The K-means and TSP Based Mobility Protocol Modeling as a Probabilistic Combinatorial Optimization Problem

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***Abstract*— Fault tolerance is considered as a critical issue and a very interesting subject of research in Wireless Sensor Networks (WSN). Some sensors may be blocked or may fail due to a lack of energy or because of their manufacture. The external interactions (interferences, malicious attacks) can also be the source of malfunctions. The failure of sensors should not affect the network performance. This is a problem of reliability or fault tolerance which is the ability to maintain network functionality without interruptions due to a failure of a sensor node. It therefore aims to reduce the influence of these failures on the overall task of a wireless sensor network. Protocols and fault- tolerant approaches must be used to ensure reliable delivery of data packets to the base station and to guarantee reliable functioning even after the vulnerability of some network components. In this paper, we describe the K-means And Traveling Salesman Problem-based mobility protocol used to assure the proper functioning of the networks; we also propose a theoretical modeling of a probabilistic combinatorial optimization problem, which is explored through this method in order to minimize the energy consumption and improve fault tolerance for WSN.**

Keywords- Wireless sensor networks; failure; fault-tolerance; modeling; probabilistic.

I. Introduction

Wireless sensor networks represent a very promising domain that has become a new research focus in communication and computer fields. They can be used in different environments in a random way and in a large variety of applications due to their easy deployment and their low cost of construction. To achieve some applications, such as medical care, military surveillance, disaster relief and environmental monitoring, reliability is fundamental and essential. A wireless network is composed of plenty of sensors that are deployed in the monitoring field, and that have perception, processing and communication ability. However, the limitation of energy in wireless nodes, and hostile environments in which they could be used, are factors that make such networks very vulnerable [1][2]. In fact, they are subject to different forms of breakdowns that affect their reliability. These problems include computer attacks and hardware failures which are, nowadays, a real threat constantly growing. The failures of sensors can be caused by different reasons, including the physical damages, malicious attacks, environmental interferences, communication link errors or the depletion of energy [2]. Without successful transmission and secure routing, many applications of wireless sensor networks cannot work.

Because of their sensitivity, several research projects have been conducted in order to find solutions to these networks in the presence of failures and intrusions [3]. In fact, a sensor network must be able to maintain and keep its functionality without interruptions caused by the failures of nodes. In other words, these breakdowns should not affect the overall functioning of the network. This problem of fault tolerance has seen a great significance among various fields of research in wireless sensor networks. So, one of the basic challenges is to guarantee the safety and the proper functioning of equipments by developing fault-tolerant and robust algorithms that offer reliability of transmitted data. The purpose of this paper is to propose a possible direction of future researches by including a probabilistic modeling of the K-means And Traveling Salesman Problem-based mobility protocol [4], which is a fault tolerant protocol in wireless sensor networks. The rest of the paper will be organized as follows: In the second Section, we will present an overview of the fault tolerance problem. Section 3 will describe the principle of this relevant a priori protocol. A theoretical probabilistic modeling of it will be also proposed in Section 4. Section 5 presents the simulation results and analysis. Finally, we will conclude the paper in Section 6.

II. Fault Tolerance in WSN : Problem Presentation

The communications between the collector node and the other sensors of a network need the implementation of routing protocols that are based on multi-hop communication. Each sensor then acts as a router in addition to its role as a data source. However, faults can happen because of some problems such as the lack or the loss of energy, the interferences of the environment (heat, rain) or by a destructive agent (like animals), attacks (Sybil, Wormhole, Selective forwarding, sinkhole, etc.) and also the physical damages. In this case, it will be possible that one or many sensors do not operate. This causes the loss of communication links that leads to a significant change in the entire network topology. The fault is the primary source of an error that causes the system failure [5]. So, the network connectivity can be affected and its life will be decreased. In this case, it must be able to detect this error and to remedy it, by finding another way to transmit information and to maintain the network always operational.

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We can say that the goal of fault tolerance is to avoid the total flaw of the system despite the existence of errors in a subset of its elementary components. The tolerance degree depends on the nature of the application, its degree of criticality and on the exchanged data. Then, it is essential to provide fault tolerant protocols that allow us to choose the best paths in order to route information from the source to the collector. They also permit the selection of an alternative path if there was a failure while sending data on the initial route, in case of an interruption at one or several sensor nodes of it. In addition, the implementation of fault tolerant clustering protocols is useful in order to provide a better routing management [6]. Then, there are algorithms used to determine many routes from each node to the sink, which guarantees the presence of more than one reliable path for transmission. This provides a fast transfer resumption in case of failure on the first selected route (selecting one of the remaining routes). Other protocols realize a better management of energy use, with the aim of increasing the network lifetime [7]. Many works in the literature suggest fault tolerant methods in sensor networks, such as the Energy Aware Routing "EAR" protocol [8] and the Periodic, Event-driven, Query-based "PEQ" protocol [8], for many objectives. As each has been developed to achieve a specific purpose, they vary widely in many settings, including security, accuracy, configurability, cost and reliability. In this work, a probabilistic derivation of the reliability problem is considered.

Actually, although there are a large number of theoretical models for problems coming from the real world, the application of these models, in a direct way, is difficult and sometimes impossible due to the vagueness, the inaccuracy or the lack of data. In some contexts, based on estimations or statistical measures, a solution may be required even before the specification of information. In fact, there are several applications where obtaining current data in a certain way is not possible due to the volatile nature of data. The community of operational research has introduced several optimization frameworks to address these constraints [9]. In our work, a sensor network is modeled by a graph, and we are operating in a situation where the vertices do not exist in a deterministic way in this graph, but they are present in a probabilistic manner. In other words, a probability of presence will be associated with each vertex. This work is located in a study framework of combinatorial optimization problems when their proceedings are changing in a probabilistic way [10].

The aim is to present a priori strategy which ensures the reliability of data transmission in the presence of faults: on any instance of the problem, we avoid the total flaw of the system by changing the graph structure (transformation in a subgraph) according to a modification strategy that will be specified in advance. Fault tolerance in wireless sensor networks can then be presented as a probabilistic optimization problem that will be modeled based on different protocols and particularly the K-means And Traveling Salesman Problem-based mobility protocol, in which we will be interested in the following sections.

III. K-MEANS AND TSP-BASED MOBILITY "KAT-MOBILITY" PROTOCOL

Nakayama et al. [12] proposed the K-means And TSP-based mobility "KAT-mobility" protocol based on optimization algorithms of routing and aggregation. It is a model of mobility of sinks that can effectively collect detected data used in a wireless sensor network, even if some sensors are destroyed [11]. The system is composed of two modules: the clustering algorithm and the approximate solution for the TSP "Traveling Salesman Problem" [12]. The sensors are firstly divided into groups by using the clustering algorithm, from which the centers of the groups are determined as anchor points. The route of the mobile sink is determined as an approximate solution of the TSP. Here, it is assumed that an administrator distributes sensors to supervise the targeted area, and the sensors are scattered at random positions and do not move afterwards. After the grouping of sensor nodes, this method reaches the mobile sink to make a course through the centers of groups according to the trajectory of an optimized path. The mobile sink collects then the data coming from sensors on the level of the visited groups. Its trajectory is assumed to be random in order to mitigate malicious attacks. The compromise between the flow rate and the energy consumption is regarded as the efficiency metric during the evaluation. During this time, the KAT-mobility protocol can calculate the migration route for the sink in order to get around the damaged area or malfunctioned sensors due to attacks while preserving its random behavior. In other words, after the reorganization of the network into groups, the proposed approach pilots the mobile collector to move through the centers of the groups by taking the optimal route. Thus, the mobile collector recovers the information from the sensors of the visited groups. The principle of this protocol is summarized in these two procedures: the optimization of the routing path and the clustering. Figure 1 shows this principle [12].

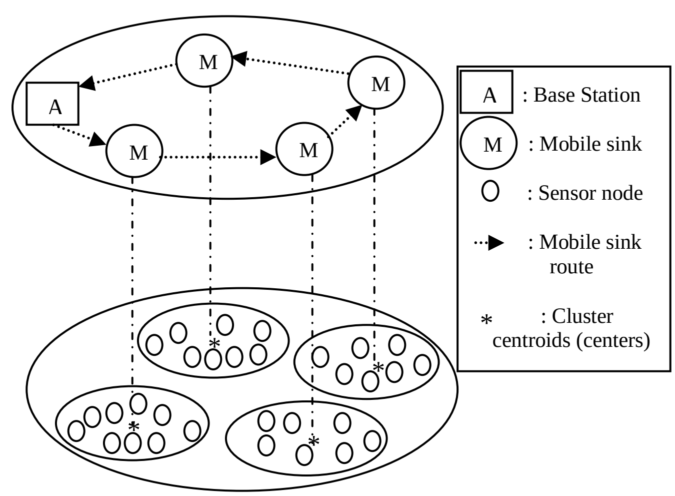

Figure 1. KAT-mobility protocol

A. Clustering Procedure

A sensor network is often composed of several thousands sensor nodes. To reduce the complexity of the routing algorithms, to facilitate data aggregation, to simplify the network management and to optimize energy consumption, sensors are grouped into clusters. The nodes that are grouped together in a cluster will be easily able to communicate with each other. A cluster head is elected to carry out several tasks, such as filtering, fusion and aggregation, with the possibility to be changed if it fails or if it reaches its power limit [13]. All communications of all nodes will be made through the head of the cluster to which they belong. The KAT-mobility protocol is based on clustering and especially on the K-means method [14][15], in which the cost of a group is estimated by the approximation error between the nodes and the collector. This algorithm divides the set of sensors virtually into K clusters (C_1, C_2 … C_K) geographically close. We denote by N the number of nodes in the network, usually N >> K. Let m_j (j = 1, 2 ... K) be a collector and x_i (i = 1, 2 ... N) be a sensor node, which is represented by a 2-dimensional vector (i.e., position of the node). d (x_i, m_j), which is indicated by the Euclidian distance between the collector (group center) and the sensor, represents the approximation error. The goal is to assign each node to a cluster C_j by reducing the total error of the clusters in order to reduce energy consumption [16]. The sum of approximate errors is expressed in the following formula:

$$DT\left(C_1(m_1), C_2(m_2), ..., C_K(m_K)\right) = \sum_{i=1}^{K} \sum_{x_i \in C_j} d(x_i, m_j) \quad (1)$$

The goal is to minimize the energy of communications by clustering in order to reduce the battery consumption of each sensor, which is proportional to d (x_i, m_j) [17]. The final objective is to assure the configuration of C_j such that DT is minimized. The clustering module of the KAT-mobility protocol can be summarized as follows [4]:

1. Initialize the location m_j randomly, t=0.

2. Define the threshold "THR" which is the stopping criterion of the following iterative process.

   - When $d^{(t)}(x_i, m_j) < d^{(t)}(x_i, m_{j*})$, $\forall$ j ≠ j*, assign a node $x_i \in C_j^{(t)}$.

   - Set the collector positions at the center of each group.

$$m_j^{(t+1)} = \frac{1}{\left|C_j^{(t)}\right|} \sum_{x_i \in C_j^{(t)}} x_i \quad (2)$$

   - Calculate the sum of approximation errors $DT^{(t+1)}$ at time (t+1), and if

$$\frac{\left|DT^{(t+1)} - DT^{(t)}\right|}{DT^{(t)}} \rangle \; THR \quad (3)$$

   is true, t can be updated and the iteration will be continued. Otherwise, the iterative process is stopped and the final center is set to $m_j^{(t+1)}$.

After the realization of the clustering procedure, the second module of the KAT-mobility protocol begins.

B. Routing Path Optimization

Finding the best route for the mobile node is analogous to the TSP. A sink represents then the traveling salesman and the cluster centroids define cities. The mobile sink passes through the clusters and gathers data coming from various nodes. As it is possible to increase efficiency by reducing the travel time, it is preferable that the sink traces the shortest route through the cluster centroids. The path optimization of the mobile collector to visit once and only once every cluster centroid is equivalent to searching for the shortest trip of the traveling salesman in order to visit each city once [4]. However, this problem is NP-hard [18]; therefore, the conclusion of the optimal trajectory can not be realized in an easy way. In fact, it is one of the most studied combinatorial optimization problems that its difficulty reveals especially through the large number of solutions. In order to solve this problem, a particular family of algorithms called the heuristics, ensuring the obtaining of almost optimal solutions, is proposed. For the KAT-mobility protocol, the local search algorithms Or-Opt and 2-Opt, based on the modification of a current solution to TSP by heuristics, are implemented [19][20]. Fortunately, for wireless sensor networks, nodes can communicate together, and the mobile sink does not need to visit all the nodes. After clustering the nodes, we only need to optimize the paths among cluster centroids. The mobile sink traces then the trajectory of the optimal TSP solution. Using the same formula mentioned in [21], the objective is finding the Hamiltonian path π that reduces the tour lengths.

$$\sum_{j=0}^{k-1} d\left(m_{\Pi(j)}, m_{\Pi(j+1)}\right) + d\left(m_{\Pi(k)}, m_{\Pi(0)}\right) \quad (4)$$

where the initial location of the mobile sink is m_0. This quantity mentioned in (4) denotes the tour length of a sink that will be carried out by visiting the centers in the specified order according to the permutation and while returning to the starting position [21]. The KAT-mobility method assumes that the mobile collector has a priori knowledge of the locations of its member sensors [4]. It is possible that the collector lose communications with its blocked or faulty member nodes. In this case, it can stay at the center of its cluster to discover broken nodes and its trajectory can then be recalculated as soon as it reaches the access point. Updating this path is preceded by a modification of the centers positions in the network, i.e., a new clustering procedure through the subset of functional nodes is performed [4]. Consequently, it will be more interesting to

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model the KAT-mobility method as a probabilistic combinatorial optimization problem.

IV. PROBABILISTIC MODELING

We model the wireless network by a graph G (V, E), where V is the set of cluster centroids and $E \subseteq V^2$ represents the set of edges reflecting the possible communications between these points. The pair (m_1, m_2) belongs to E if and only if m_2 is the neighbor of m1. We denote by K the number of vertices in the graph (|V| = K), which is considering here as the problem size. In wireless sensor networks, one or several sensors may not function correctly and in order to avoid the total flaw of the system despite the presence of faults in a subset of its elementary components, two approaches can be used: the re-optimization strategy treated by Nakayama et al. [4] and the a priori strategy that represents our proposal through this paper.

A. Re-optimization Strategy

Frequently in applications, after having solved a particular exemplary of a given combinatorial optimization problem, we must solve repeatedly many copies of the same problem. These additional copies are generally simple variations of the original problem; however, they are sufficiently different to require an individual treatment [8]. The most natural approach used to address this kind of situation consists in solving in an optimal way the different potential copies. We call this strategy "the re-optimization strategy" [22]. However, it has many disadvantages and the most important one is the high cost. This is the case of the KAT-mobility method consisting of repeating the clusters configuration and calculating the new solution through the set of surviving nodes after detecting failures in any part of the network. It is therefore necessary to adopt a different strategy. Rather than re-optimizing each successive exemplary, we can try to determine a priori solution of the initial problem that can be successively modified in a simple way to solve the following copies. We call this strategy "a priori strategy" [23].

B. Proposed a priori Strategy

A sensor network is modeled by a graph G, and we are operating in a situation where the vertices do not exist in a deterministic way in this graph, but they are present in a probabilistic manner. In other words, a probability of presence will be associated with each vertex. This work is located in a study framework of combinatorial optimization problems when their proceedings are changing in a probabilistic way [10][24]. The aim is to present a priori strategy which ensures the reliability of data transmission in the presence of faults: on any instance of the problem, we avoid the total flaw of the system by changing the graph structure (transformation into a subgraph) according to a modification strategy that will be specified in advance. In fact, we associate the probability p_i to each vertex $m_i \in V$ (the probability of remaining operational) taking into account that a center is considered as absent when all its cluster members are faulty. By applying the KAT-mobility protocol, finding a route for the mobile sink is analogous to the traveling salesman problem. A very natural probabilistic extension of this problem was introduced in 1985 for the first time by "Jaillet", when he assumed that the number of cities is a random variable [10]. Concerning the routing path optimization, we propose that only some centers among the K vertices will really require a visit according to their probabilities of occurring. In other words, we specify a strategy μ, called modification strategy, which removes absent centers from the initial a priori tour. Our purpose is to obtain a tour among the initial vertices such that the graph G is transformed into the subgraph G'= G [V'] where $V' \subseteq V$ is the set of present cluster centroids and the new route through its vertices will be in the same order as that established by the a priori tour [25]. This route is illustrated in Figure 2.

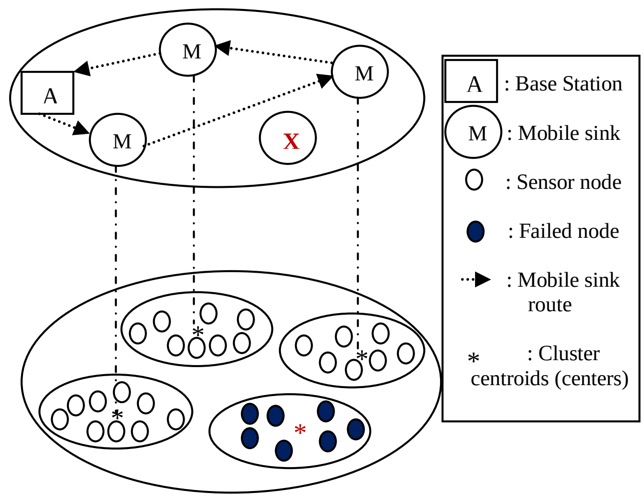

Figure 2. New routing path

This is a problem of finding a priori tour that minimizes the functional of covered distances [18]. Given the probability law ℙ, the set of cluster centroids, the set of all the subsets of V, i.e., each instance $V' \subseteq V$ has a probability of presence ℙ (V'). For a given tour R through the vertices defined on V, the modification method μ consists in deleting or gumming those who are absent from the a priori tour. Let $L_{(R,\mu)}$ be the random variable defined on 2^V, which for all V' of 2^V and with a tour R, associates the length $L_{(R,\mu)}$ (V') through V', induced of the tour R by the modification method. Consequently, the path optimization of the mobile collector to visit once and just once every sensor is equivalent to find the trajectory that minimizes the functional of $L_{(R,\mu)}$ [26][27].

$$\min_R \left(\mathbb{E}\,(L_{(R,\mu)}) = \sum_{V' \subseteq V} \mathbb{P}(V')\ L_{(R,\mu)}(V') \right) \quad (5)$$

The use of this a priori strategy allows the wireless sensor network to collect effectively detected information from external environments and deliver it to the required applications with reducing energy consumption, even if some nodes are destroyed. It was proposed to ensure transmission reliability, to increase the network lifetime and also to offer a real time solution, valid in all situations. To ameliorate this work, we propose the improvement of the route optimization in a particular case of this probabilistic problem which is the deterministic case (p_i=1) by implementing the local search method "Tabu" instead of the used algorithm 2-Opt.

V. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

In this section, simulation results are presented and analyzed. We simulated the KAT-mobility protocol and the proposed approach, based on the optimization of the routing path by using the Tabu algorithm, to evaluate their performance. We deploy 100 sensor nodes which are uniform-randomly distributed inside the simulation area. We consider the metric trajectory cost (expressed in meter) for evaluating the tour lengths. To perform our simulations, we used the software Java.

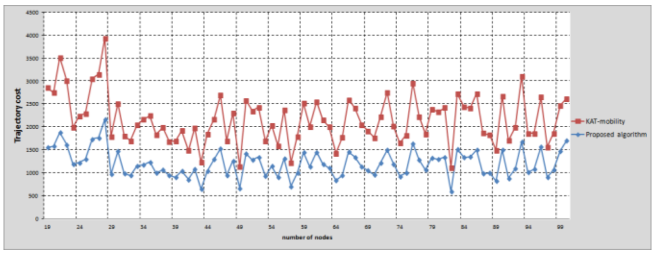

Figure 3. Trajectory cost

As can be seen from the simulation results of Figure 3, which presents the trajectory cost depending on the number of sensor nodes, randomly distributed, that varies from 19 to 100, our improved algorithm achieves better performance than the KAT-mobility algorithm because it guarantees the minimization of the tour length for any employed distribution.

Since the number of nodes may be disturbed and can vary from day to the other, we have proposed our a priori strategy shown at the previous section and to confirm the performance of this probabilistic model, we realized the comparison of the two algorithms.

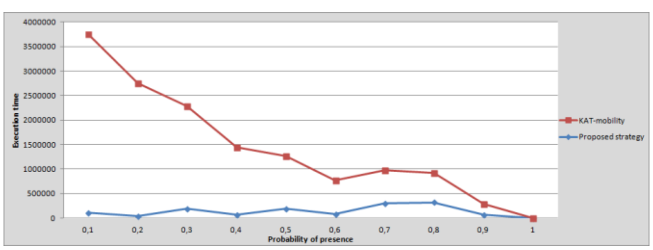

Figure 4. Execution time

It is shown in the simulation results of Figure 4, which presents the time execution (expressed in nanoseconds) depending on the probability of presence of the vertices, that our proposed strategy ensures the obtaining of solutions in real time once the problem is disturbed by the breakdowns of some sensors (the probability of remaining operational is less than 1). This proposed model is much more realistic and provides less execution time than the conventional strategy. It is clear from our simulations that the improvement provided by this new strategy is effective and that this new algorithm is valid in all situations.

VI. CONCLUSION AND FUTURE WORKS

The data routing in wireless sensor networks is considered as a complex problem because it is essential to ensure reliable delivery of information while consuming less energy. We have presented then in this paper the fault tolerant KAT- mobility protocol, as well as a proposed theoretical probabilistic modeling by considering the fault tolerance as a probabilistic combinatorial optimization problem. Our proposed strategy aims to provide not only better energy efficiency within the wireless network, but also better reliability compared with the conventional approach based on K-means clustering and the approximate solution for Traveling Salesman Problem, especially in the presence of faults. It is shown in the simulation results that this strategy was proposed to provide a better and practical solution by improving the route optimization of the collector. To accomplish this work, we aim to use new methods that navigate the mobile sink to go through the cluster centers according to the optimized route by implementing the "branch & bound" technique instead of the used algorithm 2-Opt, and it will be so interesting when the risk of sensor failures becomes important. Our goal is to propose strategies that guarantee the obtaining of solutions in real time once the problem is disturbed by the failures of some nodes.

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