Exploring Analog Emulation of Quantum Computation Using Quadrature Modulation

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***Abstract*—With the capability to emulate quantum computing, our benefactors will be able to explore the behaviors of the same algorithms being tested in leading research centers, and develop new algorithms to approach new problems. A driven pair of coupled oscillators produce normal modes that emulate the behavior of a two-level system. A system of coupled LC oscillators produces a general Hilbert space achievable within classical systems. While operating at a fraction of the true quantum speed, and perhaps trailing in the number to IBM's and Google's $\geq$ 50, we will achieve exploration of the design space, utilizing a hybrid electronics solution, operating at room temperatures. In the collaborative effort, SDSU furthers the innovation of oscillator models for the qubit emulation, while Faster Logic designs an architecture to provide a coprocessor as a product.**

***Keywords*–LC Oscillators; Qubit; Emulation; Innovation.**

I. Introduction

Whereas quantum computers stand to drastically transform computation for a number of existing and future problems and are much studied [1]; their near-term realization comes with challenges. The quantum computing (QC) industry has devised a metric called the total quantum factor (TQF) to aid comparison of the size and performance of the quantum circuit[2]. Simulations can provide noise modeling, a step that is recommended prior to migrating circuit designs to the IBMQ system[3]. As a means of reducing risk of quantum-computing adoption, simulation and emulation techniques make it possible to consider the advantages in the real-world applications of cryptography, machine learning, signal processing, and cyber-security.

The quantum state has a continuum of possible superpositions. The properties of linear superpositions in a Hilbert space provide quantum computers their enhanced efficiencies relative to classical computers. Other researchers [4] [5] have recognized this since the Hilbert space and the linearity of the operations upon it impart efficiency to quantum computation. The allowed eigenstates in the representative Hilbert space scale exponentially with the number of implemented qubits. Therefore, it makes sense to explore more readily available classical systems which have an adequately similar mathematical structure. It may be possible to exploit such systems in order to emulate properties of quantum computers.

Our advancement under Navy Small Business Technology Transfer (STTR) funding will be to interface these classical electronic circuits into a coprocessor architecture that works together with the more ubiquitous digital computer. Because of their natural parallelism, quantum computers show promise for solving difficult optimization problems related to sensor signal processing. Through the utilization of existing analog and mixed-signal integrated circuit platforms, an alternative to quantum computing could be possible.

The Navy seeks a quantum emulation device that can be integrated as a co-processor into shipboard computing platforms. While there exist devices which manifest actual quantum behavior, such as superconductors and trapped ions, quantum states in such devices can be extremely fragile and are easily affected and contaminated by external disturbances. Our effort will produce a classical, emulated system using mixed-signal integrated circuit platforms, as an appealing alternative to actual quantum computing and as a surrogate tool that serves well in the interim.

Beginning with the concept development of a circuit level simulation platform for the quantum emulation device, we will design a 5-qubit quantum emulation device co-processor that connects to a computer via the Peripheral Component Interconnect Express (PCIe) standard interface. A selection of computational benchmarks are addressed based on previous simulations and the active use of existing quantum computing systems. We address the means to evaluate the performance of a quantum emulation device; we have a capability that can compare how algorithms will scale on QC systems.

The business direction is to fabricate and test the components used to assemble the 5-qubit quantum emulation device. Our research includes analyzing the scaling within our constructions, and exploring other limitations such as memory and processor speed. Based upon benchmark results, we will extend the co-processor's capability to 10 qubits or more. Office of Naval Research (ONR) is looking to offer this extended qubit co-processor card for use in a variety of Navy applications. The emulation devices also offer a low-cost educational tool for teaching quantum computing methods and for testing quantum computing algorithms.

These techniques open the doors to learning and educational outreach. Like the simple, intuitive systems used to model more abstract physics, coherent quantum states are emulated in classical analog electronics. This modeling has been a crucial step in our San Diego State University (SDSU) laboratory work, as we have been preparing to build and deploy actual low - temperature superconducting devices.

In the paper, Section II describes the theory and background behind emulation. Section III provides the project accomplishments, including several simulation efforts being worked by our research team; it also describes the planned work ahead. Section IV explains how the work will produce a result of interest to the customer and the QC industry.

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II. An Emulated Qubit: An Electronic Two-Level System

The arguably simplest localized system in quantum mechanics, a two-level atom, also serves as the ideal qubit. It is restricted to two discrete energy levels and is characterized by a two-dimensional Hilbert space. Its wavefunction is a linear combination representing the occupation probabilities of the two states. Because of their simplicity and adaptability, two-level systems are the basis of some of the most fundamental studies of quantum mechanics. A two-level atom in a coherent state is depicted by the following

$$|\Psi(t)\rangle = a(t)\,|-\rangle + b(t)\,|+\rangle\,. \tag{1}$$

where a and b are time-dependant, complex amplitudes of the ground $|-\rangle$ and excited $|+\rangle$ states respectively. When used to describe a single two level system, the normalization condition $aa^* + bb^* = 1$ is required to ensure the system is normalized.

The density matrix, ρ, clearly depicts the state of a quantum ensemble. It is an Hermitian matrix defined as the outer product of the wave functions of the system

$$\hat{\rho} = \sum_i p_i \,|\Psi_i(t)\rangle\,\langle\Psi_i(t)| \tag{2}$$

For the two-level system, specifically in a coherent state,

$$\hat{\rho} = \begin{pmatrix} b^*b & ba^* \\ ab^* & aa^* \end{pmatrix} = \begin{pmatrix} |b|^2 & |a||b|e^{i\phi} \\ |a||b|e^{-i\phi} & |a|^2 \end{pmatrix}. \tag{3}$$

Complex elements of this density matrix can be rearranged to depict a real 3-space by the following components.

$$\begin{aligned} u &= \rho_{12} + \rho_{21} \\ v &= i\,(\rho_{12} - \rho_{21}) \\ w &= \rho_{22} - \rho_{11}. \end{aligned} \tag{4}$$

The time evolution of this vector is given by the well-known Bloch equations.

$$\frac{d}{dt}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} -1/T_2 & -\delta & 0 \\ \delta & -1/T_2 & A \\ 0 & -A & -1/T_1 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix}. \tag{5}$$

Here δ is the detuning, A is the Rabi frequency, and T_1 and T_2 are relaxation times. The Rabi frequency is the frequency at which the system rotates around the u-axis as the field is applied. Motion of the coherent state may be depicted on the Bloch Sphere, as in Figure 1. Our prototype includes this mapping for our data.

Our emulated two-level system is based on the dynamics of an electronic system containing two normal modes, from the coupled action of two inductor-capacitor LC oscillators, as in Figure 2. By introducing symmetrical voltage-controlled inductors, the equivalent to atom-photon interaction is introduced, an external coupling to our qubit. As with true two-level atoms, the state of this emulated atom can be manipulated around points on the Bloch sphere by way of resonant pulse sequences.

The coupling capacitor present between these oscillators of equal resonant frequency will create hybridized states and an energy splitting. Therefore, electronic states which emulate the presence of a “ground” and “excited” state are produced. The influence of an external control in this “emulated atom” is provided by the action of symmetric voltage-controlled inductors. The coupling produced by these dynamical inductors emulates an atom-photon interaction, and provides external control of this system. Classical analogs to quantum phenomena are produced; one example being the manifestation of Rabi oscillations. Measuring our atom’s output ports details the voltage modulations at the resistor in the feedback loop. By overlaying the modulations upon the source-carrier frequency, we can visualize our classical analog to Rabi oscillations (Figure 3).

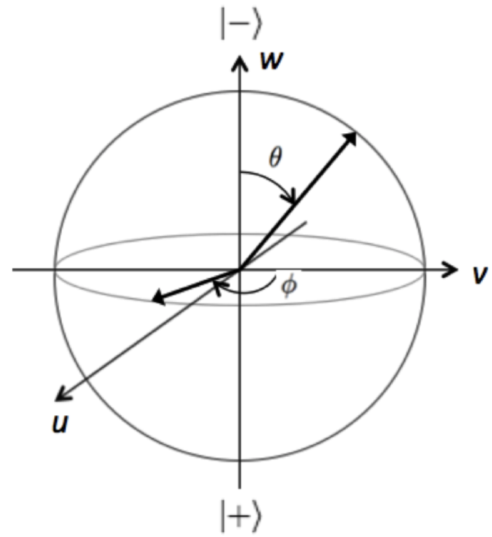

Figure 1. Coherent states of a single qubit map onto the Bloch Sphere.

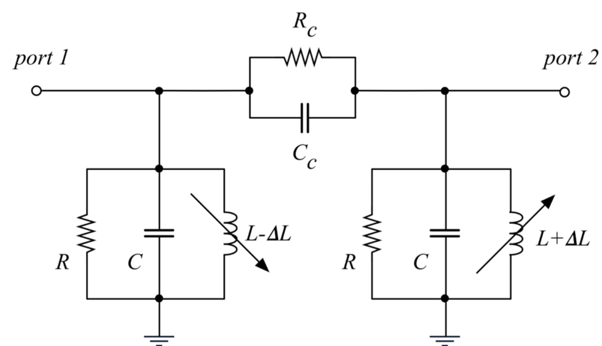

Figure 2. Coupled oscillators emulate the behavior of a two-level system.

III. Details

In the interim report of our contracted research, in a format meeting the requirements of DI-MGMT-80555A, we provided the Office of Naval Research, our Government customer, with the means to evaluate program progress made by the Faster Logic contractor and SDSU subcontractor.

A. Research to emulate

Our research and experimentation includes numerous software packages for personal computers which have been developed to simulate quantum computation. The software simulators are a family of solutions used to explore quantum computation memory and processor architectures; the FPGA simulators are well-suited for the heavily parallel problems required in quantum state evolution. Progressing from simulation to emulation addresses the quantum gate matrix operations of complex numbers.

In a recent work [6], a discussion of hierarchical modeling considers approaches and improved re-usability in previous works. The modeling of each quantum gate as an individual component requires greater resources, reduces accuracy, and limits scalability. Their approach to pipelining the entire matrix architecture significantly reduces the resource utilization and emulation times, thus improving scalability and allowing the use of floating-point precision improving accuracy.

B. Schedule

The Small Business Technology Transfer (STTR) program offers a six-month phase I timeline, truly a challenge even in normal non-pandemic times. Upon award of Phase II, the company and academic collaborator will proceed from concept to prototype. The phases of Small Business Innovation Research (SBIR) are tied to the Technology Readiness Levels (TRL)[7].

C. Simulation Studies

Classical simulation techniques, while inadequate for general quantum computation, serve to guide a path toward quantum supremacy (super-classical behavior of controllable quantum systems). In some particular cases, such simulation is known to be easy. Yet, both digital and analog quantum simulation aid the investigation of coupled quantum phenomena. [8]

We wish to ultimately produce a full-scale emulation of a Hilbert space structure capable of acting as a basis for quantum computation. As this goal requires several layers of engineering sophistication, we have found that we gain insight through several variations of independent, numerical simulation. While we recognize classical *emulation* as a goal far superior to (and separate from) *simulation*, we also acknowledge simulation as a necessary staging tool in our path forward. Here, we summarize three independent student-simulation efforts that emphasize different aspects of our work.

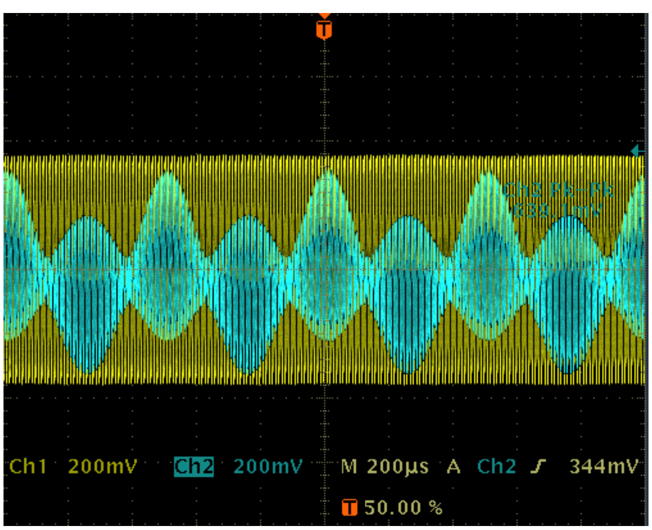

Figure 3. Laboratory qubit emulation results: Rabi oscillations.

1) Cirq: Introduced in 2018, Google Cirq, [9] a software library for simulation, implements the behavioral functionalities of a quantum computer. Working from available Python source models for Deutsch-Jozsa [10], Grover's Algorithm [11] and the QFT [12], three of the most frequently-studied algorithms; we varied the number of qubits. We also added supporting code [Listing 1] to measure simulation compute time [13] as our measurement and indication of complexity.

Listing 1: measuring simulation run times

```
start_time = process_time()
result = simulator.run(cirq.Circuit
  (circuit_under_test()),repetitions=N)
end_time = process_time()
simulation_time = end_time - start_time
```

Simulation runtime irregularities, likely a factor of Python's built-in caching and memory management [14] on our cloud-compute platform [15], and timing measurement precision were remedied by large repetition counts and repeated experiments subject to a median filter to remove outliers. As the quintessential illustration, we generalized Deutsch's algorithm to multi-Qubit inputs. The algorithm determines if a given function U_f, also known as the oracle, is balanced or a constant. The schematic diagram for the algorithm is obtained from [6] and visually comments our code.

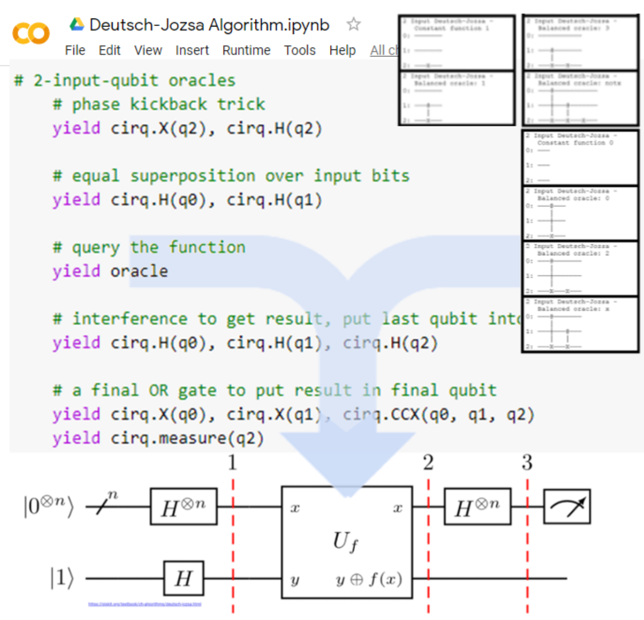

Figure 4. Deutsch-Jozsa in Cirq Jupyter notebook

For benchmarking, we avoid timing the set-up time (oracle creation) for U_f, instead measuring just the simulation. We extended the Cirq Workshop Bootcamp Final [16] from its 1-qubit and 2-qubit examples to address more qubits. Our cloud simulation processes [15] will support simulations beyond the 5-qubit requirement of the contract sponsor. Other simulators

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have been compared [17]. The classical computer's function evaluations, of which at least $2^{n-1}+1$ out of 2^n are required, could be replaced with a single process on the actual quantum computer, giving an exponential speedup [18]. Figure 4 shows the constructed Deutsch-Jozsa oracles within Cirq, and illustrates our approach to group them for comparison. Every n-input-qubit oracle has one or two meaningful comparison approaches to observe the scaling results with $m > n$ input qubits.

2) Verilog on Intel (Altera): Simulation efforts are also being investigated using FPGAs (Field-Programmable Gate Arrays). FPGAs allow for rapid calculations. As they are inherently parallel in architecture, FPGAs achieve some of the massively parallel needs required for quantum computing simulations. Our FPGA solution can also serve as a test architecture for our future low-temperature circuitry.

We are presently investigating the progress made in matrix manipulation algorithms and tensor products on FPGAs, which simulate combinations of gate operations [19]. Matrix-matrix multiplication, a computationally expensive part of simulations, permits exploring algorithmic trade-offs which may give insight into speeding up quantum computing and simulations in general.

FPGA chips do have hardware limitations. While the parallel nature of an FPGA lends itself to simulating quantum entangled systems, it can ultimately only perform classical computations. The speed of these operations are determined by memory and processor speed.

3) National Instruments' LabVIEW: Based on the emulation scheme in [5], we use quadrature modulation (*e.g.,* I and Q) to represent qubit states. We identify multiple qubits by characteristic frequencies. Hilbert states are formed by products of these waveforms. The resulting quantum state may be decomposed for gate operations by projecting onto subspace projection signals. In order to investigate the octave spacing scheme described by La Cour (2015), a quadrature modulation I&Q scheme was used to represent qubit states using National Instruments' LabVIEW Virtual Instruments. This approach offers a practical block-level description of the synthesis process for constructing arbitrary quantum states, as well the possibility for exploring physically feasible methods for performing single or multi-qubit gate operations and measurements. Figure 5 shows an important laboratory result, a 100 MHz carrier signal modulated by the wave function describing a 3-qubit state. The real and imaginary parts of the wave function can be extracted from the carrier signal using I&Q demodulation. The harmonic components representing distinct Hilbert states can be analyzed or operated on using gates to produce new states. Our wavefunction is

$$\begin{aligned}|\Psi\rangle = &|000\rangle + |001\rangle + |010\rangle + |011\rangle \\ &+ |100\rangle + |101\rangle + |110\rangle + |111\rangle\,.\end{aligned}$$

The n qubits are identified with octave-spaced frequencies ω_n below the carrier frequency ω_c. The qubit states $|0\rangle_n$ and $|1\rangle_n$ are represented by the positive or negative frequency components of $e^{\pm i\omega_n t}$. Each of the 2^N computational basis states are then $e^{i\Omega_x t}$, where each Ω_x is a unique permutation of sums and differences of the n qubit frequencies. The quantum state $|\Psi\rangle$ is a linear combination of these distinct frequency

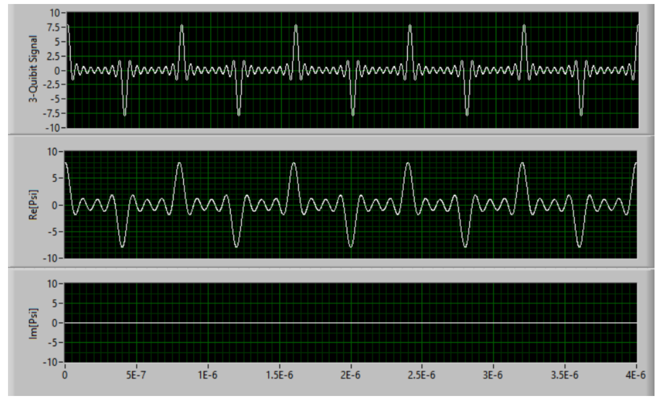

Figure 5. LabVIEW simulation output results for the 3-qubit state

components, with complex coefficients. Using I&Q modulation the real and imaginary parts of $|\Psi\rangle$ can be constructed explicitly in LabVIEW using a system of sine and cosine wave generators, amplifiers, inverters, and adders.

D. Experimental Work / Test Procedures

Based on the work of [4], a time-domain voltage waveform can represent an entire 2-level coherent state by way of an I and Q quadrature representation. This was later adopted and extended with modulation techniques as per La Cour [5]. Using a carrier and n-frequencies to represent n qubits, the Hilbert-space compound states can be represented as signals with staggered frequencies.

Available in our laboratory, we have several signal generators, oscilloscopes, PCB fabrication tools, computer interfacing modules enabled by LabVIEW, and the NI Engineering Laboratory Virtual Instrumentation Suite (ELVIS) board. These allow us to construct and manipulate analog signals for staging our designs.

Using LabVIEW virtual instruments along with the Siglent signal generator, models 1025 and 1032X, we set the frequencies 366 KHz, 732 KHz, 1.464 MHz 2.928 MHz, 5.856 MHz, 1.365 MHz, finding this to be a solution within the output capacity of our in-lab Siglent 1025 signal generators and having unique values for all possible permutations of the frequency sum. The potential laboratory installation concept is shown in Figure 6.

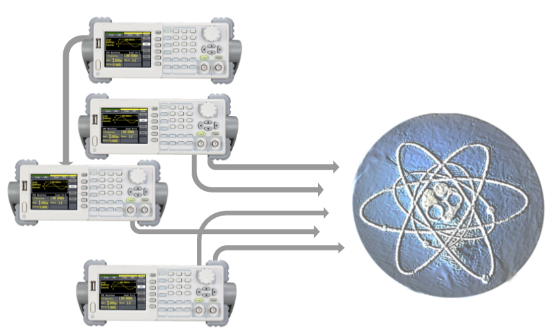

Figure 6. Signal Generators provide stable coordinated oscillations in the laboratory

As we continue with the Phase I, we will progress to experimental work with the Siglent waveform generators. These bench-top systems are ISO 9001 manufactured; they address the record-keeping and traceability of measurement so that we can architect an ISO/IEC 17025 testing and calibration approach. Our laboratory is equipped with the Voltera V-One circuit board printer, already employed in one master's student's fabrication accomplishments.

E. Designs

The description of our emulation architecture design is still in progress. Utilizing frequency-mixing protocols described in [5], we interface with a digital computer, and represent coherent qubit states with separate physical ports for ground and excited-state waveforms. We believe this is a useful compromise in reducing the complexity in constructing Hilbert space states from frequency-mixed states in return for additional layout complexity.

F. Test equipment

Our teaching laboratory has 12 of the Siglent 1025 systems. The Siglent 1032X waveform generator, which we plan to upgrade entirely to, offers LAN connectivity. We are considering an advanced model with I/Q modulation.

The Digital to Analog interface is a Pmod DA1 Four-Channel 8-bit DAC. There is no special test equipment required in the near-term plan; all of our test equipment is commercial off-the-shelf (COTS).

IV. Conclusion

While the quantum computing revolution has shown promise over the last two decades, we have seen an opportunity to advance emulation in just the course of this year. Recommended to consider the PICe coprocessor interfacing, we will experience a doubling of speed as PCIe version 4 reaches the desktop community. Exploring the advantages of quantum computation gives the learner and developer a new interest in the use of classical analog electronics. Our SDSU laboratory work continues on course as the quantum computing program introduces new course offerings.

In our SDSU laboratory work, we have successfully demonstrated the analog to a two-level system (and potential qubit) with pair of driven LC oscillators. Advancement under Navy STTR funding will allow us to better interface and control these classical electronic circuits.

Quantum computer models that can be realized using existing analog and mixed-signal integrated circuit platforms [20] and, when integrated as a fieldable solution, meets the Navy need, integrated as a co-processor into shipboard computing platforms.

A selection of quantum computational benchmarks have been modeled in the Google Colab environment using iPython Jupyter notebooks. These will resurface to evaluate the performance of the quantum emulation device; by varying the number of qubits, we can compare how algorithms will scale on QC systems.

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