# A non-Linear MIMO-OFDM Preprocessor for non-Gaussian Channels

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*Abstract*—Multiple-Input and Multiple-Output (MIMO) and Orthogonal Frequency-Division Multiplexing (OFDM) are crucial technologies inside the 5G mobile communication systems and beyond. Design and evaluation of detector techniques over realistic channel conditions are essential in order to transmit signals at high rates and with high reliability in such technologies. In this paper, we present the evaluation of MIMO-OFDM preprocessors over non-Gaussian impulsive noise. Also, we propose a nonlinear sigmoid preprocessor without a threshold parameter as an alternative to the traditional preprocessors. The simulation results show that the Symbol Error Rate (SER) performance depends on not only the preprocessors used and their thresholds but also the impulsiveness level in the noise.

Keywords—Impulsive noise; sigmoid function; non-linear preprocessors; non-linear MIMO.

#### I. INTRODUCTION

Multiple-Input and Multiple-Output (MIMO) systems have received much attention in recent years due to the increasing demand of high transmission rates and high quality of service for wireless communications. MIMO-OFDM techniques are applied to many applications and contexts, such as 4G and 5G networks, 802.11ac, and vehicular environments [1]. With the increasing number of mobile users in the same time-frequency resource, the array efficiency of MIMO allows us to reduce the transmit power and improve energy efficiency [2]. In addition, MIMO is robust in the face of hardware imperfections, such as multiplicative phase drifts and additive distortion noise [2].

One of the greatest challenges of MIMO-OFDM systems is to detect signals with high performance and relatively low computational complexity. However, information signals are degraded by many different undesirable wireless channel effects in which noise assumptions have been demonstrated as one of the greatest challenges faced by MIMO systems. Thus, the design conception of such technology must consider realistic channel and noise models in order to represent well the current applications. On the other hand, various wireless channels have been demonstrated to suffer from impulsive noise which is more accurately characterized as non-Gaussian processes [3]. Those effects are commonly caused by manmade sources, electrical devices, ignition noise in vehicles, and bursty radio frequency emissions typical in urban environments [1].

In severe impulsive noise scenarios, the effects of the MIMO performance may be misread as low Signal-to-Noise Ratio (SNR), when in fact there is a certain impulsiveness level degrading overall system performance. Especially for classical MIMO detectors that rely on second-order statistics noise models, the Gaussian model assumption of wireless

noise behavior leads to meaningful degradation or does not work well. Thus, one way to improve the performance of MIMO-OFDM systems is minimizing undesirable effects of channel and noise by designing detectors considering non-Gaussian noise. Notably, detectors have been proposed over non-Gaussian noise models with considerable improvements compared to traditional ones in those scenarios.

Several papers show non-linear preprocessors with threshold level in order to mitigate the impulsive noise in receivers [4], [5]. Performance evaluation has been done with those nonlinear techniques in OFDM receivers [5], reducing adverse effects of impulsive noise. Recently, an adaptive MIMO receiver was proposed using an impulsive noise level detector. Other adaptive techniques were also presented based on Recursive Least Mean Square (RLS), adaptive Normalized Least Mean Square (NLMS), and Variable Step-size adaptive Normalized Least Mean (VSNLMS), thereby mitigating the impulsive noise effects [6]. The Support Vector Machine (SVM) has been investigated with non-linear complex Multiple Support Vector Machine regression (M-SVM) in this environment. Furthermore, a MIMO detector was proposed based on the maximum complex correntropy criterion using channel estimation to fit a parameter of its technique [7].

Those methods present improvements in detection over impulsive non-Gaussian noise as compared to traditional detectors. However, they usually have too high computational complexity making them often infeasible, due to the adding of an adaptive step or the making of a channel estimation to fit a parameter of the detector. Moreover, many detector solutions require a parameter usually based on a prior noise information. In this context, this work introduces a non-linear preprocessor based on sigmoid functions without free parameter for MIMO detector over non-Gaussian channels.

This paper is organized as follows. In Section II, we describe the MIMO-OFDM system, presenting the channel and noise model. MIMO-OFDM preprocessors are presented in Section III. In Section IV, we propose a MIMO preprocessor based on sigmoid function. In Section V, we evaluate its performance over non-Gaussian scenarios by simulations. In Section VI, we present our final remarks.

### II. MIMO-OFDM SYSTEM

Consider a MIMO system with  $N_R$  antennas at the receiver and  $N_T$  antennas at the transmitter, illustrated in Figure 1. The transmitter consists of MIMO-OFDM modulation over Nsubcarriers [5]. In the Orthogonal Frequency-Division Multiplexing (OFDM) transmitter, the bits are mapped into baseband symbols  $S_k$  using Phase Shift Key (PSK) or Quadrature Amplitude Modulation (QAM) scheme. Then, the complex baseband OFDM signal is computed by means of inverse Discrete Fourier Transform (iDFT) as:

$$s_n(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} S_k e^{j\frac{2\pi kt}{T_s}}$$
(1)

where N is the number of subcarriers, and  $T_s$  is the active symbol interval.



Figure 1. MIMO system architecture.

The  $N_R$  antennas are spaced such that the received signals may be considered independent of each other. The k-th symbol received by the m-th antennas is given by:

$$y_m(t) = \sum_{n=1}^{N_T} s_n(t) h_{mn}(t) p(t) + w_m(t), \qquad (2)$$

where  $s_n(t)$  represents the transmitted symbol from the *n*-th antenna,  $h_{mn}(t)$  represents the channel coefficient between the *n*-th transmitting antenna and *m*-th receiving antenna,  $w_m(t)$  corresponds to the channel noise, and p(t) is a rectangular pulse.

The channel may be described as:

$$h_{mn}(t) = h_{mn,r}(t) + jh_{mn,q}(t),$$
 (3)

where  $h_{mn,r}(t)$  and  $h_{mn,q}(t)$  are Gaussian processes with mean zero and variance equal to 1/2. We also assume that the differences in propagation times of the signals from the transmitters to the receivers are small relative to the symbol duration.

### A. Noise Model

We assume that the noise is uncorrelated, and its distribution can be represented by  $\alpha$ -stable distributions, which are based on crucial properties such as generalized central limit theorem and stability. According to the generalized central limit theorem, if the sum of independent and identically distributed random variables with or without finite variance converges, then the limit distribution must be  $\alpha$ -stable. Another relevant property, known as stability property, states that the sum of two independent random variables with the same characteristic exponent ( $\alpha$  value) is also  $\alpha$ -stable.

There are different parametrizations of  $\alpha$ -stable distribution for different specifications of the characteristic function. We assume the parameters  $\theta_{\alpha} = (\alpha, \beta, \gamma, \delta)$  and the following characteristic function [8]:

$$\varphi(\omega; \boldsymbol{\theta}_{\alpha}) = \exp(-\gamma^{\alpha} |\omega|^{\alpha} [1 - j\Theta(\omega; \alpha, \beta)] + j\delta\omega), \quad (4)$$

with

$$\Theta = \begin{cases} \beta(\tan\frac{\pi\alpha}{2})(\operatorname{sign}\omega), & \alpha \neq 1\\ -\beta\frac{2}{\pi}(\ln|\omega|), & \alpha = 1, \end{cases}$$
(5)

where

 $\alpha$  is the *characteristic exponent* such that  $0 < \alpha < 2$ ,

 $\beta$  is the symmetry parameter such that  $-1 \leq \beta \leq 1$ ,

 $\gamma$  is the dispersion or scale parameter such that  $\gamma > 0$ ,  $\delta$  is the location parameter such that  $-\infty < \delta < \infty$ .

 $\omega$  is the independent variable of the characteristic function.



Figure 2. Probability distribution function of symmetrical  $\alpha$ -stable with  $\beta = \delta = 0$  and  $\gamma = 1$ .

We also assume a Symmetric  $\alpha$ -Stable (S $\alpha$ S) class because it has proved to be very useful in modeling impulsive noise [3]. For such distribution class,  $\beta = 0$  and  $\delta = 0$  [9]. Figure 2 shows the  $\alpha$  value variation versus the random variable representing the impulsiveness level of the distribution, where a low value of  $\alpha$  suggests high impulsiveness and a non-Gaussian behavior, and a high value of  $\alpha$  means that the distribution is close to the Gaussian behavior, where  $\alpha = 2$  is the Gaussian case.

#### **III. NON-LINEAR PREPROCESSORS**

In order to mitigate impulsive noise effects, non-linear preprocessors are applied at the receiver as illustrated in Figure 1. Those memoryless preprocessors are non-linear transformations over the signal amplitude. The most common non-linear preprocessors are blanking and clipping based on thresholds.

## A. Blanking

The blanking non-linear mapping can be described as:

$$y_k = \begin{cases} r_k, & |r_k| \le T \\ 0, & |r_k| > T \end{cases}, \quad k = 0, 1, \dots M - 1 \quad (6)$$

where T is the blanking threshold and M is the signal length.

### B. Clipping

Similar to blanking, the clipping technique maintains the amplitude when the signal is below a threshold. However, when the signal is above the threshold, then the amplitude is saturated by the threshold keeping its phase. This function can be described as:

$$y_{k} = \begin{cases} r_{k}, & |r_{k}| \leq T \\ T e^{j \arg(r_{k})}, & |r_{k}| > T \end{cases}, \quad k = 0, 1, \dots M - 1$$
(7)

where T is the clipping threshold and M is the signal length.

#### IV. PROPOSED PREPROCESSOR

In this proposed technique, we compute the MIMO-OFDM using a non-linear preprocessor function based on a class of functions called sigmoid. These functions have essential characteristics as non-linear functions, such as monotonically increasing and anti-symmetry.

The non-linear functions aim to ensure the existence of higher-order statistics. The most common functions in the sigmoid family are the hyperbolic tangent functions, described as follows.

$$y_k = \tanh(r_k) = \frac{\mathrm{e}^{r_k} - \mathrm{e}^{-r_k}}{\mathrm{e}^{r_k} + \mathrm{e}^{-r_k}}.$$
 (8)

These functions are commonly used to compute covariance by using non-linear data transformation, allowing to access information from the signal, even when it is contaminated by non-Gaussian noise [10].

#### V. RESULTS AND DISCUSSIONS

This section presents numerical simulation results for the performance evaluation of the MIMO-OFDM system using different preprocessors. We examined the Symbol Error Rates (SER) versus the quality of signal metrics in a 2x2 MIMO system. The simulations assess the results using the Monte Carlo method with curves computed with at least 50 errors in the estimation, using 104 subcarriers and 1000 frames. All simulations consider baseband signal using Quadrature Phase Shift Keying (QPSK) modulation and unity energy with the antennas statistically independent of each other. Also, Rayleigh flat fading was assumed as the multipath propagation model in the wireless channel.

The performance metrics are usually computed versus the Signal-to-Noise Ratio (SNR). However, the infinite variance of non-Gaussian  $S\alpha S$  processes prevents to compute the signal-to-noise ratio as a measurement of signal quality. In this work,

we use the Geometric Signal-to-Noise Ratio (GSNR) [11] instead of the SNR. The GSNR is given by

$$\text{GSNR} = \frac{1}{2C_g} \left(\frac{A}{S_0}\right)^2,\tag{9}$$

where the normalization constant  $C_g = e^{C_e} \approx 1.78$  is the exponential of the Euler constant  $(C_e)$ , used to ensure that GSNR corresponds to SNR when the channel is Gaussian ( $\alpha = 2$ );  $S_0$  is the geometric power of a S $\alpha$ S random variable; and A is the root-mean-square value of the signal.

Figure 3 shows the performance of MIMO-OFDM receivers over non-Gaussian S $\alpha$ S noise with impulsiveness level of  $\alpha = 1.3$  and threshold T = 2 for blanking and clipping preprocessors. This scenario represents an environment with high impulsiveness noise where the performance of the MIMO-OFDM system is very low compared to the Gaussian case. However, one can see the preprocessors deliver better performance than the case without the preprocessors, mainly for high GSNR values. Thus, although all preprocessors increase the performance of the MIMO-OFDM system, their performance depends on the impulsiveness level of the noise.



Figure 3. Performance comparison among the preprocessor techniques over  $S\alpha S$  noise with  $\alpha = 1.3$  and T = 2.

#### A. Impulsiveness Analysis

Figure 4 presents the performance of preprocessors over  $S\alpha S$  noise model over different impulsiveness levels, i.e., many different values of  $\alpha$ . More impulsiveness level is close to the Gaussian case ( $\alpha = 2$ ), less is the increase in performance due to preprocessors nonlinearity over impulsive noise. This behavior occurs because in this case, the noise is less impulsive than with low values of  $\alpha$ .

Although all preprocessors increase performance over high impulsive noise, that also depends on the threshold level used for the blanking and clipping methods.

#### B. Threshold Analysis

Figure 5 presents the performance of blanking and clipping preprocessors over different threshold levels. For an intermediate range of threshold values, these techniques have better



Figure 4. Performance of preprocessors over S $\alpha$ S noise with many different values of  $\alpha$ , GSNR = 15 dB and T = 2.

performance than the sigmoid preprocessor. However, this range changes with impulsiveness level and GSNR, making this region of values difficult to be set.



Figure 5. Performance varying threshold level of blanking and clipping preprocessors over S $\alpha$ S noise with  $\alpha = 1.3$  and GSNR = 15 dB.

#### VI. CONCLUSIONS

In this paper, we evaluated traditional preprocessors in the detection of signals in MIMO-OFDM systems over non-Gaussian channels. We analyzed those preprocessors by different aspects, such as the impulsiveness level of the noise, the threshold level of the preprocessors, and the quality of the signal. Also, we presented a non-linear sigmoid function as an alternative to the classical preprocessors, comparing their performance over all aspects mentioned before. The traditional blanking and clipping preprocessors depend on the threshold level, which, in turn, also depends on the impulsiveness level in the environment. On the other hand, the sigmoid function does not have any parameters, being an alternative in the tradeoff of preprocessors in the MIMO-OFDM detection systems.

Future works may use those results to investigate adaptive and machine learning solutions based on non-Gaussian noise parameters such as GSNR and  $\alpha$  values. Thus, the preprocessors using such techniques must present a different performance, thereby being an alternative to the traditional ones.

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