Blind Estimation Schemes for Frequency Offset of OFDM Systems in Non-Gaussian Noise Environments

Jong In Park, Changha Yu, Youngpo Lee, and Seokho Yoon†
College of Information and Communication Engineering
Sungkyunkwan University
Suwon, South Korea
Email: {pji17, dbckdgk, leeyp204, and †syoon}@skku.edu
†Corresponding author

***Abstract*—In this paper, the blind frequency offset estimation schemes robust to the non-Gaussian noise for orthogonal frequency division multiplexing (OFDM) systems are addressed. Based on the cyclic prefix structure and a maximum-likelihood (ML) estimation, the estimation schemes in non-Gaussian noise are proposed. Simulation results show that the proposed blind estimation schemes offer a robustness and a substantial performance improvement over the conventional blind estimation scheme in non-gaussian noise environments.**

Keywords-blind estimation; frequency offset; maximum-likelihood; non-Gaussian noise; OFDM.

I. Introduction

Orthogonal frequency division multiplexing (OFDM) has been adopted as a physical layer implementation in a wide variety of wireless systems such as long term evolution (LTE), wireless local area network (WLAN), and worldwide interoperability for microwave access (WiMAX) due to its immunity to multipath fading and high spectral efficiency [1]-[3]. However, OFDM is very sensitive to the frequency offset (FO) caused by Doppler shift or oscillator instabilities [1], [4]. In this paper, we focus on FO estimation based on the blind approach, which does not require an additional training symbol [4].

Assuming that the ambient noise is a Gaussian process, in [5], an optimal FO estimation scheme was proposed using the cyclic prefix (CP) of OFDM symbols without requiring the training symbol. However, it has been observed that the ambient noise often exhibits non-Gaussian nature in wireless channels, mostly due to the impulsive nature originated from various sources such as car ignitions, narrowband interferences, moving obstacles, and reflections from sea waves [6], [7]. The conventional estimation scheme developed assuming the Gaussian noise could suffer from severe performance degradation under the non-Gaussian noise environments.

In this paper, we propose robust blind FO estimation schemes in non-Gaussian noise environments. Using the CP structure of OFDM, we first derive a blind maximum-likelihood (ML) FO estimation scheme in non-gaussian noise modeled as a complex isotropic Cauchy noise, and then, propose a simpler blind estimation scheme reducing the size of the candidate set. From simulation results, the proposed schemes are confirmed to offer a substantial performance improvement over conventional blind estimation scheme in non-Gaussian noise environments.

The rest of this paper is organized as follows. Section II introduces the signal and noise model. In Section III, the novel blind FO estimation schemes are proposed, and then, in Section IV the simulation results are demonstrated. Section V concludes this paper.

II. Signal Model

The kth received OFDM sample $r(k)$ can be expressed as

$$r(k) = x(k)e^{j2\pi k\varepsilon/N} + n(k) \tag{1}$$

for $k = -G, \cdots, -1, 0, 1, \cdots, N-1$, where $x(k)$ is the kth sample of the transmitted OFDM symbol generated by the inverse fast Fourier transform (IFFT) of size N, G is the size of the CP, ε is the FO normalized to the subcarrier spacing $1/N$, and $n(k)$ is the kth sample of additive noise.

In this paper, we adopt the complex isotropic symmetric α stable (CISαS) model for the noise samples $\{n(k)\}_{k=0}^{N-1}$ and assume that they are independent and identically distributed; this model has been widely employed due to its strong agreement with experimental data [8], [9]. For example, the interference due to the multiple access is often modeled as the SαS noise [10], [11] as well as the ambient noise in the shallow water channel of underwater communication systems [12]. The probability density function (pdf) of $n(k)$ is then given by [8]

$$f_n(\rho) = \frac{1}{4\pi^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{-\gamma\left(u^2+v^2\right)^{\frac{\alpha}{2}} - j\Re\{\rho(u-jv)\}} du dv, \tag{2}$$

where $\Re\{\cdot\}$ denotes the real part, the dispersion $\gamma > 0$ is related to the spread of the pdf, and the characteristic exponent $\alpha \in (0, 2]$ is related to the heaviness of the tails of the pdf: A smaller value of α indicates a higher degree of impulsiveness, whereas a value closer to 2 indicates a more Gaussian behavior.

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A closed-form expression of (2) does not exist except for the special cases of $\alpha = 1$ (complex isotropic Cauchy) and $\alpha = 2$ (complex isotropic Gaussian). For complex isotropic Cauchy and Gaussian cases, (2) can be re-written in the closed form as

$$f_n(\rho) = \begin{cases} \frac{\gamma}{2\pi}\left(|\rho|^2+\gamma^2\right)^{-\frac{3}{2}}, & \text{when } \alpha = 1 \\ \frac{1}{4\pi\gamma}\exp\left(-\frac{|\rho|^2}{4\gamma}\right), & \text{when } \alpha = 2. \end{cases} \quad (3)$$

In this paper, we concentrate on the case of $\alpha = 1$, since it is known that the receiver which performs well in Cauchy noise also works well in other cases of SαS noise [11]. We shall see in Section IV that the estimation schemes obtained for $\alpha = 1$ are not only more robust to the variation of α, but they also provide a better performance for most values of α, than the conventional estimation scheme.

III. Proposed Schemes

A. Maximum-likelihood Blind Estimation Scheme

In estimating the FO, we consider a property of the CP structure of OFDM, i.e., $x(k) = x(k+N)$ for $k = -G, -G+1\cdots, -1$ as in [5]. Then, from (1), we have

$$r(k+N) - r(k)e^{j2\pi\varepsilon} = n(k+N) - n(k)e^{j2\pi\varepsilon} \quad (4)$$

for $k = -G, -G+1\cdots, -1$. Observing that $n(k+N) - n(k)e^{j2\pi\varepsilon}$ obeys the complex isotropic Cauchy distribution with dispersion 2γ (since the distribution of $-n(k)e^{j2\pi\varepsilon}$ is the same as that of $n(k)$), we obtain the pdf

$$f_{\boldsymbol{r}}(\boldsymbol{r}|\varepsilon) = \prod_{k=-G}^{-1} \frac{\gamma}{\pi\left(\left|r(k+N)-r(k)e^{j2\pi\varepsilon}\right|^2+4\gamma^2\right)^{\frac{3}{2}}} \quad (5)$$

of $\boldsymbol{r} = \{r(k+N) - r(k)e^{j2\pi\varepsilon}\}_{k=-G}^{-1}$ conditioned on ε. The ML estimation is then to choose $\hat{\varepsilon}$ such that

$$\begin{aligned} \hat{\varepsilon} &= \arg\max_{\tilde{\varepsilon}}[\log f_{\boldsymbol{r}}(\boldsymbol{r}|\tilde{\varepsilon})] \\ &= \arg\min_{\tilde{\varepsilon}} \Lambda(\tilde{\varepsilon}), \end{aligned} \quad (6)$$

where $\tilde{\varepsilon}$ denotes the candidate value of ε and the log-likelihood function $\Lambda(\tilde{\varepsilon}) = \sum_{k=-G}^{-1} \log\left\{\left|r(k+N) - r(k)e^{j2\pi\tilde{\varepsilon}}\right|^2 + 4\gamma^2\right\}$ is a periodic function of $\tilde{\varepsilon}$ with period 1: The minima of $\Lambda(\tilde{\varepsilon})$ occur at a distance of 1 from each other, causing an ambiguity in estimation. Assuming that ε is distributed equally over positive and negative sides around zero, the valid estimation range of the ML estimation scheme can be set to $-0.5 < \varepsilon \leq 0.5$, as in [5]. The estimation scheme (6) will be called the Cauchy ML blind estimation (CMBE) scheme.

B. Low-complexity Blind Estimation Scheme

The CMBE scheme is based on the exhaustive search over the whole estimation range ($|\varepsilon| < 0.5$), which requires high computational complexity. Thus, we propose a low-complexity FO estimation scheme with the reduced set of the candidate values.

In order to obtain the reduced set of the candidate values, we exploit the fact that $\varepsilon = \frac{1}{2\pi}\angle\{x^*(k)x(k+N)\} = \frac{1}{2\pi}\angle\{r^*(k)r(k+N)\}$ for $k = -G, -G+1\cdots, -1$ in the absence of noise. Based on this property, we obtain the set of the candidate values

$$\bar{\varepsilon}(k) = \frac{1}{2\pi}\angle\{r^*(k)r(k+N)\}, \text{ for } k = -G, -G+1\cdots, -1. \quad (7)$$

Exploiting the set of the candidate values in (7), the FO estimate $\hat{\varepsilon}_L$ can be obtained as follows

$$\hat{\varepsilon}_L = \arg\min_{\bar{\varepsilon}(k)} \Lambda\big(\bar{\varepsilon}(k)\big), \text{ for } k = -G, -G+1\cdots, -1. \quad (8)$$

In the following, (8) is denoted as the low-complexity CMBE (L-CMBE) scheme. Using only $N/2$ candidate values, the L-CMBE scheme can offer an almost same performance as the CMBE scheme with the exhaustive search, which is verified by simulation results in Section IV.

IV. Simulation Results

In this section, the proposed CMBE and L-CMBE schemes are compared with the Gaussian ML blind estimation (GMBE) scheme in [5] in terms of the mean squared error (MSE). We assume the following parameters: The IFFT size $N = 64$, FO $\varepsilon = 0.25$, the search spacing of 0.001 for the CMBE scheme, and a multipath Rayleigh fading channel with length $L = 8$ and an exponential power delay profile of $\mathbf{E}[|h(l)|^2] = \exp(-l/L)/\{\sum_{l=0}^{L-1}\exp(-l/L)\}$ for $l = 0, 1, \cdots, 7$, where $h(l)$ is the lth channel coefficient of a multipath channel and $\mathbf{E}[\cdot]$ denotes the statistical expectation. Since CISαS noise with $\alpha < 2$ has an infinite variance, the standard signal-to-noise ratio (SNR) becomes meaningless for such a noise. Thus, we employ the geometric SNR (GSNR) defined as $\mathbf{E}[|x(k)|^2]/(4C^{-1+2/\alpha}\gamma^{2/\alpha})$, where $C = \exp\{\lim_{m\to\infty}(\sum_{i=1}^{m}\frac{1}{i} - \ln m)\} \simeq 1.78$ is the exponential of the Euler constant [13]. The GSNR indicates the relative strength between the information-bearing signal and the CISαS noise with $\alpha < 2$. Clearly, the GSNR becomes the standard SNR when $\alpha = 2$. Since γ can be easily and exactly estimated using only the sample mean and variance of the received samples [14], it may be regarded as a known value: Thus, γ is set to 1 without loss of generality.

Figs. 1-3 show the MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 5, 10, and 15 dB, respectively. From the figures, we can clearly observe that the proposed schemes not only outperform the conventional scheme for most values of α, except for those close to 2, but also provide a robustness

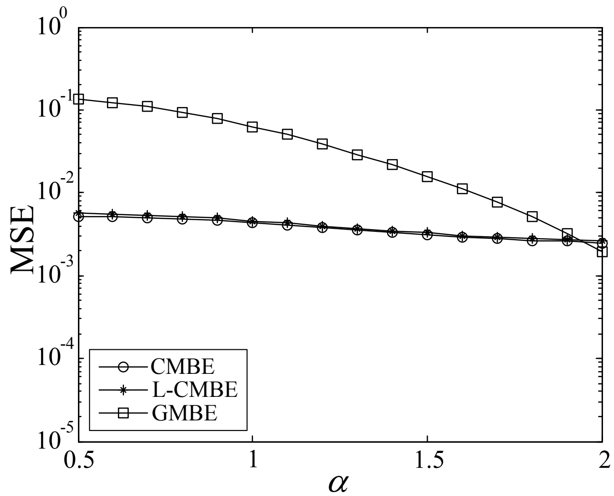

Figure 1. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 5 dB.

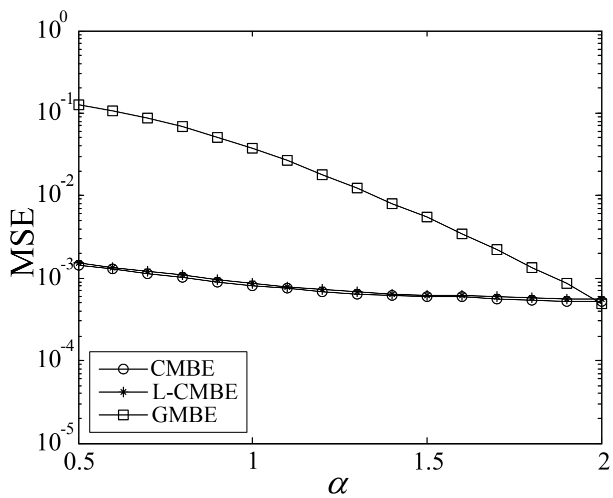

Figure 2. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 10 dB.

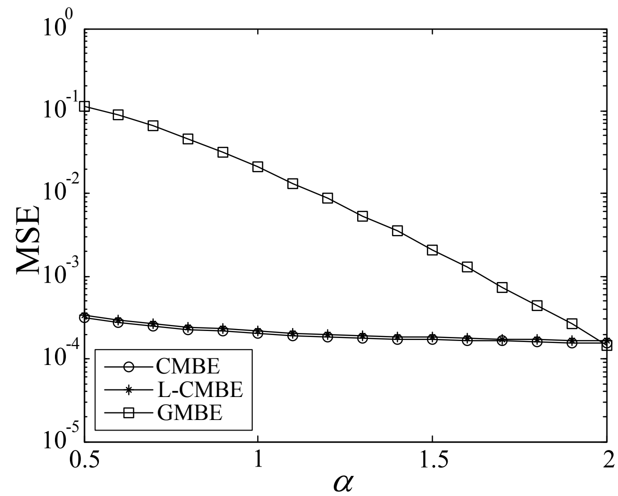

Figure 3. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of α when the GSNR is 15 dB.

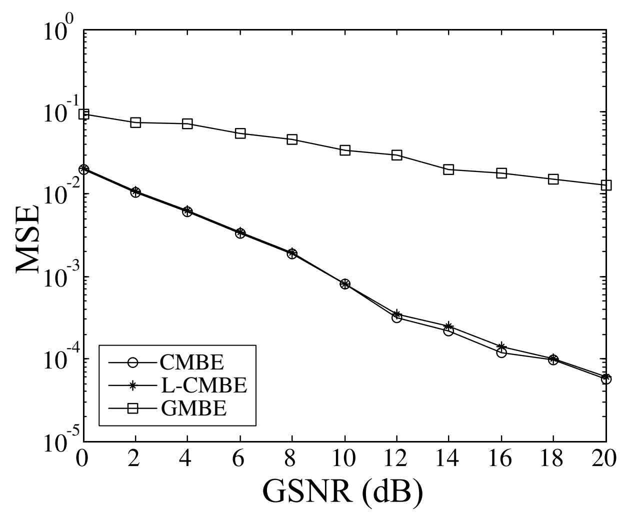

Figure 4. The MSE performances of the CMBE, L-CMBE, and GMBE schemes as a function of GSNR when $\alpha = 1$.

to the variation of the value of α. Another important observation is that the estimation performance of the L-CMBE scheme is almost same as that of the CMBE scheme. From this observation, it is confirmed that the candidate values for the L-CMBE scheme is reasonable.

Fig. 4 shows the MSE performances of the proposed and conventional schemes as a function of GSNR when $\alpha = 1$. From the figure, we can clearly observe that the proposed schemes outperform the conventional scheme regardless of value of GSNR. Moreover, the MSE performance of L-CMBE is very similar to that of the CMBE, the optimal FO estimation scheme when $\alpha = 1$, but also provide a robustness to the variation of the value of α.

V. Conclusion

In this paper, we have proposed blind FO estimation schemes in non-Gaussian noise environments. Based on the CP structure of OFDM, we first have derived a ML FO estimation scheme in non-gaussian noise modeled as a complex isotropic Cauchy noise, and then, derived a simpler blind estimation scheme with a lower complexity. From simulation results, it has been confirmed that the proposed schemes offer a robustness and a substantial performance improvement over the conventional estimation scheme in non-gaussian noise environments.

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