

# A Design of Full-Rate Distributed Space-Time-Frequency Codes with Randomized Cyclic Delay Diversity

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**Abstract**—Cyclic Delay Diversity (CDD) method was introduced to cooperative communications for improving the system diversity performance. Since the CDD with fixed cycle delay cannot obtain the optimal system performance in all situations, this paper presents a full-rate distributed space-time-frequency codes scheme in the full-rate cooperative communication model by taking advantages of Randomized Cyclic Delay Diversity (RCDD) method and Linear Constellation Precoding (LCP) technology. The proposed scheme is more practical and has the advantage of low detection complexity. Compared with the full-rate cooperative communication scheme with Fixed Cyclic Delay Diversity (FCDD), the proposed scheme can achieve Better Bit Error Rate (BER) performance in the case of large number of subcarriers.

**Keywords**—randomized cyclic delay diversity; cooperative communication; OFDM; space-time-frequency code

## I. INTRODUCTION

The traditional two-hop cooperative mode requires two time slots to complete data transmission. It increases the system diversity gain at the cost of half of the system transmission rate [1][2]. In order to solve the problem, the Non-orthogonal Amplify and Forward (NAF) transmission mode with single relay was proposed by Nabar et al. [3]. With the help of the relay node, the source node transmits two symbols to the destination node within two adjacent time slots. Thus it can achieve full-rate data transmission. However, only the data sent from the source node in the odd time slot is forwarded at the relay node results in the imbalance of error rate between odd and even time slots, which is also known as "short-board effect". In order to overcome this phenomenon, linear constellation precoding is used for data transmitted within two adjacent time slots by Zhang et al. [4][5]. This method can achieve full diversity gain and improve the system spectral efficiency, but increases the decoding complexity. The cyclic delay diversity method is introduced in the full-rate cooperative transmission model by Kwon et al. [6]. It reduces the system detection complexity and increases the channel frequency selectivity and the system frequency diversity as well. In system with CDD, the BER performance is affected by the cyclic delay value [7][8]. To realize the time diversity in the system with CDD further, the concept of randomized cyclic delay diversity was proposed by Plass et al. [9][10]. In the

system with randomized cyclic delay diversity, the OFDM signals transmitted through each antenna are cyclically delayed in time domain respectively, where the cyclic delay value is selected randomly. This scheme not only obtains both the system frequency diversity and the system time diversity, but also improves the system BER performance without increasing the detection complexity at the receiving end. However, there exists a widespread problem that the system spectral efficiency is low. To improve the system diversity gain, the RCDD method is introduced to the multi-relay cooperative communication by Choi et al. [11][12], but the system cannot achieve full-rate data transmission and thus reduces the system spectrum efficiency.

In order to maximize the utilization of wireless network spectrum resources and improve the system diversity gain, this paper presents a distributed space-time-frequency coding scheme with randomized cyclic delay method and linear constellation precoding technology in the NAF full-rate multi-relay cooperative communication model.

The rest of the paper is organized as follows. Section II describes the NAF full-rate distributed cooperative communication model. The decoding scheme of the full-rate distributed space-time-frequency codes is given in Section III. Section IV analyses the performance of the proposed scheme. Finally, Section V presents the conclusion and future work.

## II. SYSTEM MODEL

As shown in Fig. 1, the non-orthogonal amplify and forward (NAF) full-rate cooperative transmission model was used in this paper. It consists of a source node  $S$ ,  $M$  relay nodes  $R_i$ ,  $i = 1, 2, \dots, M$ , and a destination node  $D$ . Each node is equipped with a single antenna, and operates in half-duplex mode. In the first time slot, the source node broadcasts the first signal to all the relay nodes and the destination node. In the second time slot, the source node continues to transmit the next signal to the destination node. Meanwhile the relay node forwards the signal received in the first time slot to the destination node after some processing. Source node completes the transmission of two symbols in two time slots, so that it can achieve full-rate transmission.

In order to overcome the "short-board effect", the symbol transmitted from the source node  $S$  is coded by linear constellation precoding. Assume that  $S_1$ ,  $S_2$  are the

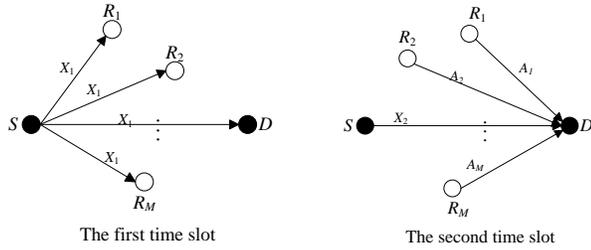


Figure 1. NAF full-rate cooperative transmission model.

symbols transmitted in two adjacent time slot from the source node in the frequency domain, respectively.  $\mathbf{X}_1, \mathbf{X}_2$  are the symbols after linear constellation precoding. Where  $\mathbf{S}_1, \mathbf{S}_2, \mathbf{X}_1$  and  $\mathbf{X}_2$  are length- $N_F$  column vectors,  $N_F$  represents the number of OFDM subcarriers. The relationship between  $\mathbf{S}_1, \mathbf{S}_2$  and  $\mathbf{X}_1, \mathbf{X}_2$  is given as (1).

$$\begin{bmatrix} \mathbf{X}_1(p) & \mathbf{X}_2(p) \end{bmatrix}^T = \mathbf{\Theta} \begin{bmatrix} \mathbf{S}_1(p) & \mathbf{S}_2(p) \end{bmatrix}^T \quad (1)$$

where,  $S_1(p)$  and  $S_2(p)$  correspond to the data transmitted on  $p$ -th subcarrier of  $\mathbf{S}_1$  and  $\mathbf{S}_2$ ,  $p \in [1, N_F - 1]$ .  $\mathbf{\Theta}$  is  $2 \times 2$  linear constellation precoding matrix. In the first time slot, the source node  $S$  broadcasts  $\mathbf{X}_1$  to all the relay nodes and the destination node  $D$ . Thus, the received signals at  $i$ -th relay node and destination node in  $p$ -th subcarrier are shown in (2) and (3).

$$R_i(p) = \sqrt{P_1} f_i(p) X_1(p) + N_i(p) \quad (2)$$

$$Z_1(p) = \sqrt{P_1} h(p) X_1(p) + W_1(p) \quad (3)$$

where,  $P_1$  is the transmit power of the source node,  $h(p)$  and  $f_i(p)$  represent the complex channel fading coefficients of  $S$  to  $D$  and  $S$  to the  $i$ -th relay node  $R_i$  in the frequency domain, respectively.  $X_1(p)$  is the data transmitted on the  $p$ -th subcarrier of  $X_1$ ,  $N_i(p)$  and  $W_1(p)$  are both complex additive white Gaussian noise (AWGN) with zero-mean and variance  $N_0$ .

In the second time slot, the relay node  $R_i$  forwards the signal  $R_i$  to the destination node  $D$  after randomized cyclic delay. Thus, the signal transmitted from the relay node  $R_i$  is

$$A_i(p) = \alpha R_i(p)^{CDD} \quad (4)$$

To ensure the transmit power of the relay node, the amplifying power factor gain  $\alpha = \sqrt{P_2 / (N_0 + P_1)}$ , where  $P_2$

is the transmit power of the relay node,  $R_i(p)^{CDD}$  is  $R_i(p)$  with random cyclic delay.

$$R_i(p)^{CDD} = R_i(p) e^{-j \frac{2\pi}{N_F} p \delta_i} \quad (5)$$

where,  $\delta_i$  is the cyclic delay value. The randomized cyclic delay scheme is used in this paper, so  $\delta_i$  is selected randomly between  $[0, N_F - 1]$ . Because delay in time domain is equivalent to the phase shift in the frequency domain, so in the frequency domain it is expressed as  $e^{-j(2\pi p \delta_i) / N_F}$ .

The source node broadcast  $\mathbf{X}_2$  to the destination node  $D$  in the second time slot as well. Thus, the received signal at the destination node in the  $p$ -th subcarrier is:

$$Z_2(p) = \alpha \sum_{i=1}^M \left( g_i(p) R_i(p)^{CDD} \right) + \sqrt{P_3} h(p) X_2(p) + W_2(p) \quad (6)$$

where,  $P_3$  is the transmit power of the source node in the second time slot,  $X_2(p)$  is the data transmitted on  $p$ -th subcarrier of  $\mathbf{X}_2$ ,  $g_i(p)$  is complex channel fading coefficient of the  $i$ -th relay node  $R_i$  to the destination node  $D$  in the frequency domain,  $W_2(p)$  is complex additive white Gaussian noise with zero-mean and variance  $N_0$ .

By combining (5) and (6) we can obtain:

$$Z_2(p) = \alpha \sqrt{P_1} \sum_{i=1}^M \left( g_i(p) f_i(p) e^{-j \frac{2\pi}{N_F} p \delta_i} \right) X_1(p) + \sqrt{P_3} h(p) X_2(p) + \tilde{W}_2(p) \quad (7)$$

where,

$$\tilde{W}_2(p) = \alpha \sum_{i=1}^M g_i(p) N_i(p) e^{-j \frac{2\pi}{N_F} p \delta_i} + W_2(p) \quad (8)$$

For a large value of  $M$ ,  $\sum_{i=1}^M |g_i(p)|^2 \approx M$  with a high probability. So, there has the result

$$\mathbf{E}[\tilde{\mathbf{W}}_2 \tilde{\mathbf{W}}_2^H] \approx N_0 (1 + M \alpha^2) \mathbf{I}_N \quad (9)$$

where  $\tilde{\mathbf{W}}_2 = [\tilde{W}_2(0), \dots, \tilde{W}_2(N_F - 1)]$ ,  $(\bullet)^H$  denotes the complex transpose conjugate and  $\mathbf{E}[\bullet]$  represents the expectation operation. Let us set  $K = N_0 (1 + M \alpha^2)$ . In order

to normalize the noises of the signals received in two time slots to be zero-mean and unit variance complex Gaussian, we divide (7) and (8) by  $\sqrt{N_0}$  and  $\sqrt{K}$  respectively. The normalized signals are given by (10) and (11).

$$\tilde{Z}_1(p) = \sqrt{c_1 \rho} h(p) X_1(p) + \tilde{W}_1(p) \quad (10)$$

$$\begin{aligned} \tilde{Z}_2(p) = & \sqrt{c_2 \rho} \sum_{i=1}^M \left( g_i(p) f_i(p) e^{-j \frac{2\pi}{N_f} p \delta_i} \right) X_1(p) \\ & + \sqrt{c_3 \rho} h(p) X_2(p) + \tilde{V}(p) \end{aligned} \quad (11)$$

where  $\tilde{W}_1(p)$  and  $\tilde{V}(p)$  are both complex additive white Gaussian noise with zero-mean and variance  $N_0$ . Let  $\rho = P/N_0$  be the total transmit signal-to-noise ratio of the system, where  $P = P_1 + MP_2 + P_3$ . Finally, the power allocation coefficients  $c_1, c_2, c_3$  are

$$c_1 = \frac{P_1}{P} \quad (12)$$

$$c_2 = \frac{P_1 P_2}{P \left( \sum_{i=1}^M |g_i(p)|^2 \cdot P_2 + P_1 + N_0 \right)} \quad (13)$$

$$c_3 = \frac{P_3 (P_1 + N_0)}{P \left( \sum_{i=1}^M |g_i(p)|^2 \cdot P_2 + P_1 + N_0 \right)} \quad (14)$$

### III. DECODING

From (10) and (11), we can obtain that the received signals at the destination node on the  $p$ -th subcarrier are

$$\begin{aligned} Y(p) = & \sqrt{\rho} \begin{bmatrix} \sqrt{c_1} X_1(p) h(p) \\ \sqrt{c_3} X_2(p) h(p) + \sqrt{c_2} X_1(p) \Psi(p) \end{bmatrix} \\ & + \tilde{N}(p) \end{aligned} \quad (15)$$

where,

$$\Psi(p) = \sum_{i=1}^M g_i(p) f_i(p) e^{-j \frac{2\pi}{N_f} p \delta_i} \quad (16)$$

$$Y(p) = \begin{bmatrix} \tilde{Z}_1(p) & \tilde{Z}_2(p) \end{bmatrix}^T \quad (17)$$

$$\tilde{N}(p) = \begin{bmatrix} \tilde{W}_1(p) & \tilde{V}(p) \end{bmatrix}^T \quad (18)$$

Using (1), we can rewrite (15) as (19).

$$\begin{aligned} Y(p) = & \sqrt{\rho} \begin{bmatrix} \sqrt{c_1} h(p) & \\ \sqrt{c_2} \Psi(p) & \sqrt{c_3} h(p) \end{bmatrix} \begin{bmatrix} X_1(p) \\ X_2(p) \end{bmatrix} + \tilde{N}(p) \\ = & \sqrt{\rho} \begin{bmatrix} \sqrt{c_1} h(p) & \\ \sqrt{c_2} \Psi(p) & \sqrt{c_3} h(p) \end{bmatrix} \Theta S(p) + \tilde{N}(p) \end{aligned} \quad (19)$$

where  $S(p) = \begin{bmatrix} S_1(p) & S_2(p) \end{bmatrix}^T$ , from the above equation, we can see that the received signals at the destination node are the linear transformation of the signals transmitted through the source node, that is to say, the proposed scheme has a one-dimensional equivalent channel, and with the number of the relay node increasing, the detecting complexity does not increase. Denote

$$\Gamma(p) = \sqrt{\rho} \begin{bmatrix} \sqrt{c_1} h(p) & \\ \sqrt{c_2} \Psi(p) & \sqrt{c_3} h(p) \end{bmatrix} \Theta \quad (20)$$

Then, (19) is equivalent to

$$Y(p) = \Gamma(p) S(p) + \tilde{N}(p) \quad (21)$$

Assume that the Channel Status Information (CSI) is perfectly known at the destination node, the signals  $\tilde{S}$  transmitted through the source node can be detected by using the Maximum Likelihood (ML) detection method.

### IV. SIMULATION RESULT AND ANALYSIS

The BER performance of the proposed scheme is simulated by Matlab in this section. Signals transmitted from the source node are firstly encoded by convolutional code and modulated into QPSK, where the rate of convolutional code is 1/2. Then, the signals are encoded with linear constellation precoding matrix  $\Theta$ . The channel model is frequency selective fading channel with the carrier frequency of 3.5GHz, and the multipath number is 2. The signal received at the relay node will be randomly cyclically delayed and then amplified and forward to destination node. The power allocation scheme in [4] was used in the paper. At the destination node, we use the ML detection method for decoding. The BER performance of system with RCDD and system with FCDD are shown in Fig. 2, Fig. 3 and Fig. 4, where the number of the relay node is 1, 2, 4, and the number of OFDM subcarriers is 32, 128 and 1024.

From Fig. 2, we can see that, in the case of small number of subcarriers, the BER performance of system with RCDD is substantially the same as the system with FCDD, and it improves as the number of the relay node increases.

From Fig. 3 and Fig. 4, we can see that, in the case of

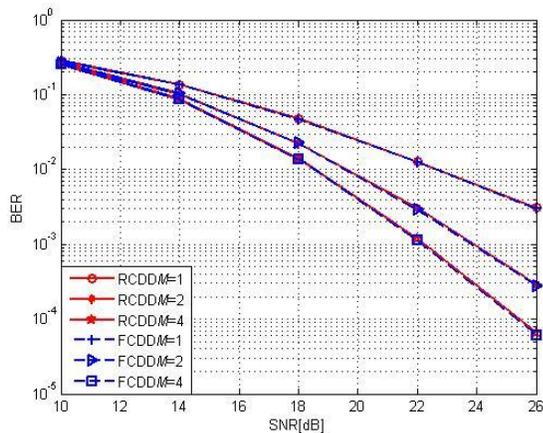


Figure 2. BER performance vs. SNR  $N_f = 32$  and  $M = 1, 2, 4$ .

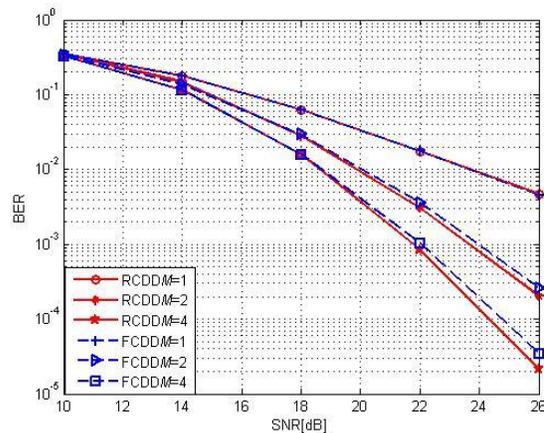


Figure 4. BER performance vs. SNR  $N_f = 1024$  and  $M = 1, 2, 4$ .

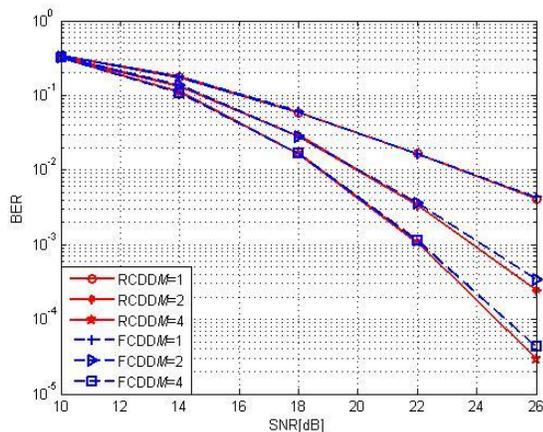


Figure 3. BER performance vs. SNR  $N_f = 128$  and  $M = 1, 2, 4$ .

large number of subcarriers, the proposed scheme can achieve better BER performance. From Fig. 2, Fig. 3 and Fig. 4, we can see that randomized cyclic delay can excavate the system diversity gain further as the number of subcarriers increases.

### V. CONCLUSION AND FUTURE WORK

Unlike fixed cyclic delay coding scheme used in NAF full-rate transmission system, the cyclic delay value is selected randomly in the proposed full-rate distributed space-time-frequency codes scheme, which is more practical than fixed cyclic delay. The proposed scheme is able to achieve better BER performance than the FCDD as the number of OFDM subcarriers increase, and it has the advantages of low detection complexity, i.e., the decoding complexity does not increase as the number of relay nodes increases.

In the future, we are planning to extend the present study to bi-directional distributed cooperative communication systems.

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