Non-binary Encoded STBC-CPM Signals for Wireless Slicing Networks

Piotr Remlein
Chair of Wireless Communications,
Poznan University of Technology
Poznan, Poland
e-mail: piotr.remlein@put.poznan.pl

Abstract—**The paper considers a wireless transmission system in a multiple-user environment that can support slicing networks. It involves a combination of non-binary convolutional codes and Space Time Block Codes (STBC) technique with narrowband Continuous-Phase Modulation (CPM) for transmission in an uplink channel using the Frequency Division Multiple Access (FDMA) method. Energy efficiency signals, e.g., Continuous Phase Modulation could be used for transmission managing or feedback information signals in return channels of slicing networks. We analyse performance of a transmission in such return channels which apply FDMA method with coded CPM signals via computer simulations. In order to reduce the Bit Error Rate (BER) a concatenation of CPM with non-binary convolutional codes over ring and STBC technique has been used. The spectral efficiency of investigated system is improved by reducing the inter-carrier frequency spacing and using a low-complexity iterative algorithm for Inter-Carrier Interference (ICI) cancellation at the receiver. BER results obtained in computer simulations are presented for the proposed solution. The results suggest that FDMA coded STBC-CPM transmission might constitute a desirable option for return channels in slicing networks.**

Keywords-Continuous Phase Modulation; Frequency Division Multiplexed system; Slicing Networks, Non-binary codes, Inter-Carrier Interference Receiver

I. INTRODUCTION

The network slicing has recently been proposed in the industry solutions for wireless networks as an enabler of network service convergence and on-demand customized services [1]–[3]. The network slicing can support on-demand tailored services for distinct application scenarios at the same time using the same physical network. Supported by network slicing, network resources can be dynamically and efficiently allocated to logical network slices according to the corresponding Quality of Service (QoS) demands. The authors in [4] proposed a network slicing mechanism for network edge nodes to offer low-latency services to users. The mobility management schemes and an optimal gateway selection algorithm to support seamless handover was described. In [5], a resource allocation scheme with the consideration of interference management was presented. In [6], a flexible software defined networking (SDN) based 5G network architecture was proposed. The SDN was used to allocate physical network resources to slices within a local area and to perform scheduling among slices. The research on mobility management in network slicing systems has been focused mostly on SDN based control and handover procedures [6]–[8].

In this article, we propose and investigate the transmission method which can be used in such systems to provide managing and control signals in the return channels.

In the paper, the focus is on FDMA CPM systems which are power and spectral efficiency. We concentrate on the study of non-binary coded STBC-CPM signals in an uplink scenario. The CPM signals are well suitable for wireless return channel transmission through its features constant envelope and its disruptions immunity occurring when signals are transmitted [9]. Additionally, CPM signals are resistant to non-linear distortion of power amplifiers. Multiuser FDMA CPM systems wherein all users employ a portion of the spectrum have been studied in, e.g., [10] – [13].

In order to increase the reliability or bit rate of telecommunication systems, the multi-antenna Multiple Input Multiple Output (MIMO) technique is used more and more often [14]. The technique involves multiple antennas on the transmitter and receiver sides. Employing the MIMO technique in wireless systems allows the use of Space-Time Codes (STC), improving transmission quality [14]. The one of the most popular STC codes and widely used in MIMO systems are space-time block codes (STBC) [15]. These codes are orthogonal and can achieve full transmit diversity specified by the number of transmit antennas. In [16] and [17], the authors studied CPM signals encoded using the so-called orthogonal space-time codes. The CPM signals concatenated with STBC codes are characterized by a low bit error rate attainable at a simultaneously low receiver complexity. Neither of the above solutions for MIMO transmission concerned FDMA CPM systems.

In order to obtain a further improvement in energy efficiency, CPM was combined in classical systems with an external binary Convolutional Encoder (CE) and a mapper. It has been shown that a CPM scheme can be decomposed into a Continuous Phase Encoder (CPE) followed by a memoryless modulator (MM) [18], where the CPE is a CE over a ring of integers [19]. Therefore, a natural way to combine CPM with an outer CE is to use a CE over the same ring of integers. Since the CE and CPE are over the same algebra, no mapper is needed, and an extra coding gain was reported [19].

The literature has discussed the aspects of increasing the spectral efficiency of systems employing CPM and FDMA, e.g., in [12], [13] but STBC coding concatenated with convolutional codes over ring in multiuser scenario has not been taken into consideration.

This paper considers a wireless MIMO FDMA transmission in return channel of slicing networks. It involves a combination of non-binary convolutional codes with STBC and CPM signals for transmission in an uplink channel using the FDMA method. For a more efficient available band utilization, the distances between individual sub-carriers are reduced. Such a solution causes the deterioration of transmission quality due to the occurrence of ICI. In order to reduce the BER in such a system, a combination of CPM with non-binary convolutional codes over a ring has been used and a low-complexity iterative algorithm for ICI cancellation was employed on the receiver side.

The paper is organized as follows. In Section II, properties of CPM modulation are presented. The description of the convolutional codes over ring is included in Section III. Section IV presents the considered system. Section V provides the obtained simulation results. The paper is summarized and concluded in Section VI.

II. CONTINUOUS-PHASE MODULATION

A general form of a continuous-phase modulation describes the following formula [9]:

$$x(t,\alpha) = \sqrt{\frac{2E_S}{T}}\cos(2\pi f_o + \varphi(t,\alpha) + \varphi_0) \qquad (1)$$

where E_S is the energy per symbol, T is the symbol interval, f_0 is the carrier frequency, φ_0 is the initial phase, $\boldsymbol{\alpha} = (\ldots, 0_{-1}, \alpha_0, \alpha_1, \ldots)$ refers to a sequence of data symbols adopting one of the values from the set:

$$\alpha_i \in \{\pm1, \pm3, \ldots, \pm(M-1)\} \qquad (2)$$

where M is the cardinality of the set and is typically chosen as a power of 2.

Phase $\varphi(t,\alpha)$, in which the transferred information is included, may be described as follows:

$$\varphi(t,\alpha) = 2\pi h \sum_{i=-\infty}^{\infty} \alpha_i q(t - iT) \qquad (3)$$

where h is the modulation index defining the value by which the phase changes in each modulation interval.

CPM signals are characterized by the following parameters: modulation index h, pulse length L and phase response function $q(t)$, or its derivative $g(t)$, the frequency response function. One of the parameters having an influence on the spectral characteristics of CPM signals is the shape of a frequency pulse g(t) [9]. In this paper, CPM signals with a rectangular (REC) pulse are analyzed.

The CPM modulator may be performed as cascade concatenation of a continuous-phase encoder (CPE) and memoryless modulator (MM). Such a system is known as the Rimoldi decomposition [18]. The CPE is a convolutional encoder that performs the function of a CPM modulator memory. The memoryless modulator assigns relevant signal shapes to symbols received from the CPE.

To increase CPM signals' resistance to different types of interference arising during transmission, a connection of the CPM signal modulator system with an external encoder is employed. Most frequently it is a convolutional coder. Such a solution is known as Serially-Concatenated Convolutional Coder (SCCC) [11]. A block diagram of the CE and modulator's serial concatenation is shown in Figure 1.

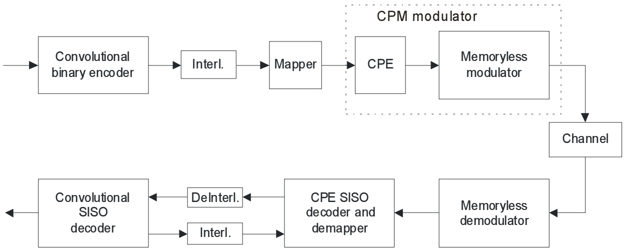

Figure 1. Serially concatenated convolutional encoder CE and CPM modulator.

The convolutional encoder is connected to the modulator by an interleaver and a mapper. The mapper is no needed if external convolutional encoder connected to CPE of CPM modulator are over the same ring of integers modulo-M. The external convolutional encoder CE with rate R_{CE} is equipped with a symbol puncturer with the rate R_{pct}. The overall encoding rate R, with the puncturing operation allowed for, is:

$$R = R_{CE} R_{pct} \qquad (4)$$

The discussed method of serially concatenation of the external convolutional encoder and CPM is called in literature a SCCC-CPM and guarantees obtaining of a low BER. There is obtained with the iterative receiver which makes many iterations between the CPE decoder and the decoder of the external convolutional code CE to make a decision on the transmitted information [11].

III. PROPERTIES OF NON-BINARY CONVOLUTIONAL CODES OVER RING

In this paper, we concentrate on the usage of non-binary convolutional codes known as convolutional codes over rings [19]-[21]. These codes in many cases have larger Euclidean distances than binary codes over GF(2) [19] [21].

In Figure 2, it is shown a realization of the systematic feedback convolutional encoder of rate $R_{CE} = k/n$, $n=k+1$. Input to the encoder, at time t, is information vector U_t with M-ary elements $u_t^{(i)}$ belonging to the ring $Z_M=\{0, 1, 2, \ldots, M-1\}$

$$U_t = (u_t^{(1),} u_t^{(2)}, \ldots, u_t^{(k)}) \qquad (5)$$

The convolutional encoder produces a coded sequence of the symbols which belong to the same ring Z_M

$$V_t = (v_t^{(1)}, v_t^{(2)}, \ldots, v_t^{(n)}) \qquad (6)$$

where $n = k+1$.

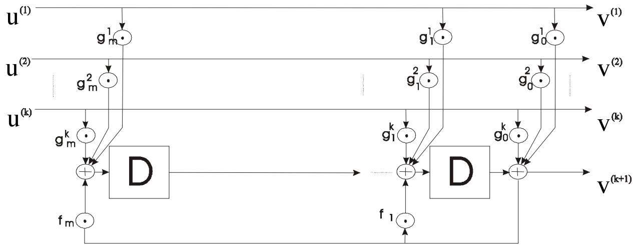

Figure 2. Systematic feedback convolutional encoder over the ring of integers modulo-M (Z_M).

The coefficients of the encoder in Figure 2 are taken from the set {0,...,M-1}. The memory cells are capable of storing the ring elements. Multipliers and adders perform multiplication and addition respectively in the ring of integers modulo-M.

The encoder output V_t at time t due to the input U_t is:

$$V_t = U_t\, G, \tag{7}$$

where G denotes the generator matrix of the encoder [21].

In Figure 3, we show an example of the systematic convolutional encoder with feedback this is the encoder over the ring Z4 of integers modulo-4.

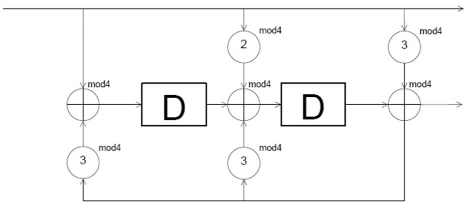

Figure 3. The systematic convolutional encoder over ring ℜ=Z4.

The code rate is R_{CE}=1/2 (k=1, n=2) and the generator matrix is:

$$G(D) = \left[1 \quad \frac{3+2D+D^2}{1+3D+3D^2} \right] \tag{8}$$

At instant t, an information vector with 4-ary symbols belonging to the ring Z_4, inputs the encoder. The convolutional encoder generates encoded vector which contains sequence of the symbols elements belonging to the same ring Z_4. The encoder coefficients in Figure 3 are from the set {0,1,2,3}. The memory cells are capable of storing the ring elements. Multiplications and additions are performed in the ring of integers modulo-4.

IV. FDMA SYSTEM WITH RING ENCODED STBC-CPM SIGNALS

In this paper, FDMA system for uplink wireless transmission, using ring encoded CPM signals is analysed. To obtain the best spectral efficiency of the examined system with multiple users, the space between carrier frequencies are minor. The tight inter-carrier frequency spacing between adjacent channels causes strong ICI. To eliminate the ICI in the receiver, an ICI cancelation algorithm is employed. Therefore, it is possible to obtain high spectral efficiency and low bit error rate [13]. Asymptotic spectral efficiency (ASE) can be determined assuming that the signal to noise ratio approaches infinity. ASE is directly proportional to the encoding rate of CE (R_{CE}) and the number of bits falling for one symbol and inversely proportional to the value of normalized spacing between carrier frequencies. ASE for MIMO systems with multiple transmit antennas (M_T) is given by means of the following formula:

$$\mathrm{ASE} = \lim_{\frac{E_b}{N_0}\to\infty} \mathrm{SE} = M_T \left(\frac{R \log_2 M}{\Delta_f T} \right) \tag{9}$$

where E_b/N_0 is the signal to noise ratio, R is the encoding rate, and $\Delta_f T$ is the normalized spacing between inter-carrier frequencies, M-ary modulation. The obtained overall spectral efficiency depends strictly on the number of transmission channels used in the defined frequency band. The encoder rate R is described by relation (4) and can be changed by the rate of the puncturer whose value is changeable in order to obtain specific spectral efficiency.

The CE encoder has a structure determined experimentally, enabling low bit error rate. Performing comparing performance of binary encoded CPM and non-binary CPM, FDMA systems requires selecting codes of equal complexity defined by the number of states. While comparing CE one has to also take into consideration that the comparison in only fair when binary and non-binary systems achieving this same ASE are investigated.

The non-binary code that contributes to acquiring the lowest BER was found for the examined FDMA CPM system. Such defined optimal CE over the ring of Z_4 that has been found in the study and is described by the generator matrix: $[1, D+3, 3D+1]$.

I this paper, we consider the reception of non-binary encodes CPM signals in an FDMA system employing the MIMO technique and STBC coding. An STBC-CPM system analyzed in this paper is based on scheme described in [15].

The transmitted CPM signal vector can be expressed as it was shown in [15].

$$x_t = \left[x_t^1, x_t^2, \ldots, x_t^{n_T} \right]^T \tag{10}$$

where n_T is the number of transmit antennas, and T denotes the transpose of the vector [ ].

The modified Alamouti scheme [15] is used for two and four transmit antennas FDMA CPM system. The transmission matrix for two antennas is showed below formula:

$$S = \begin{bmatrix} s_1 & -s_2^* \\ s_2 & s_1^* \end{bmatrix} \tag{11}$$

In this case, the signals transmitted by the first and second antennas into two time intervals are described by equations $x^1 = [s_1 \ -s_2^*]$ and $x^2 = [s_2 \ s_1^*]$, respectively.

Similarly, we can identify the signals in the case of four transmit antennas when transmission matrix is in the form:

$$S = \begin{bmatrix} s_1 & s_2 & s_3 & s_4 \\ -s_2^* & s_1^* & -s_4^* & s_3^* \\ s_3 & s_4 & s_1 & s_2 \\ -s_4^* & s_3^* & -s_2^* & s_1^* \end{bmatrix} \qquad (12)$$

Figure 4 shows a block diagram of the proposed system with STBC coding. At the input, each kth user binary information sequence $\alpha_0, \dots, \alpha_{k-1}$, is Convolutional Encoded (CE), interleaved (Int.) and converted into several (M_T) parallel streams in an STBC encoder. Each data stream is conveyed to one of CPM modulators. In order to achieve higher rates, the CE output is punctured as in [22]: a rate-matching algorithm is used to obtain an appropriate coding rate *R*. The interleaver (Int.) is a symbol, spread (S-random) interleaver [22].

Each user's transmitter consists of an CE, interleaver, STBC encoder and a CPM modulator. The efficiency of an STBC encoder depends on the number of transmit antennas. The STBC encoder considered in the paper uses two or four transmit antennas. Each user signal is characterized by a distinct phase φ_k and delay τ_k, as typically occurs in uplink systems.

The channel model used in the simulation takes into account multipath propagation. The channel model has been implemented as a Taped Delay Line (TDL) and it models a channel with flat fading and Additive White Gaussian Noise (AWGN) [14].

Each receive antenna receives a faded superposition of M_T simultaneously transmitted signals corrupted by additive white Gaussian noise. The fading is assumed to be flat and distributed according to a Rayleigh *pdf*. The random path gains between transmit antenna *m* and receive antenna *p*, $h_{m,p}(t)$ are independent complex Gaussian random variables with zero mean and variance per dimension 1/2. The fading is slow, such that the M_TxM_R fading coefficients are constant during a frame, but vary from frame to frame. The AWGN noise components $n_p(t)$ are independent zero-mean complex Gaussian random processes with power spectral density N_0.

At the receiving end, the system consists of a MIMO MMSE/STBC detector/decoder and a low-complexity iterative algorithm to ICI cancellation [13].

With the assumption of uniform parameters of CPM, and an equal number of transmit antennas M_T for each system user, the signal at the input of the *p*-th receiver antenna is described by:

$$r_p(t) = \sum_{k=0}^{K-1} \sum_{m=0}^{M_T-1} \sum_{n=0}^{B-1} g_{k,m,n}\left(t - n\tau_k, \alpha_{k,m}\ \right) \cdot e^{j(2\pi k \Delta_f t + \varphi_k)} + n_p(t), \ \ j = 0, \dots, M_R - 1 \qquad (13)$$

where *B* is the number of intervals for which the STBC-CPM signal is transmitted, for the analyzed in the paper system with STBC encoding *B*=2 or 4.

The MMSE/STBC block realizes the STBC decoding and computes the cost function, i.e., minimizes:

$$G_{MMSE} = \sqrt{\frac{n_T}{E_s}} \left(H^H H + \frac{n_T N_0}{E_s} I_{n_T} \right)^{-1} H^H \qquad (14)$$

where *H* is the matrix of channel impulse response estimates, *I* – identity matrix, N_0 – spectral density of noise power, upper index of quantity H^H denotes the Hermitian transpose of matrix *H* and it is the sum of the operations of transpose and complex conjugate of the matrix. Signal *y* from MMSE/STC reaches the ICI cancellation block. The receiver carries out ICI cancellation through a set of single-user MAP detector/remodulator blocks, as described by Perotti et al. [13]. The remodulators make use of the output of the MAP detector to compute the remodulated signal $s_k^{(i)}(t)$ relative to the *k*th user and *i*th iteration. The channel decoder performs two iterations loops. The inner loop is formed by the ICI canceller, the MAP detector, the CPE SISO decoder and the remodulator, while the outer loop involves the CPE SISO decoder, the CE SISO decoder, the interleaver and the deinterleaver between the inner CPE decoder and the outer CE decoder. ICI cancellation can be performed while executing the decoding iterations to enhance the receiver performance. In such a case, after the inner CPE decoder is executed, remodulation is performed. Then, interference cancellation is performed and the CPM receiver, including the inner CPE decoder, is again executed. The decoder starts decoding a received code word executing N_I inner iterations. Then, it executes N_O times an outer iteration followed by an inner iteration. This way, ICI cancellation is performed as part of the decoding iterations and it results in an improved ICI cancellation [13]. On the final outer iteration, a decision is made on the transmitted data symbols $\hat{\alpha}_0, \dots, \hat{\alpha}_{k-1}$.

V. SIMULATION RESULTS

The Monte-Carlo simulation method was used to determine the BER for the described FDMA system with STBC-CPM modulation concatenated with binary CE or CE over the ring. The simulations have been performed with the

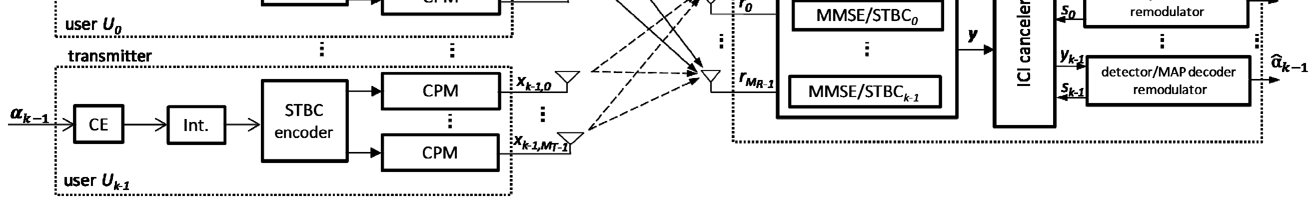

Figure 4. Block diagram of the analyzed CPM system.

aid of a simulation program written in C++ using IT++ libraries, ver. 4.2 [23]. The analysis assumed that each user transmits CPM signals with the same modulation and coding parameters, and of equal strength. We established the ideal power control, this means that the received signal power is the same for all users. It was assumed that the standardized value of the interval between the carrier frequencies of consecutive channels $\Delta_f T$ in the system is equal to 3/4, and that the transmission is performed via 4 neighboring channels (FDMA users). The signals are transmitted through AWGN channel with spectral power density $N_0/2$. The transmitted packets were 1000 bits long. The simulation was stopped if at least 100 errors occurred.

The analysis, for the FDMA system with CPM signals and CE over the ring Z_4, has been performed with reference to binary encoded CPM modulation with the parameters h=1/4, L=1 and the REC shape frequency pulse. The number of iterations in the receiver was experimentally fixed as a good trade-off between receiver performance and complexity.

Two iterations (N_I=2) in the interference cancellation loop have been made and five iterations (N_O=5) in soft output non-binary MAP decoder. The CPM modulator is concatenated with CE over the ring $[1, D + 3, 3D + 1]$ or the binary [7, 5] encoder (from [19], described in octal notation), both with four states. The obtained BER results for the CPM-FDMA systems with CE over a ring were compared with BER for CPM-FDMA systems employing binary CE, which has the same spectral efficiencies.

Figure 5 presents BER for systems employing convolutional encoded CPM modulation concatenated with STBC encoding for different numbers of receive antennas. In this case, each of the four users transmitted signals via one, two or four transmit antennas, and the receiver used one, two or four receive antennas. In this scenario, the error rate at the BER level of 10^{-5} obtained for the 2x2 system was worse than the one obtained for the 4x4 system by about 3 dB and better at the BER level of 10^{-5}than the one obtained for the 1x1 system by about 4dB.

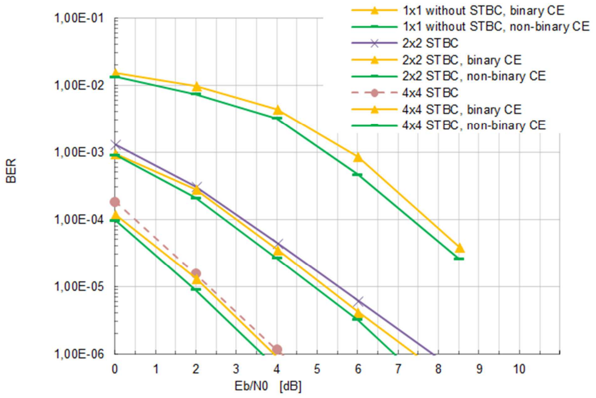

Figure 5. BER for STBC system (CPM: $h = 1/4$, $M = 4$, L = 1, $\Delta_f T = 3/4$), 4-users, with ICI cancelation and different numbers of transmit and receive antennas.

In all cases, we observe that a slight improvement in BER is achieved for the systems using binary encoded STBC CPM signals comparing to the systems without convolutional encoding. We can see that the best results of BER can be obtained when concatenation STBC and non-binary CE was used. It should be noted that, the SE for all the systems 1x1 was equal 1 b/s/Hz, for systems 2x2 amounted 2 b/s/Hz and for 4x4 the SE was 4 b/s/Hz. BER results obtained in computer simulations shoved that proposed solution with non-binary CE gives noticeable improvements compared with results for FDMA systems without convolutional encoding.

VI. CONCLUSIONS

In this paper, a multiuser FDMA STBC-CPM transmission with non-binary encoding has been proposed. Through MMSE-based multiuser detection and low-complexity iterative ICI cancellation, considerable improvements in both BER are achieved with respect to single antenna systems, while the multiuser receiver complexity is kept low. A performance evaluation has been presented to demonstrate the superiority of the proposed multiuser FDMA STBC-CPM MIMO system. The study shows that it is possible to increase transmission efficiency by using CPM modulation and concatenation non-binary convolutional encoding with STBC coding for MU transmissions. The presented results show that proposed method can be used in the slicing networks to the transmission of the managing and the control signals.

ACKNOWLEDGMENT

This work has been partially funded by the Polish Ministry of Science and Higher Education within the status activity task 08/81/DSPB/8134 in the year 2018.

REFERENCES

[1] P. Rost et al., “Mobile network architecture evolution toward 5G,” IEEE Commun. Mag., vol. 54, no. 5, pp. 84–91, May 2016.

[2] M. Jiang, M. Condoluci, and T. Mahmoodi, “Network slicing management & prioritization in 5G mobile systems,” European Wireless 2016; 22th European Wireless Conference, Oulu, Finland, pp. 1-6, 2016.

[3] Ericsson, Ericsson White Paper: 5G system, Jan. 2015.

[4] J. Heinonen, P. Korja, T. Partti, H. Flinck, and P. Poyhonen, “Mobility management enhancements for 5G low latency services,” 2016 IEEE International Conference on Communications Workshops, Kuala Lumpur, pp. 68–73, 2016.

[5] H. Zhang et al., “Resource allocation in spectrum-sharing OFDMA femtocells with heterogeneous services,” IEEE Trans. Commun., vol. 62, no. 7, pp. 2366–2377, July 2014.

[6] V. Yazici, U. C. Kozat, and M. O. Sunay, “A new control plane for 5G network architecture with a case study on unified handoff, mobility, and routing management,” IEEE Commun. Mag.,, vol. 52, no. 11, pp. 76–85, Nov. 2014.

[7] H. Song, X. Fang, and L. Yan, “Handover scheme for 5G C/U plane split heterogeneous network in high-speed railway,”

IEEE Trans. Veh. Technol. vol. 63, no. 9, pp. 4633–4646, Nov. 2014.

[8] S. Kuklinski, Y. Li, and K. T. Dinh, “Handover management in SDN-based mobile networks,” 2014 IEEE Globecom Workshops, Austin, TX, pp. 194–200, 2014.

[9] J.B. Anderson, T. Aulin, and C.-E. Sundberg, Digital Phase Modulation. New York, London: Plenum Press, 1986.

[10] D. Bokolamulla and T. Aulin, “Multiuser detection for continuous phase modulation over Rayleigh fading channels,” IEEE Communications Letters, vol. 9, pp. 906-908, October 2005.

[11] P. Moqvist and T. Aulin, “Multiuser serially concatenated continuous phase modulation,” International Symposium on Turbo Codes, (Brest, France), pp. 211-214, 2003.

[12] A. Barbieri, D. Fertonani, and G. Colavolpe, “Spectrally-efficient continuous phase modulations,” IEEE Transactions on Wireless Communications, vol. 8, pp. 1564-1572, March 2009.

[13] A. Perotti, S. Benedetto, and P. Remlein, “Spectrally efficient multiuser CPM Systems,” IEEE International Conference on Communications (ICC 2010), Cape Town, South Africa, pp. 1-5, 2010.

[14] E. Biglieri, MIMO wireless communications. Cambridge University Press, 2007.

[15] S. M. Alamouti, “A simple transmitter diversity scheme for wireless communications,” IEEE J. Select. Areas Commun., vol. 16, no. 8, pp. 1451–1458, October 1998.

[16] B. Özgül, M. Koca, and H. Deliç, “Orthogonal Space-Time Block Coding for Continuous Phase Modulation with Frequency-Domain Equalization,” IEEE Trans. Commun., vol. 57, no. 12, pp. 3579–3584, Dec. 2009.

[17] G. Wang and X.-G. Xia, “Orthogonal space-time coding for CPM system with fast decoding,” Int. Symp. Inform. Theory, p. 107, 2002.

[18] B. E. Rimoldi, “A Decomposition Approach to CPM,” IEEE Transactions on Information Theory, vol. 34, pp. 260-270, 1988.

[19] M. Xiao and T. Aulin, “Serially Concatenated Continuous Phase Modulation with Convolutional Codes Over Rings,” IEEE Trans. Commun., vol. 54, no. 8, pp. 1387-1396, 2006.

[20] J.L. Massey and T. Mittelholzer, “Convolutional codes over rings,” 4th Joint Swedish-USSR Int. Workshop Information Theory, pp. 14-18, 1989.

[21] R. A. Carrasco and M. Johnston, Non-binary error control coding for wireless communication and data storage. Wiley, 2009.

[22] D. Divsalar and S. Dolinar, “Weight Distributions for Turbo Codes Using Random and Nonrandom Permutations,” JPL TDA Progress Report, vol. 42-122, pp. 56-65, Apr-Jun 1995.

[23] IT++ project, http://itpp.sourceforge.net/4.3.1/, accessed August 2018.