Decentralized State-Space Control Involving Subsystem Interactions

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***Abstract*—The paper presents new points of view to the problems concerning the robust stability of a class of large-scale systems with subsystems interactions. The asymptotic stability conditions are formulated in terms of LMI while the impact of interconnection uncertainties is minimized using H_∞ approach. As results, a sufficient condition for the existence of solutions to this constrained stabilization problem is provided and a non-iterative algorithm for the control design solution is given.**

***Keywords*-Decentralized control; stabilizing conditions; linear matrix inequalities; control of power systems.**

I. Introduction

The control of large-scale linear systems has been studied by many researchers. If the linear model of a large dynamic system is partitioned into interconnected subsystems, the interactions of the subsystem play significant role in global system stability and, if interactions contain uncertainties, expected performances cannot be attained if the control is designed only for the nominal models. The success of these methods can be improved if the system state are grouped so that subsystem interaction is minimized and the decentralized controllers are optimized with respect to interaction uncertainties. The first results for the existence of robust decentralized controllers mostly involve the conditions under which the interconnection matrix in the considered system satisfies the prescribed matching condition [14].

Recently, a number of efforts have been made to extend the application of robust control techniques using convex optimization, involving linear matrix inequalities (LMI). It is well known that LMI-based approaches [3] are powerful for a centralized control design, but, in the decentralized case, the control design task may not be reducible to a feasibility problem because of control law structural constraints.

To meet modern system requirements, controllers have to quarantine robustness over a wide range of system operating conditions and this further highlights the fact that robustness to interconnections and interaction uncertainties is one of the major issues. Applying for power systems control, the most important terms are robustness and a decentralized control structure [10]. The robustness issue arises to deal with uncertainties which mainly come from the varying network topology and the dynamic variation of the load. On the other hand, since a real-time information transfer among subsystems is unfeasible, decentralized controllers must be used. To achieve less-conservative control gains design conditions, norm-bounded unknown uncertainties in subsystem interactions, or nonlinear bounds of interconnections, are included in terms of design [6].

This paper is sequenced in eight sections and one appendix. Following the introduction in Section I, the second section places the results obtained within the context of existing requests. Section III briefly describes the problems concerning with control of the large-scale dynamical systems with subsystem interactions. The preliminaries, mainly focused on the H_∞ design approach and the bounded real lemma, are presented in Section III. Section IV provides the quadratic stability analysis of the controlled system by use of LMIs, and states the newly proposed conditions for the state controller design. Section V illustrates the controller design task by the numerical solution and the system stability analysis and Section VI draws some concluding remarks. Appendix is devoted to a model of the multi-area power systems, used in the illustrative example.

II. The State of the Art

During the past two decades, there has been significant but scattered activity in control of the systems with interactions. A necessary and sufficient condition for solvability, for the case of fixed interconnections, has been found, e.g., in [4], [5], [15], where a homotopic method was used to reduce the control design to a feasibility problem of a bilinear matrix inequality (BMI). Moreover, if the LMI method is adopted by using a single Lyapunov function [1], [13], it leads to very conservative results.

The paper reflects the problems concerning with the system robust stability for one class of disturbed large-scale systems, in the presence of interconnection uncertainties among subsystems. The used approach is concentrated on

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performance improvement of control systems and is a continuation of the earlier work started in [9], [12], especially motivated by the techniques presented in [2]. Comparing with the above mentioned articles, the merit of the results proposed in this paper relies on the conservatism reducing, the disturbance transfer function norm minimization, the system dynamics improvement and the decentralized control design simplification. This represents issues which lead to a newly formulated set of LMIs, giving the sufficient conditions for design of the decentralized controllers. Results are illustrated using the load frequency control model of the multi-area power systems.

III. Problem Formulation

To formulate the control design task, it is assumed that the subsystems are given adequately to (A.10), (A.11), i.e. for $i = 1, 2, ..., p$ it is

$$\dot{\boldsymbol{q}}_i(t) = \boldsymbol{A}_i \boldsymbol{q}_i(t) + \boldsymbol{b}_i u_i(t) + \sum_{l=1}^{p} \boldsymbol{G}_{il} \boldsymbol{q}_l(t) + \boldsymbol{f}_i d_i(t) \tag{1}$$

$$y_i(t) = \boldsymbol{c}_i^T \boldsymbol{q}_i(t) \tag{2}$$

where $\boldsymbol{q}_i(t) \in I\!R^{n_i}$ is the vector of the state variables of the i-th subsystem, $u_i(t), y_i(t) \in I\!R$ are input and output variables of the i-th subsystem, respectively, $\boldsymbol{A}_i, \boldsymbol{G}_{il} \in I\!R^{n_i \times n_i}$ are real matrices, $\boldsymbol{b}_i, \boldsymbol{c}_i, \boldsymbol{f}_i \in I\!R^{n_i}$ are real column vectors.

It is supposed that all states variables of a subsystem are observed or measured, pairs $(\boldsymbol{A}_i, \boldsymbol{b}_i)$ for all i are controllable, and the i-th subsystem is controlled by the local control law

$$u_i(t) = \boldsymbol{k}_i^T \boldsymbol{q}_i(t) \tag{3}$$

where $\boldsymbol{k}_i \in I\!R^{n_i}$ is a constant vector.

Writing, in general, the subsystem interconnections as

$$\boldsymbol{G}_i \boldsymbol{h}_i(\boldsymbol{q}(t)) = \sum_{l=1}^{p} \boldsymbol{G}_{il} \boldsymbol{q}_l(t) \tag{4}$$

where $\boldsymbol{h}_i(\boldsymbol{q}(t)) \in I\!R^{n_i}$ is a vector function, it is supposed that

$$\boldsymbol{h}_i^T(\boldsymbol{q}(t)) \boldsymbol{h}_i(\boldsymbol{q}(t)) \leq \varepsilon_i^{-1} \boldsymbol{q}^T(t) \boldsymbol{w}_i^T \boldsymbol{w}_i \boldsymbol{q}(t) \tag{5}$$

where $\varepsilon_i^{-1} > 0$, $\varepsilon_i \in I\!R$ is a scalar parameter, related to interconnection uncertainties in the system, and $\boldsymbol{w}_i$ are constant vectors of appropriate dimensions, as well as that

$$\boldsymbol{q}^T(t) = \begin{bmatrix} \boldsymbol{q}_1^T(t) & \boldsymbol{q}_2^T(t) & \cdots & \boldsymbol{q}_p^T(t) \end{bmatrix} \tag{6}$$

Using the above defined overall system state variable vector $\boldsymbol{q}(t)$, (5) can be written as

$$\begin{aligned} \sum_{l=1}^{p} \boldsymbol{h}_l^T(\boldsymbol{q}(t)) \boldsymbol{h}_l(\boldsymbol{q}(t)) = \boldsymbol{h}^T(\boldsymbol{q}(t)) \boldsymbol{h}(\boldsymbol{q}(t)) \leq \\ \leq \boldsymbol{q}^T(t) \left[\sum_{l=1}^{p} \varepsilon_l^{-1} \boldsymbol{w}_l^T \boldsymbol{w}_l \right] \boldsymbol{q}(t) \end{aligned} \tag{7}$$

The global system model with the subsystem interactions takes the form

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) + \boldsymbol{G}\boldsymbol{h}(\boldsymbol{q}(t)) + \boldsymbol{F}\boldsymbol{d}(t) \tag{8}$$

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) \tag{9}$$

where

$$\boldsymbol{u}^T(t) = \begin{bmatrix} u_1(t) & u_2(t) & \cdots & u_p(t) \end{bmatrix} \tag{10}$$

$$\boldsymbol{y}^T(t) = \begin{bmatrix} y_1(t) & y_2(t) & \cdots & y_p(t) \end{bmatrix} \tag{11}$$

$$\boldsymbol{d}^T(t) = \begin{bmatrix} d_1(t) & d_2(t) & \cdots & \boldsymbol{d}_p(t) \end{bmatrix} \tag{12}$$

$$\boldsymbol{A} = \operatorname{diag} \begin{bmatrix} \boldsymbol{A}_1 & \cdots & \boldsymbol{A}_p \end{bmatrix}, \ \boldsymbol{B} = \operatorname{diag} \begin{bmatrix} \boldsymbol{b}_1 & \cdots & \boldsymbol{b}_p \end{bmatrix} \tag{13}$$

$$\boldsymbol{C} = \operatorname{diag} \begin{bmatrix} \boldsymbol{c}_1^T & \cdots & \boldsymbol{c}_p^T \end{bmatrix}, \ \boldsymbol{F} = \operatorname{diag} \begin{bmatrix} \boldsymbol{f}_1 & \cdots & \boldsymbol{f}_p \end{bmatrix} \tag{14}$$

$$\boldsymbol{G} = \operatorname{diag} \begin{bmatrix} \boldsymbol{G}_1 & \cdots & \boldsymbol{G}_p \end{bmatrix} \tag{15}$$

where $\sum_{i=1}^{p} n_i = n$, $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t), \boldsymbol{y}(t) \in I\!R^r$, $\boldsymbol{A}, \boldsymbol{G} \in I\!R^{n \times n}$, $\boldsymbol{B}, \boldsymbol{F} \in I\!R^{n \times r}$ and $\boldsymbol{C} \in I\!R^{r \times n}$.

The goal is the designing the parameters of the control law

$$\boldsymbol{u}(t) = \boldsymbol{K}\boldsymbol{q}(t) = \operatorname{diag} \begin{bmatrix} \boldsymbol{k}_1^T & \boldsymbol{k}_2^T & \cdots & \boldsymbol{k}_p^T \end{bmatrix} \boldsymbol{q}(t) \tag{16}$$

$\boldsymbol{K} \in I\!R^{r \times n}$, which rises up the stable large-scale system.

IV. Preliminary Results

Definition 1: Let the state-space model of the linear MIMO system is described by the vector differential equation

$$\dot{\boldsymbol{q}}(t) = \boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{17}$$

and by the output relation

$$\boldsymbol{y}(t) = \boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{D}\boldsymbol{u}(t) \tag{18}$$

where $\boldsymbol{q}(t) \in I\!R^n$, $\boldsymbol{u}(t) \in I\!R^r$, and $\boldsymbol{y}(t) \in I\!R^m$ are vectors of the state, input and output variables, respectively, and $\boldsymbol{A} \in I\!R^{n \times n}$, $\boldsymbol{B} \in I\!R^{n \times r}$, $\boldsymbol{C} \in I\!R^{m \times n}$ and $\boldsymbol{D} \in I\!R^{m \times r}$ are real matrices.

The transfer function matrix $\boldsymbol{G}(s)$ of the system (17), with the output relation (18), is given as

$$\boldsymbol{G}(s) = \boldsymbol{C}(s\boldsymbol{I} - \boldsymbol{A})^{-1}\boldsymbol{B} + \boldsymbol{D} \tag{19}$$

Note, this definition is used only in the Section IV.

Proposition 1: If $\boldsymbol{M}$, $\boldsymbol{N}$ are matrices of appropriate dimension, and $\boldsymbol{X}$ is a symmetric positive definite matrix, then

$$\boldsymbol{M}^T\boldsymbol{N} + \boldsymbol{N}^T\boldsymbol{M} \leq \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{N} + \boldsymbol{M}^T\boldsymbol{X}^{-1}\boldsymbol{M} \tag{20}$$

Proof: [11] Since $\boldsymbol{X} = \boldsymbol{X}^T > 0$, then

$$\left(\boldsymbol{X}^{-\frac{1}{2}}\boldsymbol{M} - \boldsymbol{X}^{\frac{1}{2}}\boldsymbol{N}\right)^T \left(\boldsymbol{X}^{-\frac{1}{2}}\boldsymbol{M} - \boldsymbol{X}^{\frac{1}{2}}\boldsymbol{N}\right) \geq 0 \tag{21}$$

$$\boldsymbol{M}^T\boldsymbol{X}^{-1}\boldsymbol{M} + \boldsymbol{N}^T\boldsymbol{X}\boldsymbol{N} - \boldsymbol{M}^T\boldsymbol{N} - \boldsymbol{N}^T\boldsymbol{M} \geq 0 \tag{22}$$

It is evident that (22) implies (20). ■

Proposition 2: (Quadratic performance) If a stable system is described by the stable transfer function matrix of the form (19) of the dimension $m \times r$, there exists such $\gamma > 0$, $\gamma \in I\!R$ that

$$\int_0^\infty (\boldsymbol{y}^T(v)\boldsymbol{y}(v) - \gamma \boldsymbol{u}^T(v)\boldsymbol{u}(v))\mathrm{d}v > 0 \tag{23}$$

where $\boldsymbol{y}(t) \in I\!R^m$ is the vector of the system output variables, $\boldsymbol{u}(t) \in I\!R^r$ is the vector of the system input variables and γ is square of the H_∞ norm of the transfer function matrix of the system.

Proof: [8] It is evident from (19) that

$$\widetilde{\boldsymbol{y}}(s) = \boldsymbol{G}(s)\widetilde{\boldsymbol{u}}(s) \tag{24}$$

where $\widetilde{\boldsymbol{y}}(s)$, $\widetilde{\boldsymbol{u}}(s)$ stands for the Laplace transform of m dimensional output vector and r dimensional input vector, respectively. Then (24) implies

$$\|\widetilde{\boldsymbol{y}}(s)\| \le \|\boldsymbol{G}(s)\|\|\widetilde{\boldsymbol{u}}(s)\| \tag{25}$$

where $\|\boldsymbol{G}(s)\|$ is the H_2 norm of the system transfer function matrix $\boldsymbol{G}(s)$. Since H_∞ norm property states

$$\frac{1}{\sqrt{m}}\|\boldsymbol{G}(s)\|_\infty \le \|\boldsymbol{G}(s)\| \le \sqrt{r}\|\boldsymbol{G}(s)\|_\infty \tag{26}$$

where $\|\boldsymbol{G}(s)\|_\infty$ is the H_∞ norm of the system transfer function matrix $\boldsymbol{G}(s)$, using notation $\|\boldsymbol{G}(s)\|_\infty = \sqrt{\gamma}$, the inequality (26) can be rewritten as

$$0 < \frac{1}{\sqrt{m}} \le \frac{\|\widetilde{\boldsymbol{y}}(s)\|}{\sqrt{\gamma}\|\widetilde{\boldsymbol{u}}(s)\|} \le \frac{\|\boldsymbol{G}(s)\|}{\sqrt{\gamma}} \le \sqrt{r} \tag{27}$$

Thus, based on Parceval's theorem, (27) gives for $m \ge 1$

$$1 < \frac{\|\widetilde{\boldsymbol{y}}(s)\|}{\sqrt{\gamma}\|\widetilde{\boldsymbol{u}}(s)\|} = \frac{\left(\int\limits_0^\infty \boldsymbol{y}^T(v)\boldsymbol{y}(v)\mathrm{d}v\right)^{\frac{1}{2}}}{\sqrt{\gamma}\left(\int\limits_0^\infty \boldsymbol{u}^T(v)\boldsymbol{u}(v)\mathrm{d}v\right)^{\frac{1}{2}}} \tag{28}$$

and subsequently

$$\int_0^\infty \boldsymbol{y}^T(v)\boldsymbol{y}(v)\mathrm{d}v - \gamma\int_0^\infty \boldsymbol{u}^T(v)\boldsymbol{u}(v)\mathrm{d}v > 0 \tag{29}$$

It is evident that (29) implies (23). ■

Proposition 3: (Bounded real lemma) System described by (17), (18) is asymptotically stable with the quadratic performance $\|\boldsymbol{C}(s\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}\|_\infty \le \sqrt{\gamma}$, if there exist a symmetric positive definite matrix $\boldsymbol{P} \in I\!R^{n\times n}$ and a positive scalar $\gamma \in I\!R$ such that

$$\boldsymbol{P} = \boldsymbol{P}^T > 0, \qquad \gamma > 0 \tag{30}$$

$$\begin{bmatrix} \boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{C}^T \\ * & -\gamma^2\boldsymbol{I}_r & \boldsymbol{D}^T \\ * & * & -\boldsymbol{I}_m \end{bmatrix} < 0 \tag{31}$$

where $\boldsymbol{I}_r \in I\!R^{r\times r}$, $\boldsymbol{I}_m \in I\!R^{m\times m}$ are identity matrices, respectively.

Here, and hereafter, $*$ denotes the symmetric item in a symmetric matrix.

Proof: (see. e.g. [3], [11]) Defining the Lyapunov function

$$\begin{aligned} v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t)+ \\ +\int\limits_0^t (\boldsymbol{y}^T(v)\boldsymbol{y}(v) - \gamma \boldsymbol{r}^T(v)\boldsymbol{u}(v))\mathrm{d}v > 0 \end{aligned} \tag{32}$$

where $\boldsymbol{P} = \boldsymbol{P}^T > 0$, $\boldsymbol{P} \in I\!R^{n\times n}$, $\gamma > 0$, $\gamma \in I\!R$, and evaluating the derivative of $v(\boldsymbol{q}(t))$ with respect to t along the system trajectories, it yields

$$\begin{aligned} \dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^T(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t)+ \\ +\boldsymbol{y}^T(t)\boldsymbol{y}(t) - \gamma^2\boldsymbol{u}^T(t)\boldsymbol{u}(t) < 0 \end{aligned} \tag{33}$$

Thus, substituting (3), (4) into (33) gives

$$\begin{aligned} \dot{v}(\boldsymbol{q}(t)) = (\boldsymbol{A}\boldsymbol{q}(t) + \boldsymbol{B}\boldsymbol{u}(t))^T\boldsymbol{P}\boldsymbol{q}(t)+ \\ +\boldsymbol{q}^T(t)\boldsymbol{P}(\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t)) - \gamma\boldsymbol{u}^T(t)\boldsymbol{u}(t)+ \\ +(\boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{D}\boldsymbol{u}(t))^T(\boldsymbol{C}\boldsymbol{q}(t) + \boldsymbol{D}\boldsymbol{u}(t)) < 0 \end{aligned} \tag{34}$$

and with the notation

$$\boldsymbol{q}_c^T(t) = \begin{bmatrix} \boldsymbol{q}^T(t) & \boldsymbol{u}^T(t) \end{bmatrix} \tag{35}$$

it is obtained

$$\dot{v}(\boldsymbol{q}(t)) = \boldsymbol{q}_c^T(t)\boldsymbol{P}_c\,\boldsymbol{q}_c(t) < 0 \tag{36}$$

where

$$\boldsymbol{P}_c = \begin{bmatrix} \boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B} \\ * & -\gamma\boldsymbol{I}_r \end{bmatrix} + \begin{bmatrix} \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{C}^T\boldsymbol{D} \\ * & \boldsymbol{D}^T\boldsymbol{D} \end{bmatrix} < 0 \tag{37}$$

Since

$$\begin{bmatrix} \boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{C}^T\boldsymbol{D} \\ * & \boldsymbol{D}^T\boldsymbol{D} \end{bmatrix} = \begin{bmatrix} \boldsymbol{C}^T \\ \boldsymbol{D}^T \end{bmatrix} \begin{bmatrix} \boldsymbol{C} & \boldsymbol{D} \end{bmatrix} \ge 0 \tag{38}$$

applying Schur complement property to (38), (37) implies (31). ■

V. STATE CONTROL DESIGN

Theorem 1: The autonomous system from (8) is asymptotically stable with bounded quadratic performance if there exist symmetric positive definite matrices $\boldsymbol{P}_i \in I\!R^{n_i\times n_i}$ and positive scalars $\gamma_i, \lambda_i, \varepsilon_i \in I\!R$ such that

$$\boldsymbol{P}_i = \boldsymbol{P}_i > 0,\ \gamma_i > 0,\ \lambda_i > 0,\ \varepsilon_i > 0 \tag{39}$$

$$\begin{bmatrix} \boldsymbol{\Phi} & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{P}\boldsymbol{F} & \boldsymbol{C}^T & \boldsymbol{P}\boldsymbol{G} & \boldsymbol{w}_1 & \cdots & \boldsymbol{w}_p \\ * & -\boldsymbol{\Gamma}_u & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & -\boldsymbol{\Gamma}_d & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & * & * & -\varepsilon_1 & & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \\ * & * & * & * & * & * & & -\varepsilon_p \end{bmatrix} < 0 \tag{40}$$

for $i = 1, 2, \ldots, p$, where

$$\boldsymbol{\Phi} = \boldsymbol{A}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} \tag{41}$$

The matrices

$$\boldsymbol{P} = \operatorname{diag}\left[\begin{array}{lll} \boldsymbol{P}_1 & \boldsymbol{P}_2 & \cdots \boldsymbol{P}_p \end{array}\right] \tag{42}$$

$$\boldsymbol{\Gamma}_u = \operatorname{diag}\left[\begin{array}{lll}\gamma_1 & \cdots & \gamma_p\end{array}\right],\ \boldsymbol{\Gamma}_d = \operatorname{diag}\left[\begin{array}{lll}\lambda_1 & \cdots & \lambda_p\end{array}\right] \tag{43}$$

are structured matrix variables, and all system matrix parameter structures are given in (13)-(15).

Proof: Defining Lyapunov function as follows

$$\begin{gathered} v(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{q}(t)+ \\ +\int\limits_0^t \Big(\boldsymbol{y}^T(v)\boldsymbol{y}(v) - \sum_{h=1}^{p}\big(\gamma_h \boldsymbol{u}_h^T(v)\boldsymbol{u}_h(v) + \lambda_h \boldsymbol{d}_h^T(v)\boldsymbol{d}_h(v)\big)\Big)\mathrm{d}v \end{gathered} \tag{44}$$

where $v(\boldsymbol{q}(t)) > 0$, $\boldsymbol{P} = \boldsymbol{P}^T > 0$ is given in (42), and $\gamma_h > 0$, $\lambda_h > 0$, $h = 1, 2, \dots p$, are introduced in (43). Evaluating the derivative of $v(\boldsymbol{q}(t))$ with respect to t along the autonomous system trajectories, it yields

$$\begin{gathered} \dot{v}(\boldsymbol{q}(t)) = \dot{\boldsymbol{q}}^T(t)\boldsymbol{P}\boldsymbol{q}(t) + \boldsymbol{q}^{\boldsymbol{T}}(t)\boldsymbol{P}\dot{\boldsymbol{q}}(t)+ \\ +\boldsymbol{y}^T(t)\boldsymbol{y}(t) - \left[\begin{array}{ll}\boldsymbol{u}^T(t) & \boldsymbol{d}^T(t)\end{array}\right]\boldsymbol{\Gamma}\begin{bmatrix}\boldsymbol{u}(t)\\ \boldsymbol{d}(t)\end{bmatrix} < 0 \end{gathered} \tag{45}$$

where with (43)

$$\boldsymbol{\Gamma} = \operatorname{diag}\left[\begin{array}{ll}\boldsymbol{\Gamma}_u & \boldsymbol{\Gamma}_d\end{array}\right] \tag{46}$$

Thus, substituting (8), (9) into (45) gives

$$\begin{gathered} \dot{v}(\boldsymbol{q}(t)) = \boldsymbol{q}^T(t)\boldsymbol{C}^T\boldsymbol{C}\boldsymbol{q}(t)+ \\ +\big(\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{G}\boldsymbol{h}(\boldsymbol{q}(t))+\boldsymbol{D}\boldsymbol{d}(t)\big)^T\boldsymbol{P}\boldsymbol{q}(t)+ \\ +\boldsymbol{q}^T(t)\boldsymbol{P}\big(\boldsymbol{A}\boldsymbol{q}(t)+\boldsymbol{B}\boldsymbol{u}(t)+\boldsymbol{G}\boldsymbol{h}(\boldsymbol{q}(t))+\boldsymbol{D}\boldsymbol{d}(t)\big)- \\ -\left[\begin{array}{ll}\boldsymbol{u}^T(t) & \boldsymbol{d}^T(t)\end{array}\right]\begin{bmatrix}\boldsymbol{\Gamma}_u & \\ & \boldsymbol{\Gamma}_d\end{bmatrix}\begin{bmatrix}\boldsymbol{u}(t)\\ \boldsymbol{d}(t)\end{bmatrix} < 0 \end{gathered} \tag{47}$$

Subsequently, using (20) with $\boldsymbol{X} = \boldsymbol{I}$, it can be written

$$\begin{gathered} \boldsymbol{h}^T(\boldsymbol{q}(t))\boldsymbol{G}^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{G}\boldsymbol{h}(\boldsymbol{q}(t)) \leq \\ \leq \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{h}^T(\boldsymbol{q}(t))\boldsymbol{h}(\boldsymbol{q}(t)) \end{gathered} \tag{48}$$

and using (5), (48) gives

$$\begin{gathered} \boldsymbol{h}^T(\boldsymbol{q}(t))\boldsymbol{G}^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{G}\boldsymbol{h}(\boldsymbol{q}(t)) \leq \\ \leq \boldsymbol{q}^T(t)\boldsymbol{P}\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{P}\boldsymbol{q}(t)+\boldsymbol{q}^T(t)\sum_{h=1}^{p}\varepsilon_h^{-1}\boldsymbol{w}_h^T\boldsymbol{w}_h\boldsymbol{q}(t) \end{gathered} \tag{49}$$

Thus, with the notation

$$\boldsymbol{q}_c^{\bullet T}(t) = \left[\begin{array}{lll}\boldsymbol{q}^T(t) & \boldsymbol{u}^T(t) & \boldsymbol{d}^T(t)\end{array}\right] \tag{50}$$

(47) can be rewritten as

$$\dot{v}(\boldsymbol{q}(t)) \leq \boldsymbol{q}_c^{\bullet T}(t)\boldsymbol{P}_c^{\bullet}\boldsymbol{q}_c^{\bullet}(t) < 0 \tag{51}$$

where

$$\begin{gathered} \boldsymbol{P}_c^{\bullet} = \begin{bmatrix}\boldsymbol{A}^T\boldsymbol{P}+\boldsymbol{P}\boldsymbol{A} & \boldsymbol{P}\boldsymbol{B} & \boldsymbol{P}\boldsymbol{F}\\ * & -\boldsymbol{\Gamma}_u & \boldsymbol{0}\\ * & * & -\boldsymbol{\Gamma}_d\end{bmatrix}+ \\ +\begin{bmatrix}\boldsymbol{C}^T\boldsymbol{C}+\boldsymbol{P}\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0}\\ * & \boldsymbol{0} & \boldsymbol{0}\\ * & * & \boldsymbol{0}\end{bmatrix}+ \\ +\sum_{h=1}^{p}\begin{bmatrix}\boldsymbol{w}_h^T\varepsilon_h^{-1}\boldsymbol{w}_h & \boldsymbol{0} & \boldsymbol{0}\\ * & \boldsymbol{0} & \boldsymbol{0}\\ * & * & \boldsymbol{0}\end{bmatrix} < 0 \end{gathered} \tag{52}$$

Since it yields

$$\begin{bmatrix}\boldsymbol{C}^T\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0}\\ * & \boldsymbol{0} & \boldsymbol{0}\\ * & * & \boldsymbol{0}\end{bmatrix} = \begin{bmatrix}\boldsymbol{C}^T\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\left[\begin{array}{lll}\boldsymbol{C} & \boldsymbol{0} & \boldsymbol{0}\end{array}\right] \geq 0 \tag{53}$$

$$\begin{bmatrix}\boldsymbol{P}\boldsymbol{G}\boldsymbol{G}^T\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0}\\ * & \boldsymbol{0} & \boldsymbol{0}\\ * & * & \boldsymbol{0}\end{bmatrix} = \begin{bmatrix}\boldsymbol{P}\boldsymbol{G}\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\left[\begin{array}{lll}\boldsymbol{G}^T\boldsymbol{P} & \boldsymbol{0} & \boldsymbol{0}\end{array}\right] \geq 0 \tag{54}$$

$$\begin{bmatrix}\boldsymbol{w}_h\varepsilon_h^{-1}\boldsymbol{w}_h^T & \boldsymbol{0} & \boldsymbol{0}\\ * & \boldsymbol{0} & \boldsymbol{0}\\ * & * & \boldsymbol{0}\end{bmatrix} = \begin{bmatrix}\boldsymbol{w}_h\\ \boldsymbol{0}\\ \boldsymbol{0}\end{bmatrix}\varepsilon_h^{-1}\left[\begin{array}{lll}\boldsymbol{w}_h^T & \boldsymbol{0} & \boldsymbol{0}\end{array}\right] \geq 0 \tag{55}$$

then, applying Schur complement property to (53)-(55), (52) implies (40). ■

Theorem 2: The system (8), with output given by the relation (9), is stabilized with bounded quadratic performance via the controller (16) if there exist symmetric positive definite matrices $\boldsymbol{X}_i \in I\!R^{n_i \times n_i}$ and positive scalars $\gamma_i, \lambda_i, \varepsilon_i \in I\!R$ such that

$$\boldsymbol{X}_i = \boldsymbol{X}_i > 0,\ \gamma_i > 0,\ \lambda_i > 0,\ \varepsilon_i > 0 \tag{56}$$

$$\begin{bmatrix}\widetilde{\boldsymbol{\Phi}} & \boldsymbol{B} & \boldsymbol{F} & \boldsymbol{X}\boldsymbol{C}^T & \boldsymbol{G} & \boldsymbol{X}\boldsymbol{w}_1 & \cdots & \boldsymbol{X}\boldsymbol{w}_p\\ * & -\boldsymbol{\Gamma}_u & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ * & * & -\boldsymbol{\Gamma}_d & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ * & * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \cdots & \boldsymbol{0}\\ * & * & * & * & * & -\varepsilon_1 & & \boldsymbol{0}\\ \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \\ * & * & * & * & * & * & & -\varepsilon_p\end{bmatrix} < 0 \tag{57}$$

for all $i = 1, 2, \dots, p$, where

$$\widetilde{\boldsymbol{\Phi}} = \boldsymbol{X}\boldsymbol{A}^T + \boldsymbol{A}\boldsymbol{X} - \boldsymbol{Y}^T\boldsymbol{B}^T - \boldsymbol{B}\boldsymbol{Y} \tag{58}$$

The matrices

$$\boldsymbol{X} = \operatorname{diag}\left[\begin{array}{llll}\boldsymbol{X}_1 & \boldsymbol{X}_2 & \cdots & \boldsymbol{X}_p\end{array}\right] \tag{59}$$

$$\boldsymbol{\Gamma}_u = \operatorname{diag}\left[\begin{array}{lll}\gamma_1 & \cdots & \gamma_p\end{array}\right],\ \boldsymbol{\Gamma}_d = \operatorname{diag}\left[\begin{array}{lll}\lambda_1 & \cdots & \lambda_p\end{array}\right] \tag{60}$$

are structured matrix variables, and all system matrix parameter structures are given in (13)-(15).

If the above conditions hold, the set of control gain matrices is given by

$$\boldsymbol{K} = \boldsymbol{Y}\boldsymbol{X}^{-1} = \left[\begin{array}{llll}\boldsymbol{k}_1^T & \boldsymbol{k}_2^T & \cdots & \boldsymbol{k}_p^T\end{array}\right] \tag{61}$$

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Proof: Inserting the global closed-loop system matrix $\boldsymbol{A}_c = = \boldsymbol{A} - \boldsymbol{B}\boldsymbol{K}$ in (41) gives

$$\boldsymbol{\Phi} = \boldsymbol{A}^T\boldsymbol{P} - \boldsymbol{K}^T\boldsymbol{B}^T\boldsymbol{P} + \boldsymbol{P}\boldsymbol{A} - \boldsymbol{P}\boldsymbol{B}\boldsymbol{K} \tag{62}$$

Defining the congruence transform matrix

$$\boldsymbol{T} = \text{diag}\left[\boldsymbol{P}^{-1}\ \boldsymbol{I}_r\ \ \boldsymbol{I}_r\ \ \boldsymbol{I}_r\ \ \boldsymbol{I}_r\ \ 1\ \cdots\ 1\right] \tag{63}$$

and pre-multiplying both side of the (40) by (63), the next LMIs are obtained

$$\begin{bmatrix} \widetilde{\boldsymbol{\Phi}} & \boldsymbol{B} & \boldsymbol{F} & \boldsymbol{P}^{-1}\boldsymbol{C}^T & \boldsymbol{G} & \boldsymbol{P}^{-1}\boldsymbol{w}_1 & \cdots & \boldsymbol{P}^{-1}\boldsymbol{w}_p \\ * & -\boldsymbol{\Gamma}_u & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & -\boldsymbol{\Gamma}_d & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & * & -\boldsymbol{I}_r & \boldsymbol{0} & \cdots & \boldsymbol{0} \\ * & * & * & * & * & -\varepsilon_1 & & \boldsymbol{0} \\ \vdots & \vdots & \vdots & \vdots & \vdots & & \ddots & \\ * & * & * & * & * & * & & -\varepsilon_p \end{bmatrix} < 0 \tag{64}$$

$$\widetilde{\boldsymbol{\Phi}} = \boldsymbol{P}^{-1}\boldsymbol{A}^T - \boldsymbol{P}^{-1}\boldsymbol{K}^T\boldsymbol{B}^T + \boldsymbol{A}\boldsymbol{P}^{-1} - \boldsymbol{B}\boldsymbol{K}\boldsymbol{P}^{-1} \tag{65}$$

respectively. Introducing the LMI variables

$$\boldsymbol{P}^{-1} = \boldsymbol{X}, \quad \boldsymbol{K}\boldsymbol{P}^{-1} = \boldsymbol{Y} \tag{66}$$

then (66) implies (61), and (64), (65) implies (57), (58). ■

VI. ILLUSTRATIVE EXAMPLE

To demonstrate the algorithm properties, the next subsystem parameters for $i = 1, 2, 3$ are used

$$\boldsymbol{A}_i = \begin{bmatrix} -12.50 & 0.00 & -5.21 & 0.00 \\ 3.33 & -3.33 & 0.00 & 0.00 \\ 0.00 & 6.00 & -0.05 & -6.00 \\ 0.00 & 0.00 & 1.10 & 0.00 \end{bmatrix}, \ \boldsymbol{b}_i = \begin{bmatrix} 12.5 \\ 0.0 \\ 0.0 \\ 0.0 \end{bmatrix}$$

$$\boldsymbol{c}_i^T = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}, \quad \boldsymbol{f}_i^T = \begin{bmatrix} 0 & 0 & -6 & 0 \end{bmatrix}$$

and

$$\boldsymbol{G}_{ih} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -0.55 & 0 \end{bmatrix}, \ \boldsymbol{g}_x = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -0.55 \end{bmatrix}$$

$$\boldsymbol{G} = \text{diag}\left[\boldsymbol{g}_x \quad \boldsymbol{g}_x \quad \boldsymbol{g}_x\right]$$

$$\boldsymbol{w}_1^T = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{w}_2^T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\boldsymbol{w}_3^T = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, solving (56), (56) with respect to the LMI matrix variables $\boldsymbol{X}$, $\boldsymbol{Y}_i$, $\gamma_i, \lambda_i, \varepsilon_i,\ i = 1, 2, 3$ using SeDuMi package for Matlab, the feedback gain matrix design problem was feasible with the results

$$\boldsymbol{Y}_i^T = \begin{bmatrix} -12.9516 & 1.4183 & 0.1961 & -3.4268 \end{bmatrix}$$

$$\boldsymbol{X}_i = \begin{bmatrix} 14.7389 & 2.7960 & -1.9062 & 3.6230 \\ * & 5.0205 & -1.7327 & 3.4803 \\ * & * & 2.2244 & -0.6127 \\ * & * & * & 3.6596 \end{bmatrix}$$

$$\gamma_i = 16.9863, \ \lambda_i = 15.5306, \ \varepsilon_i = 11.5833$$

giving the control law gain vectors

$$\boldsymbol{k}_i^T = \begin{bmatrix} -0.4870 & 3.5065 & 1.4243 & -3.5505 \end{bmatrix}$$

The decentralized closed-loop eigenvalues spectrum is

$$\rho(\boldsymbol{A}_{ch}) = \{-0.2646\ \ -3.2126\ \ -3.1578 \pm 11.9004\mathrm{i}\}$$

and rises up the stable global system.

VII. CONCLUDING REMARKS

A new characterization for interaction bounds is presented and sufficient condition for stabilizing decentralized robust control design are formulated in the sense of the bounded real lemma. The optimization, involving structured matrix variables in the linear matrix inequalities, take into account the strong interactions among subsystems, as well as the interaction uncertainties. An illustration example is presented to show that such a procedure can simplify the decentralized control design.

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APPENDIX

The next analysis is based on the assumption that the electrical interconnections within each area of multi-area power system are so strong, at least in relation to ties with the neighboring areas that the whole area can be characterized only by a single frequency (see, e.g., [12] and the references therein). Therefore, it is supposed that the power equilibrium applied to the area i can be written as

$$\begin{aligned} T_{Pi}\tfrac{\mathrm{d}\Delta f_i(t)}{\mathrm{d}t} + \Delta f_i(t) + K_{Pk}\Delta P_{Tk}(t) = \\ = K_{Pi}\Delta P_{Gi}(t) - K_{Pi}\Delta P_{Di}(t) \end{aligned} \tag{A.1}$$

where T_{Pi} is the area model time constant (s), $\Delta f_i(t)$ is the area incremental frequency deviation (Hz), K_{Pi} is the area gain (Hz/pu MW), $\Delta P_{Ti}(t)$ is the incremental change of total real power exported from area (Hz/pu MW), $\Delta P_{Gi}(t)$ is the incremental change in generator output (Hz/pu MW), and $\Delta P_{Di}(t)$ is the unknown load disturbance (Hz/pu MW).

If the line losses are neglected, the individual line powers can be written in the form

$$\begin{aligned} P_{Ti}(t) = \tfrac{|V_i||V_v|}{X_{vi}P_{vi}} \sin(\delta_i(t) - \delta_v(t)) = \\ = P_{Tiv\,\max} \sin(\delta_i(t) - \delta_v(t)) \end{aligned} \tag{A.2}$$

$$V_i(t) = |V_i| \exp(j\delta_i(t)),\ V_v(t) = |V_v| \exp(j\delta_v(t)) \tag{A.3}$$

is the terminal bus voltage of the line, and $X_{\nu i}$ is its reactance.

If the phase angles deviate from their nominal values by amounts $\Delta\delta_i$, $\Delta\delta_\nu$, respectively, it can be obtained

$$\begin{aligned} \Delta P_{Ti}(t) = \\ = \tfrac{V_i||V_v|}{X_{vi}P_{vi}} \cos(\delta_{in}(t) - \delta_{vn}(t))(\Delta\delta_i(t) - \Delta\delta_v(t)) \end{aligned} \tag{A.4}$$

$$\begin{aligned} \Delta P_{Ti}(t) = \\ = 2\pi\tfrac{|V_i||V_v|}{X_{vi}P_{vi}} \cos(\delta_{in}(t) - \delta_{vn}(t)) \begin{Bmatrix} \int_0^t \Delta f_i(r)\mathrm{d}r - \\ -\int_0^t \Delta f_v(r)\mathrm{d}r \end{Bmatrix} \end{aligned} \tag{A.5}$$

respectively. Related to the area frequency changes, the derivative of the individual line powers with respect to time is

$$\frac{\mathrm{d}\Delta P_{Tiv}(t)}{\mathrm{d}t} = S_{iv}(\Delta f_i(t) - \Delta f_v(t)) \tag{A.6}$$

$$\frac{\mathrm{d}\Delta P_{Ti}(t)}{\mathrm{d}t} = \sum_{i\neq l} S_{il}(\Delta f_i(t) - \Delta f_l(t)) \tag{A.7}$$

respectively, where S_{il} is the synchronizing coefficient (electrical stiffness of the tie line).

The incremental generated power of the area i for small signals around the nominal settings can be represented by the equations

$$T_{Ti}\frac{\mathrm{d}\Delta P_{Gi}(t)}{\mathrm{d}t} + \Delta P_{Gi}(t) = \Delta x_{Hi}(t) \tag{A.8}$$

$$T_{Hi}\frac{\mathrm{d}\Delta x_{Hi}(t)}{\mathrm{d}t} + \Delta x_{Hi}(t) = \Delta P_{Ci}(t) - \frac{1}{R_i}\Delta f_i(t) \tag{A.9}$$

where T_{Ti} is the turbine time constant (s), T_{Hi} is the governor time constant (s) (generator response is instantaneous), R_i is a measure of static speed droop (Hz/pu MW), $\Delta P_{Ci}(t)$ is the incremental change of command signal to the speed changer (control input), and $\Delta x_{Hi}(t)$ is the incremental change in the governor value position (pu MW), all with respect to the area i.

The compact form of (A.1), (A.7), (A.8), and (A.9) is [12]

$$\dot{\boldsymbol{q}}_i(t) = \boldsymbol{A}_i\boldsymbol{q}_i(t) + \boldsymbol{b}_i u_i(t) + \sum_{l=1}^{p} \boldsymbol{G}_{li}\boldsymbol{q}_i(t) + \boldsymbol{f}_i d_i(t) \tag{A.10}$$

$$y_i(t) = \boldsymbol{c}_i^T \boldsymbol{q}_i(t) \tag{A.11}$$

where

$$\boldsymbol{q}_i(t) = \begin{bmatrix} \Delta x_{Hi}(t) & \Delta P_{Gi}(t) & \Delta f_i(t) & \Delta P_{Ti}(t) \end{bmatrix}^T \tag{A.12}$$

$$u_i(t) = \Delta P_{Ci}(t) \qquad d_i(t) = \Delta P_{Di}(t) \tag{A.13}$$

$$\boldsymbol{A}_i = \begin{bmatrix} -\frac{1}{T_{Hi}} & 0 & -\frac{1}{R_i T_{Hi}} & 0 \\ \frac{1}{T_{Ti}} & -\frac{1}{T_{Ti}} & 0 & 0 \\ 0 & \frac{K_{Pi}}{T_{Pi}} & -\frac{1}{T_{Pi}} & -\frac{K_{Pi}}{T_{Pi}} \\ 0 & 0 & \sum\limits_{l\neq i} S_{il} & 0 \end{bmatrix} \tag{A.14}$$

$$\boldsymbol{G}_{li} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -S_{li} & 0 \end{bmatrix}, \quad \boldsymbol{b}_i = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{T_{Hi}} \end{bmatrix} \tag{A.15}$$

$$\boldsymbol{f}_i = \begin{bmatrix} 0 \\ 0 \\ -\frac{K_{Pi}}{T_{Pi}} \\ 0 \end{bmatrix}, \quad \boldsymbol{c}_i = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{A.16}$$

Under above given model parameters, the stability of the overall system can be studied by the stability properties of all subsystems, and by global features of all subsystem interactions.

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