Quaternion Lifting Scheme for Multi-resolution Wavelet-based Motion Analysis

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***Abstract*—This paper considers human body motion analysis as local changes of orientations in hierarchical skeleton parts over time. Possible approaches by applying multiresolution analysis in form of a second generation wavelet transform directly on quaternion signal are shown. Quaternions in terms of motion analysis are a more efficient representation of rotation than Euler angles. This paper presents that the lifting scheme can be efficiently applied directly to quaternions. Lifting scheme building blocks for the quaternion Haar and linear transformation are presented.**

Keywords-quaternions; multi-resolution analysis; wavelet transform; lifting scheme; quaternion interpolation; motion analysis.

I. Introduction

Human body motion synthesis and analysis are very challenging tasks and a very popular research domain. The most precise measurements of motion data are obtained by motion capture systems. We cooperate with a high tech motion capture laboratory having dedicated hardware capable of performing motion acquisition. It can acquire motion data through simultaneous and synchronous measurement and recording of motion kinematics, muscle potentials by electromyography, ground reaction forces and video streams in high definition format and the HML supporting system which allows for storing, playing and browsing data. The data from the above mentioned subsystems are large and accurate, allowing for a thorough analysis of motion. Techniques of analysis of such data can be potentially applied in:

- Medicine - diagnosis and verification of a medical treatment;
- Entertainment - realistic animations;
- Sport - new training techniques;
- Security - people recognition based on their body movement.

In this paper, we present our approaches in performing motion analysis with multi-resolution techniques based on rotations of joints over time written in the form of quaternion signal. For this reason, we are trying to use a second generation wavelet transform constructed by the lifting scheme for quaternion rotation representation. Using the quaternion lifting scheme based on the quaternion algebra we can work directly on correlated motion data. This is in opposition to the methods presented in the literature where the filters work on Euler angles as three non-correlated components. Also, the example application of multi-resolution for denoising data is presented.

Section II describes the main assumptions of multi-resolution wavelet analysis of motion data and presents a short review of solutions presented in literature. Section III presents general information about the second generation wavelets and the lifting scheme as a simple construction tool of such wavelets. Section IV focuses on quaternion interpolating methods. Section V describes the construction of the lifting schema blocks for Haar and linear quaternion transformation and presents some results. Section VI presents example application of result quaternion multi-resolution representation. The last section is a conclusion.

II. Multi-resolution wavelet analysis of motion data

The main idea of the multi-resolution transformation is to represent a signal coarse to fine hierarchy. The input signal is decomposed into coarse base data (global pattern of signal) and a hierarchy of detail coefficients. The result multi-resolution representation can be based of many algorithms such as [10], [14]: denoising, filtering (smoothing, enhancement), compression, feature detection and multi-resolution editing.

Most of the solutions are based on processing orientation data as three non-correlated signals defined by Euler angles. In [1], spatial filters for orientation data are proposed. A similar solution based on a digital filter bank technique is in [2]. In [3], the cubic interpolating bi-orthogonal wavelet basis, implemented as lifting scheme blocks, are used to compression skeletal animation data. Temporal coherence is exploited by this wavelet transform. The B-spline wavelet for unit quaternion is used for smoothing motion data in [4].

Quaternion wavelets are also proposed for phase based stereo matching for uncalibrated images [5]. This solution is based on a bi-orthogonal filter bank, where the real valued image signal is convolved with an analytic quaternion wavelet filter, to construct the 2D analytic signal.

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In [6] and [7], the quaternion multiplier of plane rotations, inspired by a factorization algorithm, is implemented. This proposition considers the factorization of a quaternion multiplication matrix into lifting scheme steps. Our work is different because we work directly on quaternion signal as representing orientation changes over time and we design lifting scheme blocks using the convention of second generation wavelets. Our main task is to analyze human motion.

III. SECOND GENERATION WAVELETS AND LIFTING SCHEME

The lifting scheme [8], [9] is a simple but powerful tool to construct a wavelet transform. The main advantage of this solution is the possibility of building wavelet analysis on non-standard structures of data (irregular samples, bounded domains, curves, surfaces) while keeping all powerful properties as speed and good ability of approximation [10], [11], [12], [13]. This generalization are called as second generation wavelets [14]. They are not necessarily translated and dilated of one function (mother function). In this meaning, the lifting scheme also considers non-linear and data-adaptive multi-resolution decompositions.

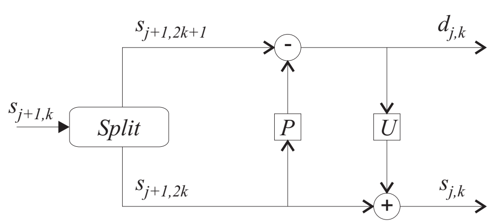

Figure 1. The forward lifting scheme

A general lifting scheme (Figure 1) consists of three types of operations:

- **Split**: splits input dataset into two disjoint sets of even and odd indexed samples. The definition of the lifting scheme does not impose any restriction on how the data should be split nor on the relative size of each subsets.
- **Predict**: predicts samples with odd indexes based on even indexed samples. Next the odd indexed input value is replaced by the offset (difference) between the odd value and its prediction.
- **Update**: updates the output, so that coarse-scale coefficients have the same average value as the input samples. This step is necessary for stable wavelet transform [14].

These calculations can be performed in-place. In all stages input samples can be overwritten by output samples of that step. The inverse transform (Figure 2) is easy to find by reversing the order of operations and flipping the signs.

IV. INTERPOLATING QUATERNIONS

Quaternions [15]–[17] are structures allowing descriptions of vectors relations. They are commonly used (mainly in computer graphics) for performing rotations.

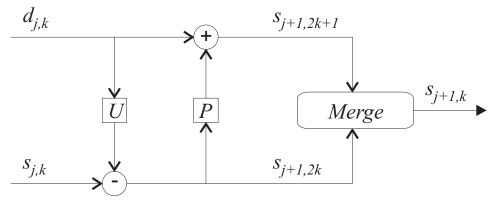

Figure 2. The reverse lifting scheme

Quaternion q is denoted as: $q = [s, v], s \in R, v \in R^3$. Here, s represents the *scalar part* and v is the *imaginary part* of the quaternion. In more details this representation can be given as:

$$q = xi + yj + zk + w,$$
$$x, y, z, w \in R, i^2 = j^2 = k^2 = ijk = -1.$$

Now, w is the scalar part and $[x, y, z]$ is the imaginary part. The space of quaternions is denoted as H.

We are working with unit quaternions; therefore, all interpolated quaternions are assumed to be unit quaternions. Interpolation step h is in range $[0; 1]$. Based on [18], the following quaternion interpolation methods can be distinguished:

- lerp - Computationally the most efficient from presented those but gives poor quality in generated rotation quaternions and does not guarantee unit quaternions as a result. Normalization is required. Generated movements have sharp ending motion so the movement of the body is not considered to be smooth.

$$lerp(q_i, q_{i+1}, h) = q_i * h + q_{i+1} * (1 - h)$$

- slerp - Ensures unit quaternion as a result. Unfortunately on the endings moves are still sharp.

$$slerp(q_i, q_{i+1}, h) = q_i(q_i^* q_{i+1})^h$$

- squad - Computational very demanding but gives very smooth movement after interpolation. No rapid changes in movement are noticeable at the interpolation range endings. This method is inspired by splines.

$$squad(q_i, q_{i+1} s_i, s_{i+1}, h) = slerp(slerp(q_i, q_{i+1}, h), slerp(s_i, s_{i+1}, h), 2h(1 - h))$$
$$s_i = q_i \exp\left(-\frac{\log(q_i^{-1} q_{i+1}) + \log(q_i^{-1} q_{i-1})}{4}\right)$$

- shoemake-bezier - Interpolation method is based on the De Castlejau algorithm. More details might be found in [19].

$$
\begin{aligned}
bezier(q_i, q_{i+1}, s_i, s_{i+1}, h) = slerp(slerp(q_{11}, q_{12}, h),\\
slerp(q_{12}, q_{13}, h), h)\\
q_{11} = slerp(q_i, s_i, h)\\
q_{12} = slerp(s_i, s_{i+1}, h)\\
q_{13} = slerp(s_{i+1}, q_{i+1}, h)\\
s_i = q_i \exp\left(-\frac{\log(q_i^{-1}q_{i+1}) + \log(q_i^{-1}q_{i-1})}{4}\right)
\end{aligned}
$$

V. Quaternion lifting scheme

Using the quaternion lifting scheme based on the quaternion algebra we can work directly on correlated motion data. This is in opposition to the methods presented in the literature where the filters work on Euler angles as three non-correlated components.

A short comment about the interpretation of details for the lifting scheme must be given here, as currently the coefficients are represented by quaternions. The goal of the prediction step is to produce values as close as possible to the given data. It means the smaller the difference (detail coefficient), the better the prediction step is. For quaternions this difference should tend to a zero rotation quaternion - $[1, [0, 0, 0]]$. If all coefficients are close to this value it means we have found a closed form description of the analyzed movement and this function might be used for such movement reproduction and analysis. Additionally small detail values suggest that the data is strongly correlated. On the other hand, if all coefficients have similar, but rather large values it is also possible that either the prediction step poorly describes data correlation or mean signal value is not maintained in the next resolutions.

Input motion signal with length 2^n is a set of normalized quaternions. In the split block this signal is divided into even and odd indexed samples: ..., $o_i^j, e_i^j, o_{i+1}^j, e_{i+1}^j$, ... The upper index j indicates the step scheme (the level of resolution).

In this paper, we give two propositions for the quaternion lifting schemes, which are fully and easy reversible and allow in-place computations. Those schemes also preserve the average signal at each resolution level.

A. Motion data

Data for analysis are obtained from the Human Motion Laboratory of the Polish-Japanese Institute of Information Technology in Bytom (Poland). The data (Figures 3 and 4) represent knee joint motion sampled at 100Hz. To better visualize the motion data, we have chosen the three Euler angles plots.

B. Quaternion Haar lifting schema

In literature the most basic lifting scheme is the Haar wavelet transformation. It predicts odd indexed samples with corresponding even indexed samples. The lifting scheme steps are the following:

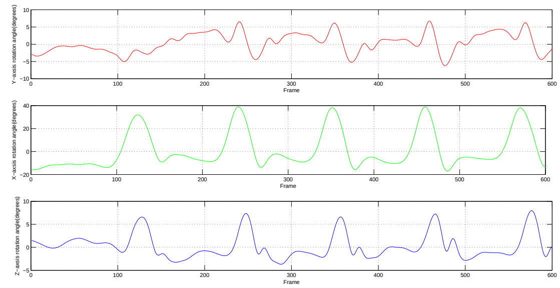

Figure 3. The Euler angles of knee joint motion data obtained from the Human Motion Laboratory of the Polish-Japanese Institute of Information Technology in Bytom (Poland).

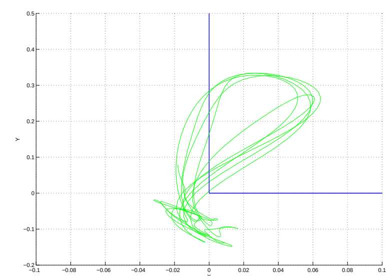

Figure 4. The quaternion curve of knee joint motion data obtained from the Human Motion Laboratory of the Polish-Japanese Institute of Information Technology in Bytom (Poland).

- Prediction step:

$$o_i^j = o_i^{j+1} \cdot SLERP(e_i^{j+1}, o_i^{j+1}, 0.5)^{-1}$$

- Update step:

$$e_i^j = o_i^j \cdot e_i^{j+1}$$

The reverse lifting scheme steps:

- Undo prediction step:

$$o_i^{j+1} = o_i^j \cdot e_i^j$$

- Undo update step:

$$e_i^{j+1} = SLERP(o_i^{j+1}, e_i^j, 2)$$

The results of the Haar transformation are presented in Figure 5 and 6. These are plots of Euler angles at the first and forth level of resolution (after first and forth step of the lifting schema) and details for such levels (differences between removed quaternions and predicted by the lifting schema).

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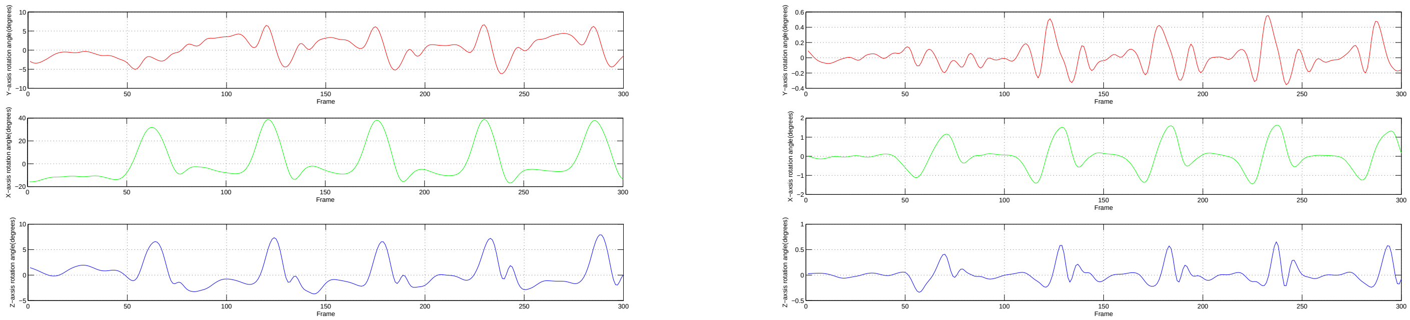

Figure 5. The first level of resolution computed by the Haar lifting schema: data (left plot) and details (right plot).

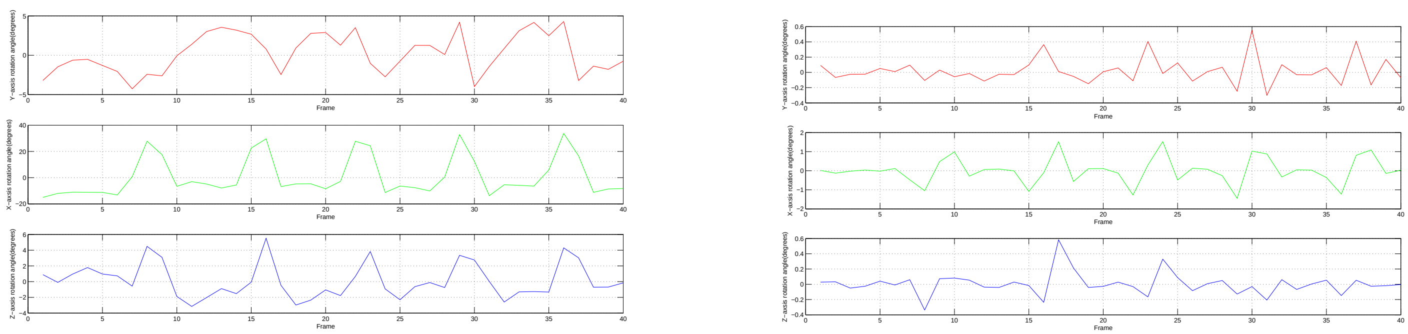

Figure 6. The forth level of resolution computed by the Haar lifting schema: data (left plot) and details (right plot).

C. Quaternion linear lifting schema

The next common prediction step is the linear interpolation between surrounding values. The lifting scheme steps are the following:

- Prediction step:

$$o_i^j = SLERP(e_i^{j+1}, e_{i+1}^{j+1}, 0.5)^{-1} \cdot o_i^{j+1}$$

- Update step:

$$e_i^j = e_i^{j+1} \cdot (SLERP(o_{i-1}^j, o_i^j, 0.5))^{0.5}$$

The reverse lifting scheme steps are:

- Undo update step:

$$e_i^{j+1} = e_i^j \cdot (SLERP(o_{i-1}^j, o_i^j, 0.5))^{0.5}$$

- Undo prediction step:

$$o_i^{j+1} = SLERP(e_i^{j+1}, e_{i+1}^{j+1}, 0.5) \cdot o_i^j$$

The results of the linear transformation are presented in Figure 7 and 8. These are plots of Euler angles of data at the first and forth level of resolution (after first and forth step of lifting schema) and details for such levels (differences between removed quaternions and predicted by the lifting schema).

VI. Example application - Denoising motion data

Denoising methods rely on removing the high frequency component of a signal, which consists of noise. The simplest method is to set to zero wavelet coefficients representing high frequencies from the first few levels of decomposition. In the quaternion lifting schema, details are set to unit quaternion. Another method is based on threshold methods, which change the wavelets coefficients selected on the basis of some threshold value. Determining the value of threshold for the quaternions domain requires further research.

Motion data with artificially added white Gaussian noise (Figure 9) was decomposed by a lifting schema into two levels of resolutions with detail quaternions coefficients. Coefficients from the first levels of transformation are small and mostly contain information about high frequency component. We can see this in Figure 10. In the reverse lifting scheme those coefficients were set to the unit quaternion. The results of the Haar and linear lifting schemes are presented in Figure 11. Because the prediction step in the linear lifting schema is based on two adjacent samples, the results of denoising for this schema are much better.

VII. Summary and future work

We have shown with results of our experiments that the lifting scheme can be efficiently applied to quaternions. This

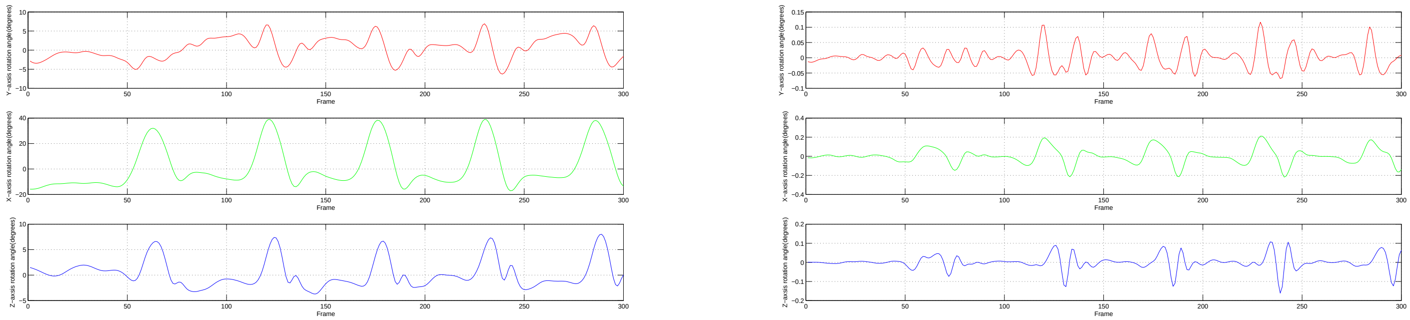

Figure 7. The first level of resolution computed by the linear lifting schema: data (left plot) and details (right plot).

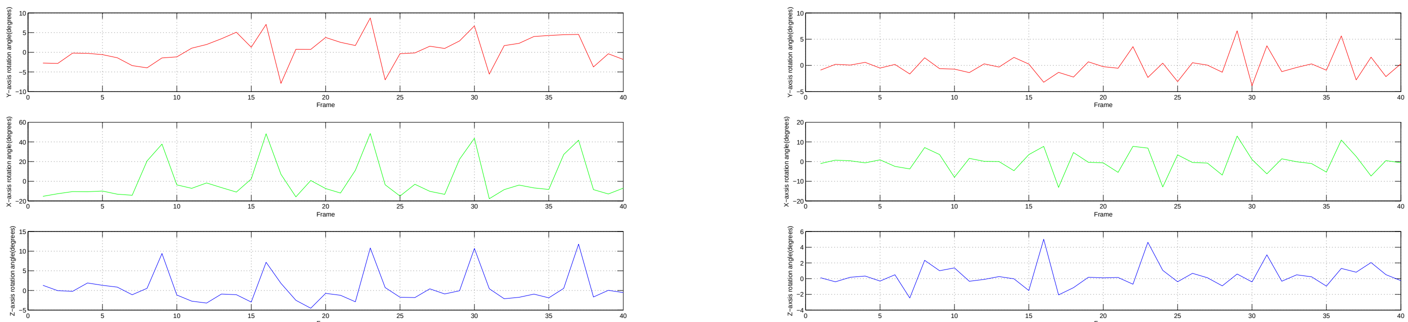

Figure 8. The forth level of resolution computed by the linear lifting schema: data (left plot) and details(right plot).

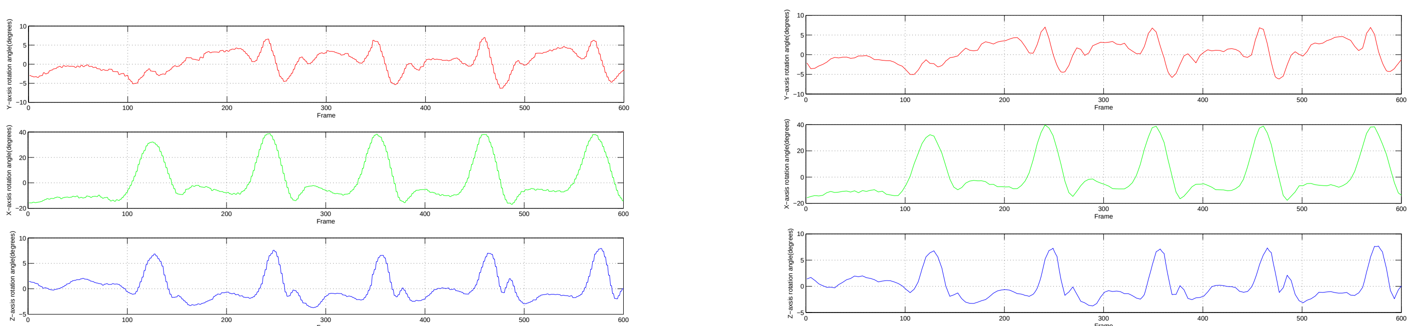

Figure 11. The motion data after removal of noise based on the Haar (left plot) and the linear lifting schema (right plot).

allows us to quickly and in place multi-resolution analysis applied directly on quaternion signal, which is a description of orientation changes over time. Using quaternion algebra properly, following quaternion space laws, data correlation would be captured at each level of resolution. This is more efficient than analyzing changes in time of each angle as a non-correlated signal.

Moreover, we are looking forward to creating proper update and predict steps for more complex quaternion interpolation methods mentioned in this paper, but not included in our research.

The results of multi-resolution representation can also be a base for different motion processing algorithms as a generalization of signal processing tools. Examples can be filtering, feature detection and compression. As an example, the very simple denoising algorithm was presented in this paper.

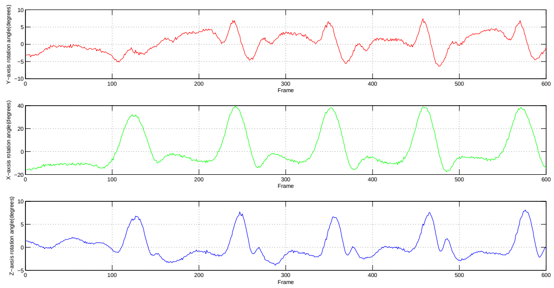

Figure 9. The signal with added white Gaussian noise.

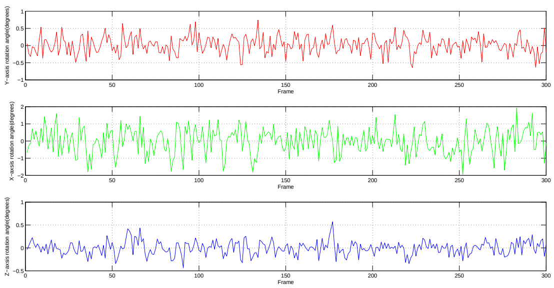

Figure 10. The Euler angles of the first level of details coefficients computed by the linear lifting scheme.

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