Higher-order Statistics of Series of Packet Losses

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***Abstract*—We analyze higher-order statistics of the length of the series of consecutive packet losses at the router's output buffer. So far, only the average length of the series of losses has been studied, in the context of the quality of real-time multimedia transmissions. In this paper, we compute the coefficient of variation, skewness and excess kurtosis of the length of the series of losses, using a complex traffic model. Then we study the influence of properties of the queue and the traffic on these higher-order statistics. In particular, we study the impact of the autocorrelation of packet interarrival times, the batch structure of the traffic, the buffer capacity and the load of the queue on the coefficient of variation, skewness and kurtosis and discuss their potential impact on the quality of multimedia transmissions perceived by end users.**

***Index Terms*—Internet; buffer overflows; series of losses; higher-order statistics**

I. Introduction

One of the consequences of the best-effort design of the Internet are packet losses, which happen at the router's output buffer, when the temporary arrival rate from all input interfaces exceeds the capacity of the output link. The mechanism of these losses is simple - the buffer gets full and newly arriving packets are deleted.

When analyzing and describing the packet loss process, the most obvious and useful characteristic is the loss ratio, i.e., the ratio of the number of lost packets to the total number of packets, considered in some, usually not short, time interval. This characteristics has been widely studied using measurements [1]–[7] and mathematical models [8]–[15].

The second well known and useful characteristic of the loss process is the burst ratio [16]. This characteristic is just the average length of the series of consecutive packet losses, properly normalized. Therefore, it describes not the bare frequency of occurrences of losses, but their statistical structure – tendency to cluster together in series. Such series are known to impair significantly the quality of real-time audio and video transmissions, via unpleasant pausing of freezing [17]. There is also a substantial literature on the burst ratio, with direct measurements [18]–[21] and mathematical models [22]–[28].

As it was said, the burst ratio is proportional to the average length of the series of losses. Hence, it does not contain any detailed information about the distribution of the series of losses, but its average value only. In particular, it does not contain any information about the variability of the series of losses and how heavy is the tail of this distribution. Intuitively, such information may be of some value when analyzing the impairment of real-time audio and video transmissions. For instance, a heavy tail of this distribution indicates that we can expect occasionally a very long series of losses.

In this paper, we compute and analyze higher-order statistics of the distribution of the series of losses, i.e., the coefficient of variation, skewness and excess kurtosis. They are based on higher moments (the second, third and fourth moment, respectively) and, combined together, contain much more detailed information about this distribution, than the bare burst ratio.

In calculations, we use a mathematical model of the buffer fed by an aggregated traffic. The Batch Markovian Arrival Process (BMAP) is used to model the traffic. It is perhaps one of the most versatile and useful models, due to it broad modeling capabilities. In particular, using BMAP we can model an arbitrary interarrival time distribution and the autocorrelation of packet interarrival times [29], [30], batch arrivals (useful in TCP modeling [31]) and several other features of traffic. Some of these features will be exploited in numerical examples. The BMAP process is widely used in the performance evaluation of networks, see [32]–[35] and the references there.

To the best of the authors' knowledge, there are no published results on higher-order statistics of the series of losses. The only results we are aware of are those devoted to the first-order statistic, i.e., the burst ratio, published in the papers mentioned above.

The rest of the paper is structured as follows. In Section II, the model of the buffer fed by the BMAP process is described. In Section III, formulas for the coefficient of variation, skewness and excess kurtosis of the series of packet losses are presented. In Section IV, numerical results are given. In particular, five different parameterizations of traffic are used and accompanied by different buffer sizes, loads of the queue and service times. Influence of all these factors on higher-order statistics of the series of losses is discussed. Concluding remarks are given in Section V.

II. The model

We use a finite-buffer queueing model with a single server. Namely, the buffer size is K, including the service position. If upon a packet arrival there are K packets present in the buffer, a newly arriving packet is deleted. The service time has general distribution given by distribution function F with the average value of $\overline{F}$. The load of the queue, ρ, is defined as

$$\rho = \Lambda\overline{F}, \tag{1}$$

 ISBN: 978-1-68558-036-0

where Λ is the rate of the arrival process.

The packet arrival process is modeled by the BMAP process [36]. BMAP is a Markov process denoted by $(N(t), J(t))$, $t \geq 0$, where $N(t)$ is the cumulative number of packets that arrived in $(0, t)$, while $J(t)$ is the state of some Markov chain (continuous-time type), called the modulating chain, with the state space $\{1, \ldots, s\}$. The infinitesimal matrix of $(N(t), J(t))$ is:

$$\begin{bmatrix} D_0 & D_1 & D_2 & D_3 & \cdot\cdot \\ & D_0 & D_1 & D_2 & \cdot\cdot \\ & & D_0 & D_1 & \cdot\cdot \\ & & & \cdot & \cdot\cdot \end{bmatrix},$$

where each D_i, $i \geq 0$, constitutes an $s \times s$ matrix. In addition, each $D_i, i \geq 1$, is nonnegative, while D_0 is negative on its diagonal elements and nonnegative outside the diagonal. Finally, $D = \sum_{i=0}^{\infty} D_i$ has to be an irreducible infinitesimal matrix and has to differ from D_0.

In the analysis of BMAP, the function $P_{i,j}(n, t)$ is used frequently:

$$P_{n,m}(k,t) = \Pr\{N(t) = k, J(t) = m | N(0) = 0, J(0) = n\}. \quad (2)$$

In what follows, $\boldsymbol{e}$ is the column vector of length s of 1's, I is the $s \times s$ identity matrix, $\mathbf{0}$ is the $s \times s$ matrix of 0's and $\mathbf{1}$ is the $s \times s$ matrix of 1's.

III. Higher-order statistics

The higher-order statistics of the length of the series of losses can be obtained in a similar way, as the burst ratio parameter was obtained in [27]. Namely, following the proof of Theorem 1 of [27], we can see that the probability that the series of consecutive packet losses is of length k is:

$$P_k = \frac{\boldsymbol{ur}(k)}{1 - \boldsymbol{ur}(0)}, \qquad k = 1, 2, \ldots, \quad (3)$$

where vector $\boldsymbol{u}$ of size s contains the distribution of the modulating chain at the moment ending the buffer overflow period in the stationary regime. It can be computed as the stationary vector of stochastic $s \times s$ matrix V, i.e., the vector fulfilling the set of equations:

$$\begin{cases} \boldsymbol{ue} = 1, \\ \boldsymbol{u}V = \boldsymbol{u}, \end{cases} \quad (4)$$

where matrix V has the following form:

$$V = W^{-1}\left(Z + \sum_{i=1}^{K} R_{K-i}\overline{A}_i - \sum_{i=1}^{K}\sum_{j=1}^{i} Y_{K-i}R_{i-j}\overline{A}_j\right), \quad (5)$$

with

$$A_k = \left[\int_0^{\infty} P_{n,m}(k,t)dF(t)\right]_{n,m}, \quad (6)$$

$$Y_k = \left[\frac{-(D_0)_{nn}p_n(k,m)}{\lambda_n}\right]_{n,m}, \quad (7)$$

$$R_0 = \mathbf{0}, \quad R_1 = A_0^{-1},$$

$$R_{j+1} = A_0^{-1}\left(R_j - \sum_{i=0}^{j} A_{i+1}R_{j-i}\right), \quad j \geq 1, \quad (8)$$

$$Z = \sum_{i=K}^{\infty} Y_i \overline{A}_0, \quad (9)$$

$$\overline{A}_i = A_0 - \sum_{j=0}^{i-1} A_j, \quad (10)$$

and

$$W = \sum_{i=0}^{K} R_{K-i}A_i - \sum_{i=1}^{K}\sum_{j=0}^{i} Y_{K-i}R_{i-j}A_j. \quad (11)$$

Moreover, $p_n(k, m)$ in (7) is the probability that in the arrival process there will be a change of the modulating chain to m together with an arrival of a batch of size k, if the modulating chain is currently in state n. This probability is equal to:

$$p_n(0, n) = 0, \quad \text{for every } n, \quad (12)$$

$$p_n(0, m) = \frac{1}{-(D_0)_{nn}}(D_0)_{nm}, \quad n \neq m, \quad (13)$$

$$p_n(k, m) = \frac{1}{-(D_0)_{nn}}(D_m)_{nm}, \quad k \geq 1. \quad (14)$$

On the other hand, vectors $\boldsymbol{r}(k)$, which are present in (3), are defined as follows. The n-th entry of vector $\boldsymbol{r}(k)$ is the probability, that during the first buffer overflow period the number of lost packets equals k, assuming $X(0) = K - 1$ and $J(0) = n$. Vectors $\boldsymbol{r}(k)$ have the following form, see [27]:

$$\boldsymbol{r}(k) = W^{-1} \cdot \left(\sum_{i=1}^{K} R_{K-i}A_{i+k} - \sum_{i=1}^{K}\sum_{j=1}^{i} Y_{K-i}R_{i-j}A_{j+k} + \sum_{i=K}^{K+k} Y_i A_{K+k-i}\right)\boldsymbol{e}. \quad (15)$$

Matrices A_k, which are present in (8), (10), (11) and (15), can be computed using the uniformization method of [36]. Exploiting this method we get:

$$A_i = \sum_{j=0}^{\infty} \frac{T_{i,j}}{j!} \int_0^{\infty} e^{-\theta t}(\theta t)^j dF(t), \quad (16)$$

with

$$\theta = \max_n\{(-D_0)_{nn}\}, \quad (17)$$

and:

$$\begin{aligned} T_{0,0} &= I, & (18) \\ T_{k,0} &= \mathbf{0}, \quad k \geq 1, & (19) \\ T_{0,j+1} &= T_{0,j}(I + \theta^{-1}D_0), & (20) \\ T_{k,j+1} &= \theta^{-1}\sum_{i=0}^{k-1} T_{i,j}D_{k-i} + T_{k,j}(I + \theta^{-1}D_0). & (21) \end{aligned}$$

 ISBN: 978-1-68558-036-0

Now, using (3) with (4), (5) and (15), we can obtain higher-order statistics of the series of losses. Namely, the coefficient of variation, C_v, of the length of the series of losses is:

$$C_v = \frac{S}{G}, \tag{22}$$

where G is the average length of the series of losses:

$$G = \frac{\sum_{k=1}^{\infty} k\boldsymbol{ur}(k)}{1 - \boldsymbol{ur}(0)}, \tag{23}$$

while S is the standard deviation:

$$S = \sqrt{\frac{\sum_{k=1}^{\infty} k^2\boldsymbol{ur}(k)}{1 - \boldsymbol{ur}(0)} - G^2}. \tag{24}$$

The skewness M of the length of the series of losses is:

$$M = \frac{\sum_{k=1}^{\infty} (k-G)^3\boldsymbol{ur}(k)}{S^3(1 - \boldsymbol{ur}(0))}. \tag{25}$$

Finally, the excess kurtosis N of the length of the series of losses is:

$$N = \frac{\sum_{k=1}^{\infty} (k-G)^4\boldsymbol{ur}(k)}{S^4(1 - \boldsymbol{ur}(0))} - 3. \tag{26}$$

IV. EXAMPLES

In these examples, we will use the same parameterizations of the system that were used in [27] to study the first-order statistic. In particular, the following five BMAP parameterizations will be used:

$BMAP_1$: $D_0 = -6.66666666 \cdot I,\ D_1 = 6.66666666 \cdot I.$

$BMAP_2$:

$$D_0 = \begin{bmatrix} -2.66407491 & 0.21318153 & 0.06876823 \\ 0.28194977 & -4.12978130 & 0.28194977 \\ 0.07564506 & 0.07564506 & -14.6406033 \end{bmatrix},$$

$$D_1 = \begin{bmatrix} 1.30324098 & 0.60086202 & 0.47802213 \\ 0.28641400 & 2.92428551 & 0.35518224 \\ 0.80716673 & 0.28882659 & 13.3933198 \end{bmatrix}.$$

$BMAP_3$: $D_0 = -I,\ D_2 = 0.02222222 \cdot \mathbf{1},\ D_4 = 0.07777778 \cdot \mathbf{1},\ D_8 = 0.23333333 \cdot \mathbf{1}.$

$BMAP_4$:

$$D_0 = \begin{bmatrix} -0.39961124 & 0.03197723 & 0.01031523 \\ 0.04229246 & -0.61946720 & 0.04229246 \\ 0.01134675 & 0.01134675 & -2.19609050 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.14544482 & 0.01134675 & 0.02166199 \\ 0.01134675 & 0.03197723 & 0.02166199 \\ 0.02166199 & 0.03197723 & 0.05260770 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.01134675 & 0.02166199 & 0.01134675 \\ 0.01134675 & 0.33111906 & 0.01134675 \\ 0.04229246 & 0.01134675 & 0.02166199 \end{bmatrix},$$

$$D_8 = \begin{bmatrix} 0.03869456 & 0.05712054 & 0.03869456 \\ 0.02026858 & 0.07554653 & 0.02026858 \\ 0.05712054 & 0.00000000 & 1.93472829 \end{bmatrix}.$$

$BMAP_5$:

$$D_0 = \begin{bmatrix} -45.5935855 & 1.95261616 & 0.19526161 \\ 0.01952616 & -4.55935855 & 0.19526161 \\ 0.00195261 & 0.01952616 & -0.45593586 \end{bmatrix},$$

$$D_2 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_4 = \begin{bmatrix} 0.06508720 & 0.52069762 & 5.20697622 \\ 0.52069762 & 0.00065087 & 0.05792761 \\ 0.05076801 & 0.00650872 & 0.00065087 \end{bmatrix},$$

$$D_8 = \begin{bmatrix} 0.35797962 & 2.86383692 & 28.6383692 \\ 2.86383692 & 0.00357979 & 0.31860186 \\ 0.27922410 & 0.03579796 & 0.00357979 \end{bmatrix}.$$

On purpose, all of these arrival processes have the same arrival rate, $\Lambda = 20/3$, but quite different internal statistical properties. In particular, $BMAP_1$ is in fact a simple Poisson process, so it has no autocorrelation of interarrival times, nor batch arrivals. $BMAP_2$ is positively, strongly autocorrelated, but has no batch arrivals. On the other hand, $BMAP_3$ is not autocorrelated, but has batch arrivals. Finally, both $BMAP_4$ and $BMAP_5$ are strongly autocorrelated and have batch arrivals. The difference is that the autocorrelation of $BMAP_4$ is positive, while $BMAP_5$ has an oscillating autocorrelation, with positive and negative signs. It is important that when batch arrivals are involved (in $BMAP_3$, $BMAP_4$ and $BMAP_5$), the same batch sizes are used in every case. Similarly, in two cases with positive autocorrelation ($BMAP_2$ and $BMAP_4$), exactly the same autocorrelation function is used.

By default, we will use $K = 50$, $\rho = 1$ and exponential service time with the mean of 3/20. There will be some exceptions, but they will be clearly stated.

In Figure 1, the distribution P_k of the length of the series of losses is presented for all considered arrival processes. As we can see, for $BMAP_1$ and $BMAP_2$, which do not have the batch structure, the distribution is regular and monotonic. A heavier tail can be observed in the correlated case, $BMAP_2$. For $BMAP_3, \ldots, BMAP_5$, which do have the batch structure, this distribution has an irregular form, with multiple spikes.

TABLE I
HIGHER-ORDER STATISTICS OF THE SERIES OF LOSSES FOR DIFFERENT ARRIVAL TRAFFIC. $K = 50$ AND $\rho = 1$.

traffic	C_v	M	N
$BMAP_1$	0.7071	2.1213	6.5000
$BMAP_2$	0.8030	2.0786	6.3582
$BMAP_3$	0.7414	1.8390	5.6675
$BMAP_4$	0.7552	2.0258	6.3752
$BMAP_5$	0.7378	1.6039	4.5386

In Table I, the higher-order statistics of the series of consecutive packet losses for $BMAP_1, \ldots, BMAP_5$ are presented. As we can see, the coefficient of variation assumes moderate

 ISBN: 978-1-68558-036-0

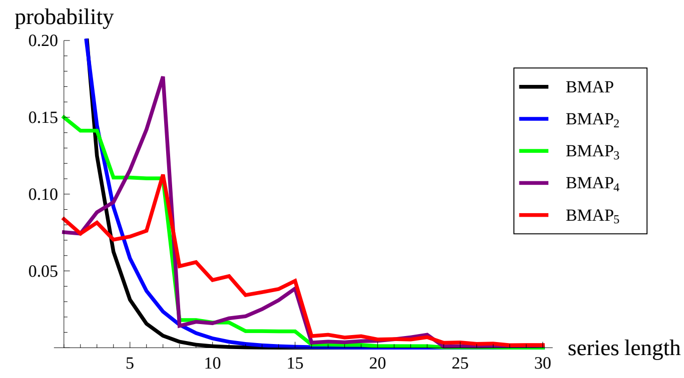

Figure 1. Distribution of the length of the series of losses for $BMAP_1,\ldots,BMAP_5$. $K=50$ and $\rho=1$.

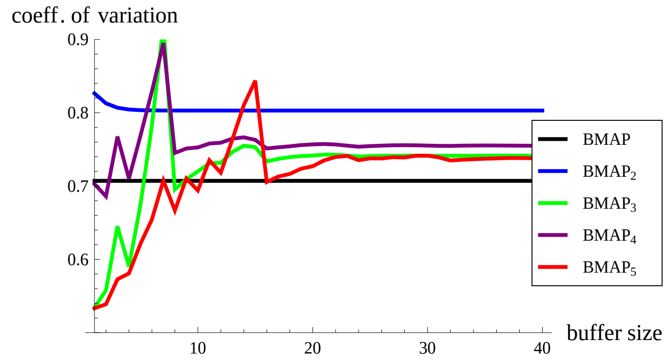

Figure 2. Coefficient of variation versus the buffer size for $BMAP_1,\ldots,BMAP_5$, $\rho=1$.

values in the range 0.7-0.8. The autocorrelated structure seems to elevate C_v slightly (compare $BMAP_2$ with $BMAP_1$). This is not so clear about the batch structure. If we compare $BMAP_3$ with $BMAP_1$, it seems that the batch structure makes the coefficient of variation greater. On the other hand, C_v is less in the case of $BMAP_4$, than in the case of $BMAP_2$, even though they share the same autocorrelation function, $BMAP_4$ has the batch structure, while $BMAP_2$ does not.

The skewness in Table I is positive in all the cases, with the values around 2. This is consistent with Figure 1, in which all the tails are on the right side.

The most interesting statistic in Table I is the excess kurtosis. As we can see, it assumes rather high, positive values in all the cases, which indicate fat tails of the distributions of the series of losses (much fatter than in the case of normal distribution which has $N=0$).

Contrary to C_v, the excess kurtosis seems to be less when a positive autocorrelation or batch structure is involved - compare again $BMAP_2$ with $BMAP_1$ and $BMAP_3$ with $BMAP_1$, respectively. In the case of the oscillating autocorrelation, $BMAP_5$, the smallest value is observed, while still rather high.

So far, only the buffer of size 50 was considered. Now we will check the dependence of the higher-order statistics on the buffer size.

In Figures 2, 3 and 4, the coefficient of variation, skewness and excess kurtosis as functions of the buffer size are presented, for all the considered arrival streams. As we can see, all three statistics are practically independent on the buffer size, when the arrival process has no batch structure - the curves are flat for $BMAP_1$ and $BMAP_2$. On the other hand, a high dependence of the three statistics on the buffer size can be observed when the arrival process does have the batch structure and the buffer is rather small - see the spikes in Figures 2-4 for $BMAP_3,\ldots,BMAP_5$. However, for a relatively small K, about 25, all three statistics stabilize and do not change anymore, when the buffer grows. Therefore $K=50$ used herein as a default value is already in the stable regime.

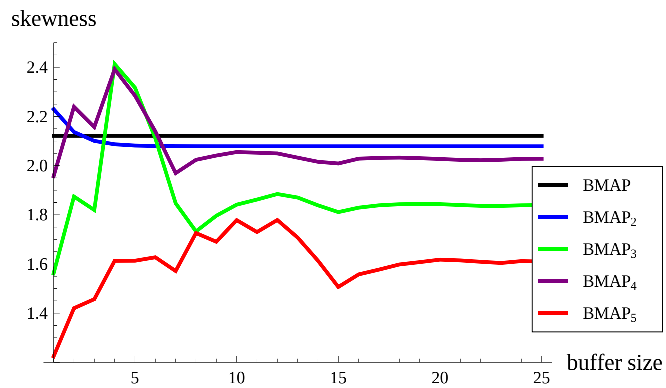

Figure 3. Skewness versus the buffer size for $BMAP_1,\ldots,BMAP_5$, $\rho=1$.

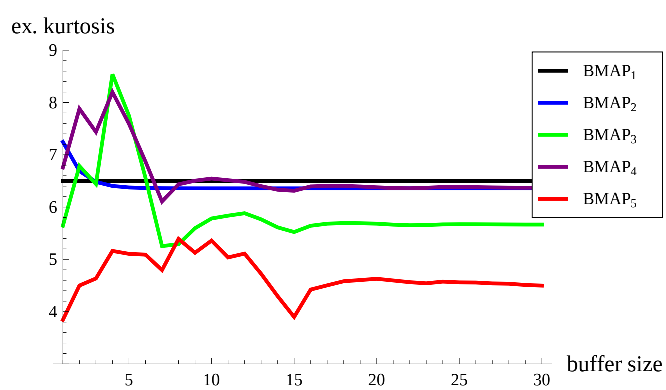

Figure 4. Excess kurtosis versus the buffer size for $BMAP_1,\ldots,BMAP_5$, $\rho=1$.

Increasing the buffer e.g. to $K=100$ would have a negligible effect on the three statistics.

Now we get back to $K=50$ and vary the load of the queue

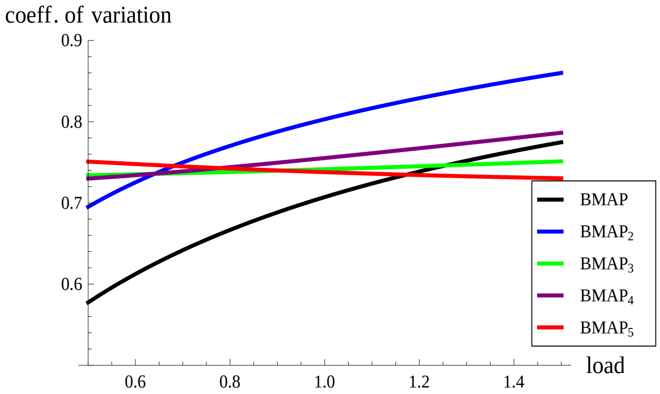

Figure 5. Coefficient of variation versus the system load for $BMAP_1, \ldots, BMAP_5$, $K = 50$.

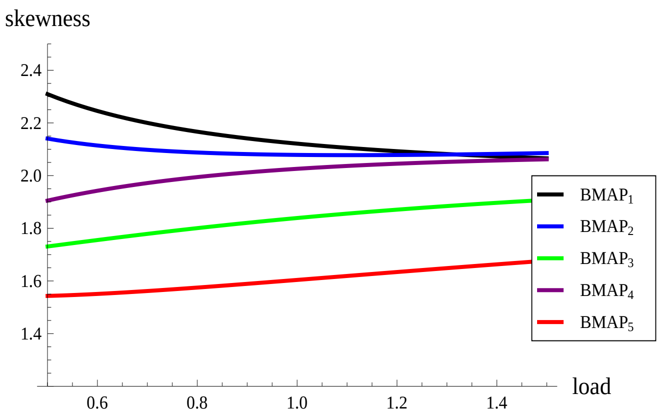

Figure 6. Skewness versus the system load size for $BMAP_1, \ldots, BMAP_5$, $K = 50$.

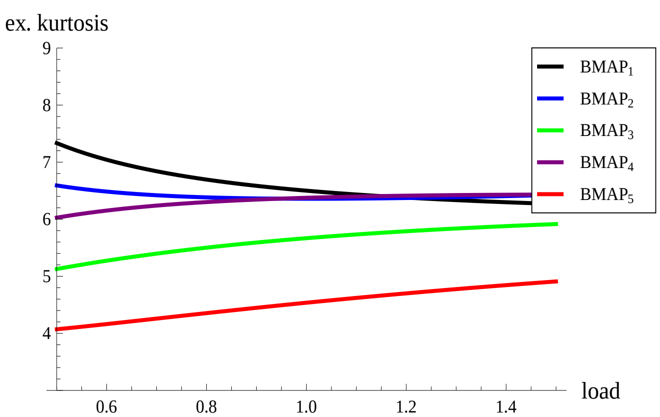

Figure 7. Excess kurtosis versus the system load for $BMAP_1, \ldots, BMAP_5$, $K = 50$.

(it was unaltered so far, $\rho = 1$).

In Figures 5, 6 and 7, the coefficient of variation, skewness and excess kurtosis as functions of the load of the queue are presented for all the considered arrival streams. As we can see in Figure 5, the behaviour of C_v depends strongly on the presence of batches. Namely, C_v grows significantly with the queue load if the arrivals are single ($BMAP_1$, $BMAP_2$), no matter if the arrival process is correlated or not. C_v changes much slower with load when the arrival process has the batch structure.

In Figure 6, we can notice that the skewness is different for all the arrival processes, when the load is low, $\rho = 0.5$. When the load is high, $\rho = 1.5$, the skewness is almost identical for $BMAP_1$, $BMAP_2$ and $BMAP_4$, but different for remaining BMAPs. A similar situation is in the case of excess kurtosis, which can be observed in Figure 7.

TABLE II
HIGHER-ORDER STATISTICS OF THE SERIES OF LOSSES FOR DIFFERENT SERVICE TIME DISTRIBUTIONS. $BMAP_5$, $K = 50$ AND $\rho = 1$ WERE USED.

service time	C_v	M	N
F_1	0.6729	1.1257	2.1510
F_2	0.6954	1.2963	2.8960
F_3	0.7090	1.4159	3.5801
F_4	0.7378	1.6039	4.5386
F_5	0.9512	2.9188	14.2978

Now we get back to $\rho = 1$ and vary the service time distribution, which was exponential in all the examples so far. Namely, we use now the following five distribution functions of the service time:

$$F_1(x) = 0 \text{ if } x < \frac{3}{20}, \quad F_1(x) = 1 \text{ otherwise}, \tag{27}$$

$$F_2(x) = \frac{20}{6}x, \quad 0 \leq x < \frac{6}{20}, \tag{28}$$

$$F_3(x) = 1 - \frac{40}{3}xe^{-\frac{40}{3}x} - e^{-\frac{40}{3}x}, \quad x \geq 0, \tag{29}$$

$$F_4(x) = 1 - \frac{20}{3}e^{-\frac{20}{3}x}, \quad x \geq 0, \tag{30}$$

$$F_5(x) = 1 - 0.95e^{-9.5x} - 0.05e^{-x}, \quad x \geq 0. \tag{31}$$

All of these distributions have the mean of 3/20, therefore all of them produce the load of 1. They differ, however, in the standard deviation, which is 0, 0.086, 0.107, 0.150 and 0.314 for F_1-F_5, respectively.

The results are shown in Table II. They can be summarized in two points. First, the variation of the service time influences significantly all three higher-order statistics. A particularly great influence can be observed in the case of the excess kurtosis. Second, the dependence is monotonic in each case, i.e., the higher the variation of the service time, the higher C_v, M and N.

V. CONCLUSIONS

We analyzed higher-order statistics of the length of the series of packet losses at a router's output buffer, using a queueing system with a complex, flexible traffic model. In particular, we presented formulas and numerical examples for

 ISBN: 978-1-68558-036-0

the coefficient of variation, skewness and excess kurtosis of the length of the series.

A few observations were made. For instance, all three statistics depended on the buffer size in a very complicated way, but only for small buffer sizes. For a moderate buffer size, they all stabilized and did not change anymore when the buffer grew. All three statistics were significantly influenced by the variance of the service time, in a monotonic manner. The coefficient of variation of the series of losses grew rather quickly with the system load when the arrival process did not have the batch structure, and much slower, when it did.

Perhaps the most important observation made was that the distribution of the series of losses was strongly leptokurtic in all the considered examples. It means that this distribution has usually a rather fat tail, so occasionally a long series of losses can be expected. Naturally, such series may influence badly the quality of real-time multimedia transmissions. Unfortunately, no quantitative measure of such influence has been proposed so far in terms of a higher-order statistic. For instance, the impairment of voice transmission is estimated using the first-order statistic only (see [17]). An interesting future work would be proposing a formula for this impairment, taking into account the kurtosis.

ACKNOWLEDGEMENT

This research was funded by National Science Centre, Poland, grant number 2020/39/B/ST6/00224.

REFERENCES

[1] J. Bolot, "End-to-end packet delay and loss behavior in the Internet", Proc of ACM SIGCOMM'93, pp. 289–298, 1993.

[2] M. Yajnik, S. Moon, J. Kurose, and D. Towsley, "Measurement and modelling of the temporal dependence in packet loss", Proc. IEEE INFOCOM, vol. 1, pp. 345–352, 1999.

[3] N. G. Duffield, F. Lo Presti, V. Paxson, and D. Towsley, "Inferring link loss using striped unicast probes", IEEE INFOCOM, pp. 915–923, 2001.

[4] J. Sommers, P. Barford, N. Duffield, and A. Ron, "Improving accuracy in end-to-end packet loss measurement", Computer Communication Review, no. 35(4), pp. 157–168, 2005.

[5] J. Sommers, P. Barford, N. Duffield, and A. Ron, "A geometric approach to improving active packet loss measurement", IEEE/ACM Transactions on Networking, no. 16(2), pp. 3 – 320, 2008.

[6] Z. Hu and Q. Zhang, "A new approach for packet loss measurement of video streaming and its application", Multimedia Tools and Applications, no. 77(10), pp. 11589–11608, 2018.

[7] H. Lan, "Strengthening packet loss measurement from the network intermediate point", KSII Transactions on Internet and Information Systems, no. 13(12), pp. 5948–5971, 2019.

[8] I. Cidon, A. Khamisy, and M. Sidi, "Analysis of packet loss processes in high-speed networks", IEEE Transactions on Information Theory, no. 39(1), pp. 98–108, 1993.

[9] X. Yu, J. W. Modestino, and X. Tian, "The accuracy of Gilbert models in predicting packet-loss statistics for a single-multiplexer network model", IEEE INFOCOM, pp.2602–2612, 2005.

[10] A. Chydzinski, R. Wojcicki, and G. Hryn, "On the Number of Losses in an MMPP Queue", Lecture Notes in Computer Science, no. 4712, pp. 38–48, 2007.

[11] G. Hasslinger and O. Hohlfeld, "The Gilbert-Elliott model for packet loss in real time services on the Internet", Proc. of MMB 2008, pp. 1–15, 2008.

[12] M. Bratiychuk and A. Chydzinski, "On the loss process in a batch arrival queue", Applied Mathematical Modelling, no. 33, iss. 9, pp. 3565–3577, 2009.

[13] A. Chydzinski, and B. Adamczyk, "Transient and stationary losses in a finite-buffer queue with batch arrivals", Mathematical Problems in Engineering, vol. 2012, ID 326830, pp. 1–18, 2012.

[14] M. Ellis, D. P. Pezaros, T. Kypraios, and C. Perkins, "A two-level Markov model for packet loss in UDP/IP-based real-time video applications targeting residential users", Computer Networks no. 70, pp. 384–399, 2014.

[15] P. Mrozowski and A. Chydzinski, "Queues with dropping functions and autocorrelated arrivals", Methodology and Computing in Applied Probability, no. 20(1), pp. 97–115, 2018.

[16] J. W. McGowan, "Burst ratio: a measure of bursty loss on packet-based networks", 16 2005. US Patent 6,931,017, 2005.

[17] ITU-T Recommendation G.107: "The E-model, a computational model for use in transmission planning", Technical report, 2014.

[18] D. Samociuk, A. Chydzinski, and M. Barczyk, "Experimental measurements of the packet burst ratio parameter", Communications in Computer and Information Science, no. 928, pp. 455–466, 2018.

[19] D. Samociuk, M. Barczyk, and A. Chydzinski, "Measuring and analyzing the burst ratio in IP traffic", Lecture Notes of the Institute for Computer Sciences, Social Informatics and Telecommunications Engineering, no. 303, pp. 86–101, 2019.

[20] A. Chydzinski and D. Samociuk, "Burst ratio in a single-server queue", Telecommunication Systems, no. 70(2), pp. 263–276, 2019.

[21] M. Barczyk and A. Chydzinski, "AQM based on the queue length: A real-network study", PLoS ONE, no. 17(2): e0263407, 2022.

[22] J. Rachwalski and Z. Papir, "Burst Ratio in Concatenated Markov-based Channels", Journal of Telecommunications and Information Technology, no. 1, pp. 3–9, 2014.

[23] J. Rachwalski and Z. Papir, "Analysis of Burst Ratio in Concatenated Channels", Journal of Telecommunications and Information Technology, no. 4, pp. 65–73, 2015.

[24] A. Chydzinski, M. Barczyk, and D. Samociuk, "The Single-Server Queue with the Dropping Function and Infinite Buffer", Mathematical Problems in Engineering, vol. 2018, ID 3260428, pp. 1–12, 2018.

[25] A. Chydzinski, D. Samociuk, and B. Adamczyk, "Burst ratio in the finite-buffer queue with batch Poisson arrivals", Applied Mathematics and Computation, no. 330, pp. 225–238, 2018.

[26] A. Chydzinski, "Impact of the Dropping Function on Clustering of Packet Losses", Sensors, no. 22(20), 2022.

[27] A. Chydzinski, "Burst ratio for a versatile traffic model", PLoS ONE, no. 17(8): e0272263, 2022.

[28] A. Chydzinski, "Per-flow structure of losses in a finite-buffer queue", Applied Mathematics and Computation, no. 428, 127215, pp. 1-15, May 2022.

[29] A. Klemm, C. Lindemann, and M. Lohmann, "Modeling IP traffic using the batch Markovian arrival process", Performance Evaluation, vol. 54, issue 2, 2003.

[30] P. Salvador, A. Pacheco, and R. Valadas, "Modeling IP traffic: joint characterization of packet arrivals and packet sizes using BMAPs", Computer Networks, no. 44, pp. 335-352, 2004.

[31] M. Garetto and D. Towsley, "An efficient technique to analyze the impact of bursty TCP traffic in wide-area networks", Performance Evaluation, no. 65, pp.181–202, 2008.

[32] H. W. Lee, N. I. Park and J. Jeon, "A new approach to the queue length and waiting time of BMAP/G/1 queues", Computers & Operations Research, vol. 30, pp. 2021–2045, 2003.

[33] T. Hofkens, K. Spaey and C. Blondia, "Transient Analysis of the D-BMAP/G/1 Queue with an Application to the Dimensioning of a Playout Buffer for VBR Video", Proceedings of Networking'04. Lecture Notes in Computer Science, vol. 3042, pp. 1338–1343, 2004.

[34] A. N. Dudin, A. A. Shaban, and V. I. Klimenok, "Analysis of a queue in the BMAP/G/1/N system", International Journal of Simulation: Systems, Science and Technology. vol. 6, no. 1–2, pp. 13–23, 2005.

[35] A. D. Banik and M. L. Chaudhry, "Efficient Computational Analysis of Stationary Probabilities for the Queueing System BMAP/G/1/N With or Without Vacation(s)", INFORMS Journal on Computing, no. 29(1), pp. 140–151, 2017.

[36] D. M. Lucantoni, "New results on the single server queue with a batch Markovian arrival process", Commun. Stat., Stochastic Models, vol. 7, no. 1, pp. 1–46, 1991.

 ISBN: 978-1-68558-036-0