

On the Performance of a Low-Complexity Data-Reuse RLS Algorithm for Stereophonic Acoustic Echo Cancellation

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Abstract—The Stereophonic Acoustic Echo Cancellation (SAEC) setup implies the estimation of four loudspeaker-to-microphone unknown impulse responses, which generate unwanted acoustic replicas (i.e., echoes). In this context, some state-of-the-art approaches that combine the Widely Linear (WL) model with various versions of the Recursive Least-Squares (RLS) algorithm have been recently proposed. This paper focuses on the most recent one – the version that uses a Data-Reuse (DR) approach over the WL-RLS based on Dichotomous Coordinate Descent (DCD) iterations, i.e., the namely WL-DR-RLS-DCD. The target of the paper is to present how this algorithm behaves in various conditions, for input signals that present a very high correlation, such as speech or Auto-Regressive (AR) sequences. Simulations results proved that the DR approach is suitable in the SAEC context.

Index Terms—Stereophonic Acoustic Echo Cancellation (SAEC); Recursive Least-Squares (RLS) algorithm; Data-Reuse (DR); Dichotomous Coordinate Descent (DCD); Widely Linear (WL) model.

I. INTRODUCTION

Stereophonic communication creates the sensation of audio directionality by employing for each terminal two loudspeakers and two microphones, respectively [1]–[3]. When performing the Stereophonic Acoustic Echo Cancellation (SAEC), a total number of four echo paths must be estimated in order to mitigate the echo effect produced by any of the four associated loudspeaker-to-microphone pairs. The standard approach for such scenarios is the implementation of four individual adaptive systems, usually based on the Least-Mean-Squares (LMS) family of algorithms [4]–[6]. However, despite their low arithmetic costs, the LMS methods provide limited performance when working with highly correlated input signals (such as speech sequences) due to gradient noise [4].

In [7]–[9], the recursive least-squares (RLS) algorithm [4] was combined with the dichotomous coordinate descent (DCD) iterations [10], and employed in SAEC scenarios in order to improve performance, especially in terms of convergence rate and computational complexity. The widely linear (WL) model described in [1][8], was used as a framework in order to improve the handling of the system, by grouping the four adaptive filters working with real valued variables into a single adaptive system working with fewer complex valued variables (CRVs). The resulting WL-RLS-DCD method has acceptable arithmetic workloads and performance levels

comparable with other consecrated RLS versions, such as the one based on Woodburry’s identity [4].

This paper analyzes the performance of an improved version of the WL-RLS-DCD algorithm designed for superior tracking capabilities, which requires minimal extra arithmetic costs. A Data-Reuse (DR) approach for the WL-RLS-DCD adaptive method will be described, which re-uses the same input data multiples times for each of the adaptive filter’s iterations. An analysis will be performed in several scenarios in order to demonstrate the capabilities of the new WL-DR-RLS-DCD method, and several other aspects will be discussed.

This paper is organized as follows. Section II describes the mathematical model of the WL framework for RLS adaptive systems. Section III presents the WL-DR-RLS-DCD algorithm, which is suitable for improving its corresponding tracking speeds. Section IV discusses simulation results for the proposed algorithm in the SAEC setup, and the paper is concluded by a few conclusions in Section V.

II. SYSTEM MODEL

In the SAEC configuration, for each discrete-time index n , we store the last L samples corresponding to $x_{L_c}(n)$ and $x_{R_c}(n)$ (i.e., the left and right channels) in the $L \times 1$ vectors denoted by $\mathbf{x}_{L_c}(n)$, respectively $\mathbf{x}_{R_c}(n)$ [1][8]. Furthermore, the echo contributions associated with the stereo channels can be obtained using the input vectors and the four possible $L \times 1$ echo path impulse responses denoted by $\mathbf{g}_{t,L_c L_c}$, $\mathbf{g}_{t,L_c R_c}$, $\mathbf{g}_{t,R_c L_c}$, and $\mathbf{g}_{t,R_c R_c}$ [8][9]:

$$y_{L_c}(n) = \mathbf{g}_{t,L_c L_c}^T \mathbf{x}_{L_c}(n) + \mathbf{g}_{t,R_c L_c}^T \mathbf{x}_{R_c}(n), \quad (1)$$

$$y_{R_c}(n) = \mathbf{g}_{t,L_c R_c}^T \mathbf{x}_{L_c}(n) + \mathbf{g}_{t,R_c R_c}^T \mathbf{x}_{R_c}(n), \quad (2)$$

where T represents the transpose operator. As a consequence, the microphone (or reference) signals can be written as

$$d_{L_c}(n) = y_{L_c}(n) + w_{L_c}(n), \quad (3)$$

$$d_{R_c}(n) = y_{R_c}(n) + w_{R_c}(n), \quad (4)$$

with $w_{L_c}(n)$ and $w_{R_c}(n)$ being environmental noise signals.

When we apply the WL model, we can use $j = \sqrt{-1}$ to combine the inputs and the outputs of the SAEC unknown echo paths into the complex input signal

$$x(n) = x_{L_c}(n) + jx_{R_c}(n), \quad (5)$$

respectively the complex output

$$y(n) = y_{L_c}(n) + jy_{R_c}(n). \quad (6)$$

When applying this approach for the vectors $\mathbf{x}_{L_c}(n)$ and $\mathbf{x}_{R_c}(n)$, then interleaving the resulting complex valued $L \times 1$ vector with its complex conjugate version, a $2L \times 1$ input vector $\tilde{\mathbf{x}}(n)$ is obtained [1][8][9]. Similarly, the unknown echo paths can be linearly combined and interleaved in order to generate a single $2L \times 1$ complex valued unknown system denoted as $\tilde{\mathbf{g}}_t$ [1][8][9].

Furthermore, the complex valued output can be written as

$$y(n) = \tilde{\mathbf{g}}_t^H \tilde{\mathbf{x}}(n), \quad (7)$$

where H denotes the Hermitian operator.

We can write the complex valued noise signal

$$w(n) = w_{L_c}(n) + jw_{R_c}(n), \quad (8)$$

and also the expression for the complex reference as

$$d(n) = y(n) + w(n). \quad (9)$$

Thus, we express the complex error as

$$e(n) = d(n) - \tilde{y}(n), \quad (10)$$

where $\tilde{y}(n)$ is the estimate of the complex valued system's output determined with the adaptive filter's set of coefficients $\tilde{\mathbf{g}}(n)$, which is employed to approximate $\tilde{\mathbf{g}}_t$.

The WL framework improves the handling of SAEC applications using adaptive algorithms. It re-casts four unknown system identification problems into a single one. The two-input/two-output system with real random variables is expressed as a single-input/single-output system with CRVs.

III. THE WL-DR-RLS-DCD ALGORITHM

The process of generating the estimate $\tilde{\mathbf{g}}(n)$ can be approached in SAEC scenarios using RLS filters due to their increased performance when working with highly correlated signals, such as speech. In [8][10], the complex valued leading DCD was employed in combination with the RLS algorithm in order to decrease the corresponding arithmetic complexity and mitigate their prohibitive nature when approaching hardware implementations. The DCD requires only additions and bit-shifts to solve an auxiliary system of equations, by exploiting the properties of the input signal [8][10].

The RLS-DCD method applied to the WL setup (i.e., the WL-RLS-DCD) is presented in Table I, where we denoted by ϵ a small positive constant employed to initialize the correlation matrix in a non-singular form, and by $\mathbf{R}^{(:,1)}(n)$ the first column of the matrix $\mathbf{R}(n)$. Steps 1 and 2 update the complex input vector $\tilde{\mathbf{x}}(n)$, respectively the $2L \times 2L$ correlation matrix estimate $\mathbf{R}(n)$, using the time-shift property of the input signal and the forgetting factor λ ($0 < \lambda \leq 1$), which determines the memory of the algorithm [1][9]. After computing the filter output and the corresponding error in steps 3 and 4, the residual component $\mathbf{p}_0(n)$ is updated in step 5 using the residual vector $\mathbf{r}(n-1)$ [8][10]. Consequently, the

DCD iterations are employed in step 6 in order to solve an auxiliary system of equations and generate the $2L \times 1$ complex valued solution vector $\Delta\tilde{\mathbf{g}}(n)$, and an updated version of $\mathbf{r}(n)$. In the final step of the WL-RLS-DCD (i.e., step 7), the solution vector is used to update the filter coefficients $\tilde{\mathbf{g}}(n)$. The overall complexity of the algorithm is proportional to the value $2L$ multiplied by a small factor, which makes it attractive for hardware implementations.

Table I. WL-RLS-DCD ALGORITHM

Step	Action
Init.	Set: $\tilde{\mathbf{g}}(0) = \mathbf{0}_{2L \times 1}$; $\mathbf{r}(0) = \mathbf{0}_{2L \times 1}$ $\mathbf{R}(0) = \epsilon \mathbf{I}_{2L}$, $\epsilon > 0$
For $n = 1, 2, \dots$, number of iterations :	
1	Update $\tilde{\mathbf{x}}(n)$
2	Update $\mathbf{R}(n)$ using time-shift $\mathbf{R}^{(:,1)}(n) = \lambda \mathbf{R}^{(:,1)}(n-1) + x^*(n)\tilde{\mathbf{x}}(n)$
3	$\tilde{y}(n) = \tilde{\mathbf{g}}^H(n-1)\tilde{\mathbf{x}}(n)$
4	$e(n) = d(n) - \tilde{y}(n)$
5	$\mathbf{p}_0(n) = \lambda \mathbf{r}(n-1) + e^*(n)\tilde{\mathbf{x}}(n)$
6	$\mathbf{R}(n)\Delta\tilde{\mathbf{g}}(n) = \mathbf{p}_0(n) \xrightarrow{\text{DCD}} \Delta\tilde{\mathbf{g}}(n), \mathbf{r}(n)$
7	$\tilde{\mathbf{g}}(n) = \tilde{\mathbf{g}}(n-1) + \Delta\tilde{\mathbf{g}}(n)$

The tracking speed of the WL-RLS-DCD can be improved using the DR approach [9][11][12], which employs the same input data multiple times per each time index n . The DCD method is run for a number of N_{it} iterations (i.e., for $\phi = 0 \dots N_{it} - 1$), and for each of the solution vectors $\Delta\tilde{\mathbf{g}}_\phi(n)$, the output signal estimate changes to $\tilde{y}_\phi(n)$ [9] (i.e., it is updated accordingly):

$$\tilde{y}_\phi(n) = \begin{cases} \tilde{\mathbf{g}}^H(n-1)\tilde{\mathbf{x}}(n), & \phi = 0, \\ \left[\tilde{\mathbf{g}}(n-1) + \sum_{k=0}^{\phi-1} \Delta\tilde{\mathbf{g}}_k(n) \right]^H \tilde{\mathbf{x}}(n), & \phi > 0. \end{cases} \quad (11)$$

It can be noticed that only the first branch of (11) is used when the new algorithm has the parameter $N_{it} = 1$. In this case, it is equivalent to the WL-RLS-DCD and only the filter coefficients from the previous time index (i.e., $n-1$) are required for any computations at time index n . However, if $N_{it} > 1$, then for any DR iteration corresponding to $\phi > 1$, the values corresponding to the filter taps have to be updated using the solution vector $\Delta\tilde{\mathbf{g}}_{\phi-1}(n)$ from the previous DR step (also, from the same time index). In a similar manner, the value of the error signal is adjusted to reflect the change in the filter coefficients [9], and to also take advantage of the results available at previous iterations:

$$e_\phi(n) = \begin{cases} d(n) - \tilde{\mathbf{g}}^H(n-1)\tilde{\mathbf{x}}(n) \triangleq e_0(n), & \phi = 0 \\ e_{\phi-1}(n) + \Delta\tilde{\mathbf{g}}_{\phi-1}^H(n)\tilde{\mathbf{x}}(n), & \phi > 0. \end{cases} \quad (12)$$

Furthermore, in order to run the DCD for each DR step, the residual contributions must be updated and can be expressed as [9]

$$\mathbf{p}_{0,\phi}(n) = \begin{cases} \lambda \mathbf{r}_{N_{it}-1}(n-1) + e_0^*(n)\tilde{\mathbf{x}}(n), & \phi = 0 \\ \mathbf{r}_{\phi-1}(n) + e_0^*(n)\tilde{\mathbf{x}}(n), & \phi > 0. \end{cases} \quad (13)$$

The first branch of (13) represents the classical transition from the previous time index, while the second branch performs the transition from the previous DR iteration. A common conclusion is also available for (11), (12), respectively (13) when considering $N_{it} = 1$: all the update expressions are simplified such that the form of the proposed algorithm reverts back to the steps corresponding to the WL-RLS-DCD method.

Table II. WL-DR-RLS-DCD ALGORITHM

Step	Action
Init.	Set: $\tilde{\mathbf{g}}(0) = \mathbf{0}_{2L \times 1}$; $\mathbf{r}(0) = \mathbf{0}_{2L \times 1}$ $\mathbf{R}(0) = \epsilon \mathbf{I}_{2L}$, $\epsilon > 0$, $\phi = 0$
For $n = 1, 2, \dots$, number of iterations :	
1	Update $\tilde{\mathbf{x}}(n)$
2	Update $\mathbf{R}(n)$ using time-shift $\mathbf{R}^{(:,1)}(n) = \lambda \mathbf{R}^{(:,1)}(n-1) + x^*(n)\tilde{\mathbf{x}}(n)$
3	$\phi = \phi + 1$
4	Determine $e_\phi(n)$ using (12)
5	Determine $\mathbf{p}_{0,\phi}(n)$ using (13)
6	$\mathbf{R}(n)\Delta\mathbf{g}_\phi(n) = \mathbf{p}_{0,\phi}(n) \xrightarrow{\text{DCD}} \Delta\tilde{\mathbf{g}}_\phi(n), \mathbf{r}_\phi(n)$
7	Determine $\tilde{\mathbf{g}}_\phi(n)$ using $\Delta\tilde{\mathbf{g}}_\phi(n)$ If $\phi < N_{it} \xrightarrow{\text{jump}}$ step 3

The resulting adaptive algorithm, namely the WL-DR-RLS-DCD, is presented in Table II. It is expected to generate improved tracking capabilities with the cost of some accuracy when the adaptive system reaches the steady-state. Moreover, the overall arithmetic effort remains proportional to the value $2L$ multiplied by a small number, which includes the contribution of N_{it} . Simulations will demonstrate that a value of N_{it} much smaller than 10 is sufficient in order to generate satisfactory performance.

IV. SIMULATION RESULTS

Simulations were performed for the SAEC context working with the WL framework. Two types of input signal sources were employed: a Gaussian noise filtered through an autoregressive (AR) system with a single pole, and a high quality speech sequence [13]. Both types are highly correlated, which challenges the convergence of the algorithms. These inputs were filtered through two distinct real acoustic impulse responses in order to achieve the effect of the *left* and *right* channels and generate the samples $x_{L_c}(n)$ and $x_{R_c}(n)$. For each acoustic channel, Gaussian noise was added to the echo signals having the signal-to-noise ratio (SNR) experimentally set to 30 dB.

The pre-distortion approach presented in [1][7][14] was also used in order to avoid the problem of the correlation between the $x_{L_c}(n)$ and $x_{R_c}(n)$ sequences. The method performs the addition of a certain level of nonlinearity to these two signals in order to obtain a unique solution and it is controlled by a parameter denoted by α_r (with $0 < \alpha_r < 1$). Consequently, starting with the so-called *half-wave rectifier* method, we can generate 2 new signals using the expressions [1][14]:

$$\mathbf{x}'_{L_c}(n) = \mathbf{x}_{L_c}(n) + \alpha_r \frac{\mathbf{x}_{L_c}(n) + |\mathbf{x}_{L_c}(n)|}{2}, \quad (14)$$

$$\mathbf{x}'_{R_c}(n) = \mathbf{x}_{R_c}(n) + \alpha_r \frac{\mathbf{x}_{R_c}(n) - |\mathbf{x}_{R_c}(n)|}{2}. \quad (15)$$

From a practical point of view, the stereo effect is not impacted if $0 < \alpha_r \leq 0.5$.

In all analyzed cases, the RLS-DCD specific parameters were chosen as $\lambda = 1 - 1/(KL)$, $H = 1$, $N_u = 4$, and $M_b = 16$. The initial convergence is not shown, as it is not considered relevant [15]. The normalized misalignment has been chosen as a performance measure:

$$\text{Mis}(n) = 20 \log_{10} \frac{\|\tilde{\mathbf{g}}_t - \tilde{\mathbf{g}}(n)\|_2}{\|\tilde{\mathbf{g}}_t\|_2} \quad [\text{dB}], \quad (16)$$

with $\|\cdot\|_2$ denoting the ℓ_2 norm [1].

In Figure 1, the input signal is a Gaussian noise filtered through an AR(1) system with the pole 0.9. For this scenario, the signs corresponding to the filter coefficients associated with the four acoustic impulse responses were suddenly changed after the adaptive system has reached steady-state. It can be noticed that, for the same values of α_r , the tracking

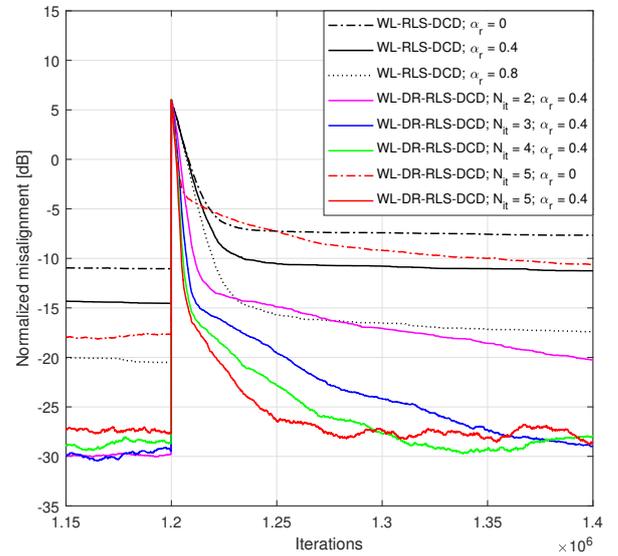


Figure 1. Performance of the WL-DR-RLS and WL-DR-RLS-DCD algorithms for different values of N_{it} and α_r . The input signal is an AR(1) sequence with the pole 0.9, and the length of the four unknown impulse responses is $L = 128$ and $\lambda = 1 - 1/(64L)$. The unknown system changes at time index 1200000.

speed is improved with the increase of N_{it} . When the input signals are not pre-distorted, the performance values are the lowest, even when using $N_{it} = 5$ DR iterations. For the value $\alpha_r = 0$, despite having good initial convergence, the WL-DR-RLS-DCD with $N_{it} = 5$, does not perform well when reaching steady-state. Moreover, when trying to compensate the extra DR iterations using $\alpha_r = 0.8$ (exceeding the recommended limit of 0.5), the tracking capabilities of the WL-RLS-DCD (i.e., the WL-DR-RLS-DCD with $N_{it} = 1$) algorithm are weaker than the ones obtained via the DR variants with $\alpha_r = 0.4$. Finally, taking into account that an additional workload is introduced by employing multiple DR

iterations, respectively that a performance cap is reached in terms of tracking capabilities for higher values of N_{it} , a value of $N_{it} = 3$ DR iterations can be considered sufficient for attaining the desired effect.

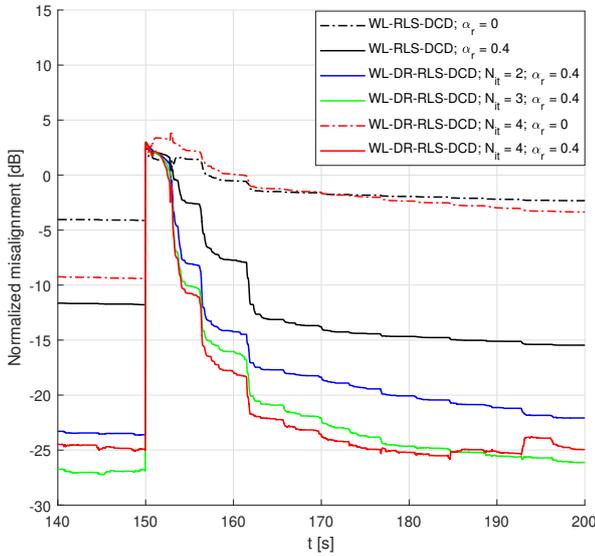


Figure 2. Performance of the WL-DR-RLS and WL-DR-RLS-DCD algorithms for different values of N_{it} and α_r . The input signal is a high quality speech sequence, and the length of the four unknown impulse responses is $L = 256$ and $\lambda = 1 - 1/(96L)$. The unknown system changes at $t = 150$ s due to the interchanging of the microphones positions.

In Figure 2, the input signal is a high quality speech sequence [13]. The four echo paths are suddenly changed at steady-state to:

$$\begin{cases} \mathbf{g}_{t,L_c L_c} = \mathbf{g}_{t,L_c R_c}, & \text{and} & \begin{cases} \mathbf{g}_{t,L_c R_c} = \mathbf{g}_{t,L_c L_c}, \\ \mathbf{g}_{t,R_c L_c} = \mathbf{g}_{t,R_c R_c}, \end{cases} \end{cases} \quad (17)$$

The adjustment is equivalent to interchanging the microphones positions from the WL model for the SAEC context. The displayed results show the performance of the WL-DR-RLS-DCD for different values of N_{it} and $\alpha_r = 0.4$. As a reference, results were also shown for $\alpha_r = 0$. As the value of the DR parameter increases, so do the tracking capabilities of the WL-DR-RLS-DCD, until a certain performance cap is revealed. It can also be noticed that the absence of the pre-processing of the input signal leads to a noticeable performance gap between the cases with $\alpha_r = 0$ and $\alpha_r = 0.4$.

V. CONCLUSIONS AND FUTURE WORK

The algorithm analyzed in this paper is an enhanced version of the combination between the exponentially weighted RLS algorithm and the DCD method [8][10]. The WL-DR-RLS-DCD is designed to approach the SAEC scenarios by exploiting the properties of the input signal and solve an auxiliary system of equations using only additions and bit-shifts. The overall complexity is proportional to the adaptive filter's length multiplied by a small number.

Simulation results were presented using Gaussian noise filtered through an AR(1) system, respectively a speech sequence, as inputs. The results have shown that, with respect to the WL-RLS-DCD, the WL-DR-RLS-DCD has a corresponding compromise between superior tracking capabilities on one side, respectively estimation accuracy at steady-state and a slight increase in arithmetic complexity on the other side. The performance trade-off is controlled from the DR portion of the algorithm through the number of iterations parameter N_{it} .

Considering the performance demonstrated by the WL-DR-RLS-DCD in simulations, the algorithm is attractive for hardware implementations in system identification scenarios, such as the SAEC configurations. A generalization to the multichannel case will be also considered.

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