramer-Rao lower Bound deducing, the simulation results can be obtained as shown in Figure 2 and Figure 3, improvement is quite obvious. According to the simulation results figures, the following table can be achieved. Comparison of two positioning methods performance is given in Table II.

TABLE II. COMPARISON RESULTS SUMMARY

| Velocity Variance | 1m/s | 0.5m/s |
|---|---|---|
| Positioning error | | |
| RFPM with 3km/h | 49.6m | 47.2m |
| RFPM with 120km/h | 46.6m | 45.7m |
| RFPM without velocity | 66.9m | 66.9m |

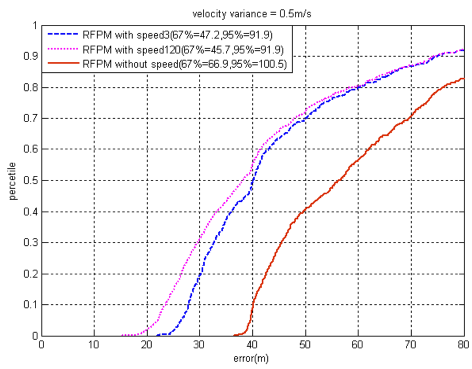


Figure 2. Comparison of methods (0.5m/s)

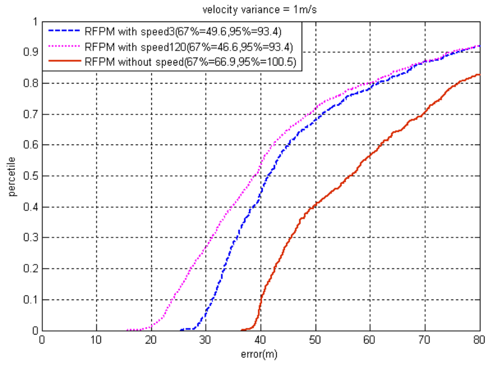


Figure 3. Comparison of methods (1m/s)

## V. CONCLUSION AND FUTURE WORKS

In this paper, several positioning error models are provided based on the current 3GPP RAN4 protocols and the Cramer-Rao lower bound is used for positioning accuracy performance evaluation. As a novel update to the conventional schemes, the UE velocity is adopted to improve the RF pattern matching efficiency, and this novel RFPM scheme can remarkably enhance the UE positioning accuracy with 25.8%~32.2% gain compared with the traditional RFPM from Cramer-Rao lower bound deducing and related simulations. Besides the velocity information, other signature information can also improve the RFPM performance, such as temperature information, and these signature information for RFPM can be included in future works.

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