

Adaptive Detection of Transients by the Complex Cepstrum of Higher Order Statistics Based on Givens Rotations

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Abstract— In this paper, the problem of detecting transient signals of unknown waveforms and arrival times embedded in white Gaussian noise is addressed. The use of the cepstrum coefficients of the 4th order correlations of the transient signal for forming a detection statistic is demonstrated. It is considered an adaptive approach for the detection of the signal which is assumed to satisfy a linear constant coefficient difference equation. The adaptive approach is a least squares realization based on Q-R decomposition of the 4th order statistics matrix involved in the computation of the cepstrum coefficients. It is shown that the adaptive approach allows for detection of short length transients which are of unknown arrival times using a single data record even before the whole amount of data becomes available.

Keywords—Detection; Transient signals; Complex Cepstrum; Q-R decomposition; Givens Rotations.

I. INTRODUCTION

Detection of transient signals of unknown waveforms and unknown arrival times is a common problem in several signal processing areas. Some applications include detecting targets by radar and sonar. Another application is in hydraulic and power systems where monitoring sudden changes protects the system. In the detection of seismic waves and in biomedicine, the signal carries important information of the disease and an early detection is essential for the treatment. Transients can be either deterministic or stochastic signals, are short in duration, and are embedded in long periods of background noise. In both cases we have a highly non-stationary problem. Classical signal detection theory has been applied to this problem mainly using the autocorrelation or data domain.

If the deterministic signal waveform is unknown, but the arrival time is known, and the signal is embedded in additive white Gaussian noise, a generalized likelihood ratio test is discussed in [1], where the signal is the impulse response of a proper rational transfer function. However, some a-priori knowledge for the signal is required. Furthermore, the detector is not of Constant False Alarm Rate (CFAR). A similar approach is presented in [2] where the noise is colored Gaussian Autoregressive of order M (AR(M)) process. For the same transient problem, but for unknown arrival times, the Gabor representation of the signals is used in [3].

Higher order statistics have been used for spectrum estimation of stochastic signals [4]-[11]. For detection problems, their use has not been very extensive.

Here we propose a new detection scheme for the detection of transient signals based on the computed cepstrum coefficients of the fourth-order statistics of the signal [12]. Cepstrum coefficients are appropriate for representation of transient signals because they contain all the information of the signal. Since they also peak around the origin, they are suitable for signal detection. The proposed method does not require knowledge of the noise variance or skewness and it is also able to detect the signal in the presence of non-Gaussian white noise as long as it is of zero mean independent, identically distributed (i.i.d.)

Higher order (3rd, 4th, etc.) cumulants are zero for Gaussian i.i.d. process [12]. This means that cumulants have the ability to suppress the noise. This fact is one of the reasons that we present herewith a detection statistic based on cepstrum coefficients and particularly the ones based on the tricepstrum sequence. However, the same detection statistic still works when noise is not Gaussian i.i.d. but non-skewed (e.g. symmetrically distributed).

The paper is organized as follows. In Section II, the adaptive approach is presented for the proposed detector. In Section III, its performance is demonstrated by means of simulation examples. Finally, conclusions are drawn in Section IV.

II. ADAPTIVE Q-R DECOMPOSITION OF THE TRISPECTRUM CEPSTRAL EQUATION

A. Problem Definition

The following detection problem is considered

$$\begin{aligned} H_0: x(n) &= w(n) \\ H_1: x(n) &= m(n) + w(n) \quad n = 0, \dots, N-1 \end{aligned} \quad (1)$$

where $w(n)$ is a stationary, zero mean, white, Gaussian noise of unknown variance σ_w^2 and $m(n)$ is a deterministic transient signal of unknown waveform. The complex cepstrum of the 4th order statistics of a random process $\{x(n)\}$ is known to satisfy the following identity [12],

$$\sum_{k=1}^p A(k) [f_x(-(m+k), -m, -m) - f_x(-(m-k), -(m-k), -(m-k))] + \sum_{k=1}^q B(k) [f_x(-(m+k), -(m+k), -(m+k)) - f_x(-(m-k), -m, -m)] =$$

$$m \cdot f_x(-m, -m, -m) = c_x(-m, -m, -m) \quad p, q \rightarrow \infty \quad (2)$$

where the minimum phase $\{A(k)\}$ and maximum phase $\{B(k)\}$ parameters are given by,

$$g_x(k, 0, 0) \begin{cases} -\frac{1}{k} \cdot A(k), & k = 1, \dots, p \\ \frac{1}{k} \cdot B(-k), & k = -1, \dots, -q \end{cases} \quad (3)$$

and $g_x(k, l, n)$ is the tricepstrum of the 4th order statistics $f_x(k, l, n)$ of the signal. In this paper, the cepstrum coefficients in (2) are being used, for the detection problem given by (1). The following assumptions are being made.

- 1) Under H_0 it is assumed that $\{A(k)\}, \{B(k)\}$ for all k are equal to zero.
- 2) Under H_1 since the process $\{x(n)\}$ is not stationary, it is assumed availability of many data records, i.e. $x^{(i)}(n) = m(n) + w^{(i)}(n)$, $i = 1, \dots, M$ is the given ensemble data set, where $\{w^{(i)}(n)\}$ are different noise realizations of identical statistical properties then,

$$f_x(k, l, m) = E \left\{ \sum_n x(n)x(n+k)x(n+l)x(n+m) \right\} \quad (4)$$

- 3) Since $\{A(k)\}, \{B(k)\}$ are decaying sequences they can be truncated (2) and p, q finite integers can be used [12].

By choosing $p = q$ then (2) can be written,

$$c_x(m, n) = \sum_{k=1}^p f_x(m, k, n) \cdot A(k, n) + \sum_{k=1}^p f'_x(m, k, n) \cdot B(k, n),$$

$$m = -p, \dots, -1, 1, \dots, p \quad (5)$$

where $\{A(k, n)\}, \{B(k, n)\}, f_x(m, k, n), f'_x(m, k, n)$, denote the estimates of the corresponding values of (2) at time instant n based on N samples. In a matrix form,

$$\mathbf{F}(p, n) \cdot \mathbf{T}(p, n) = \mathbf{C}(p, n) \quad (6)$$

where the elements of $\mathbf{T}(p, n)$ are

$$\mathbf{T}(k, n) = \begin{cases} A(k, n), & k = 1, \dots, p \\ B(k - p, n), & k = p + 1, \dots, 2p \end{cases} \quad (6.1)$$

and

$$\mathbf{F}(p, n) = \begin{pmatrix} f_x(p, 1, n) & \dots & f_x(p, p, n) & f'_x(p, 1, n) & \dots & f'_x(p, p, n) \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ f_x(-p, 1, n) & \dots & f_x(-p, p, n) & f'_x(-p, 1, n) & \dots & f'_x(-p, p, n) \end{pmatrix}, \quad (6.2)$$

$$\mathbf{C}(p, n) = (c_x(p, n), \dots, c_x(-p, n))^T, \quad (6.3)$$

"T" denotes the transpose operation. Under H_0 , the matrix $\mathbf{F}(p, n)$ is of full, $2p$ rank. Asymptotically under H_0 the estimates of the tricepstrum coefficients, $\{A(k)\}, \{B(k)\}$, are Gaussian random variables of zero mean and constant covariance matrix. It is also assumed that if $N \rightarrow \infty$ $f_x(m, k, n), f'_x(m, k, n)$ become their true values $f_x(m, k), f'_x(m, k)$. Therefore, the following variable can be used as a detection statistic:

$$L_T = \sum_{k=1}^l (A(k, n)^2 / (\sigma_{a_k}^2) + (B(k, n)^2 / (\sigma_{b_k}^2)), \quad (7)$$

where, $\sigma_{a_k}^2, \sigma_{b_k}^2$ are the variances of $A(k, n), B(k, n)$. This is a central quadratic form ($l \leq p$). For fixed probability of false alarm P_{FA} , the threshold can be computed using the cumulative distribution F_0 of L_T under H_0 ,

$$t_h = F_0^{-1}(1 - P_{FA}) \quad (8)$$

Instead of using 4th order statistics, 3rd order statistics can be used. However, in this case the noise cannot be Gaussian, i.i.d. The Additive White Non-Gaussian Noise (AWNGN) with zero mean assumption is enough to guarantee asymptotically under H_0 rank $2p$, for the matrix $\mathbf{F}(p)$.

Summarizing the algorithm for detecting deterministic transient signals embedded in additive white Gaussian noise, we have the following:

- 1) Estimate the 4th order statistics of $f_x(k, l, n)$ [12].
- 2) Estimate $A(k), B(k)$ using a least squares solution to the overdetermined system of equations (2) when $p = q, m = p, \dots, p - W, W \geq 2p$
- 3) Compute L_T and compare it with a threshold chosen according to (8).

B. The Recursive Approach of the Higher Order Cepstrum Based Detector

Instead of estimating the cepstrum coefficients $\{A(k)\}$ and $\{B(k)\}$ in one step when the whole data record is available we seek for a recursive solution of (6) which will allow for fast updating of the coefficients when new data arrive and the amount of the data is large. Another reason for developing a recursive approach is when the arrival times of the transients are unknown. It is assumed the following partition for $f_x(m, k, n), f'_x(m, k, n)$.

$$f_x(m, k, n) = \sum_{i=n_0+1}^n \lambda^{n-i} x(i) (x^2(i-m), 1) \cdot (x(i-(m+k)), -x^3(i-(m-k)))^T + \lambda^{n-n_0} f_x(m, k, n_0), \quad (9.1)$$

$$f'_x(m, k, n) = \sum_{i=n_0+1}^n \lambda^{n-i} x(i) (x^2(i-m), 1) \cdot (-x(i-(m-k)), x^3(i-(m+k)))^T + \lambda^{n-n_0} f'_x(m, k, n_0), \quad (9.2)$$

where, λ is a weight constant, $0 < \lambda \leq 1$ and $f(m, k, n_0)$, $f'_x(m, k, n_0)$ are computed values from the initialization period which will be explained in the sequel. Also note that a time recursion for $C(p, n)$ is

$$C(p, n) = \lambda \cdot C(p, n-1) + a^T(n), \quad (10)$$

$$a(n) = (px^3(n-p)x(n), \dots, (-p)x^3(n+p)x(n)) \quad (11)$$

To realize this solution matrix $F(p, n)$ is decomposed into two sub matrices, V and U and their Q-R decomposition is updated at each time instant that new information is present,

$$F(p, n) = V^T(p, n) \cdot U(p, n) \quad (12)$$

Both V^T and U start with,

$$Q(1) \cdot \hat{A}(1) = \begin{pmatrix} F(1) \\ 0 \end{pmatrix} \quad (13)$$

where $\hat{A}$ matrix will represent either V^T or U , $F(1) = x(0)$ and

$$Q(1) = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \quad \hat{A}(1) = \begin{pmatrix} 0 \\ x(0) \end{pmatrix} \quad (14)$$

Given the above initial values, new data at each iteration i , for both V^T and U , namely $u_{in1}(i)$ and $u_{in2}(i)$ are obtained. The general notation $u_{in}(i)$ is used here. Then, Q-R decomposition is applied on $\hat{A}(1)$

$$\hat{A}(2) = \begin{pmatrix} \hat{A}(1) | \mathbf{0}_{(2 \times 1)} \\ \mathbf{u}_{in}(i) \end{pmatrix} \quad (15)$$

In the subsequent steps of the initialization period, we input new data until we finally obtain Q-R decompositions for V^T and U .

It remains to describe the orthogonal transformations that are used to compute the Givens rotation parameters and then the Givens transformation matrix $G(i)$. In particular for each step, the partially triangularized matrix is assumed,

$$F(i) = \begin{pmatrix} D_{ixj}^{1/2}(i) \cdot \hat{F}_{jxj}(i) \\ \mathbf{0}_{1 \times j} \end{pmatrix}, \quad (16)$$

$$j = \begin{cases} i, & i \leq 2p \\ 2p, & i > 2p \end{cases}, \quad (17)$$

where $D^{1/2}(i)$ is diagonal matrix and $\hat{F}(i)$ is a unit upper triangular matrix. The requirement is to find rotation parameters so that we can annihilate the new input vector $u_{in}(i)$. Thus, a sequence of Givens rotations [13] is used, described by

$$G_m(i, k, l) = \begin{cases} c_m(i), k = l = m \\ s_m(i), k = i, l = i + 1 \\ -s_m(i), k = i + 1, l = m \\ c_m(i), k = l = i + 1 \\ 1, k = l, k \neq m, 1 \leq k < i \end{cases} \quad (18)$$

and the Givens transformation matrix itself is

$$G(i) = \prod_{m=1}^{l_n} G_m(i) \quad (19)$$

$$l_n = \begin{cases} k_n + 1, k_n < 2p \\ 2p, k_n = 2p \end{cases} \quad (20)$$

where k_n is the dimension of the input vector.

After the end of the initialization period, initial Q-R decompositions for $V^T(p, n)$ and $U(p, n)$ are available. It is denoted again in general each one of them by $\hat{A}(p, n)$. For $U(p, n)$ the new data sets for the next iteration are its last two lines, i.e., per iteration its Q-R decomposition is updated twice. For $V^T(p, n)$, per iteration its Q-R decomposition is updated $4p + 2$ times, using equal number of new data sets, $u_{in}(n, i)$, which are described as follows,

$$u_{in}(n, i) = \begin{cases} u_{in1}(n, i) = x(n-p+i) \cdot u_{in2}(n, i), \\ u_{in2}(n, i) = \begin{pmatrix} \frac{i-1}{0, \dots, 0}, x(n+1), \dots, x(n-(2p-i)) \end{pmatrix}, \\ i = 1, \dots, 2p+1 \end{cases} \quad (21)$$

where the step $i = p + 1$ (which corresponds to $m = 0$, row of $F(p, n)$ in (6)) is ignored. The decomposition of the first step $i = 1$ is stored for each time instant n and is used as initial for the iterations, $i = 1, \dots, 2p + 1$ of the next time instant $n + 1$ for $V^T(p, n)$. I.e., summarizing the update process for both $V^T(p, n)$, $U(p, n)$ for every time instant n we have the following,

$$Q_{jxj}(p, n) \cdot \hat{A}_{jx2p}(p, n) = \begin{pmatrix} F_{2px2p}(p, n) \\ \mathbf{0}_{(j-2p) \times 2p} \end{pmatrix}, \quad (22.1)$$

$$F_{2px2p}(p, n) = D_{2px2p}^{1/2}(p, n) \cdot \hat{F}_{2px2p}(p, n), \quad (22.2)$$

where $j = 2(n - n_0) + 8p + 2$ for $U(p, n)$ and $j = 2(n - n_0) + 4p + 2$ for $V^T(p, n)$ and it was assumed that $n_0 = 2p$. At the next time instant $n + 1$, new information is available (note that at time instant n , samples of a growing rectangular window are available up to time instant $n + 2p$, i.e., when it is said new available information it is meant that

this window is moved one position forward) and $\hat{A}(p, n)$ and $F(p, n)$ matrices are updated,

$$\hat{A}(p, n+1, i) = \begin{pmatrix} \hat{A}(p, n, i) \\ \mathbf{u}_{in}(n, i) \end{pmatrix}, \quad (23.1)$$

$$\mathbf{F}^*(n+1, i) = \begin{pmatrix} \mathbf{F}_{2p \times 2p}(p, n, i) \\ \mathbf{0}_{(j-2p) \times 2p} \\ \mathbf{u}_{in}(n, i) \end{pmatrix}, \quad (23.2)$$

The index i is used here to indicate the iterations for every time instant n . Note that, $\hat{A}(p, n) = \hat{A}(p, n, 1)$ and $F(p, n) = F(p, n, 1)$ also for $U(p, n), i = 1$, whereas for $V^T(p, n), i = 1, \dots, 2p+1$.

The rotation parameters are computed and the sequence of the square root Givens rotations are applied at time instant n on $F(p, n)$ to annihilate all $2p$ elements of the last row. Then,

$$\mathbf{G}(n+1, i) \cdot \mathbf{F}_{(j+1) \times 2p}^*(n+1, i) = \begin{pmatrix} \mathbf{F}_{2p \times 2p}(p, n, i) \\ \mathbf{0}_{(j+1-2p) \times (2p)} \end{pmatrix}, \quad (24.1)$$

$$\mathbf{Q}_{(j+1) \times (j+1)}(p, n+1, i) = \mathbf{G}(n+1, i) \cdot \begin{pmatrix} \mathbf{Q}_{j \times j}(p, n, i) & \mathbf{0}_{j \times 1} \\ \mathbf{0}_{1 \times j} & 1 \end{pmatrix}, \quad (24.2)$$

which gives the following Q-R decomposition at time instant $n+1$,

$$\hat{A}_{j \times 2p}(p, n+1) = \mathbf{Q}_{j \times j}^T(p, n+1) \cdot \begin{pmatrix} \mathbf{F}_{2p \times 2p}(p, n+1) \\ \mathbf{0}_{(j-2p) \times (2p)} \end{pmatrix}, \quad (25)$$

where now $j = 2(n+1 - n_0) + 8p + 2$ for both $V^T(p, n)$, $U(p, n)$. If indexes u, v are used to denote the corresponding Q, F matrices for $V^T(p, n)$, $U(p, n)$ then the least squares solution (6) can be realized as,

$$\mathbf{F}^{(u)} \cdot \mathbf{T} = \left(\mathbf{Q}^{(v)} (\mathbf{Q}^{(u)})^T \right)^{-1} \cdot (\mathbf{F}^{(v)})^{-T} \cdot \mathbf{C}, \quad (26)$$

The two square matrices on the right hand side of (26) are invertible because of the way that they were constructed using the Givens rotations and the above linear system of equations can be solved using back substitution.

III. SIMULATION EXAMPLES

Test Case 1 (Minimum phase transient, unknown arrival time). The z-transforms of the infinite duration signal is given by, example 1,

$$F(z) = \frac{1}{z^2 - (1.35)z + 0.75} \quad (27)$$

and we assume that it is of unknown arrival time at 200 samples. The signal plus noise records for AWGN of variance, $\sigma_w^2 = 3.162 \times 10^{-1}$, 0.1, for 15 sample signal are shown in Figure 1a, 1b. In Figure 2a, 2b, we plot the detection statistics for the tricepstrum method and for the Infinite Impulse Response (IIR) adaptive algorithm versus

time (the same way as for the definition of L_T , we use the sum of the squares of the estimated coefficients of the recursive IIR model as a detection statistic for the comparison algorithm). For the tricepstrum $p = q = 2$ and $l = 2$ and for the IIR model order, 2 were the choices for this experiment. For all the experiments below, including this one we keep l equal to the order of the model. Both methods under $H_0(0, \dots, 199)$ samples remain in the zero state and when the transient appears they jump and slowly converge again to the zero state. The weighting constant λ was for both methods 0.99. In Figure 2c, we show operation of the algorithms for noise variance $\sigma_w^2 = 1$ and $\lambda = 0.98$.

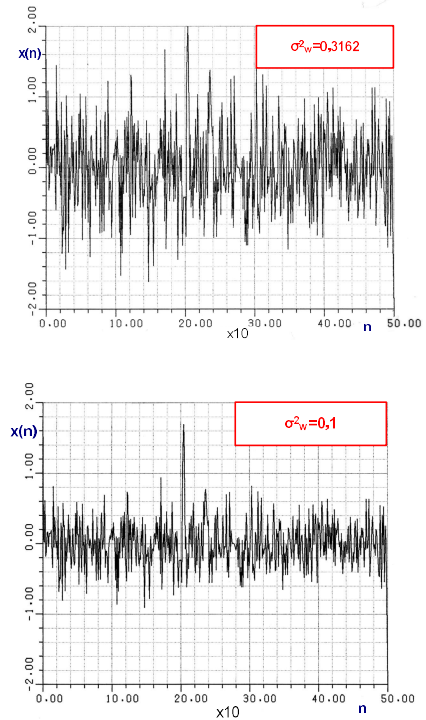
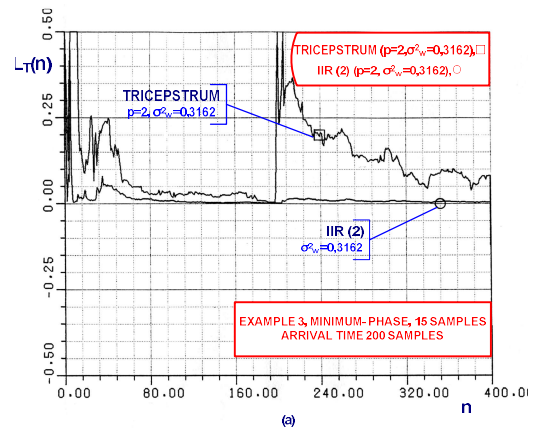


Figure 1. Signal plus noise records for the minimum-phase signal, example 1, 15 samples, arrival time, 200 samples: (a) $\sigma_w^2 = 3.162 \times 10^{-1}$, (b) $\sigma_w^2 = 0.1$.



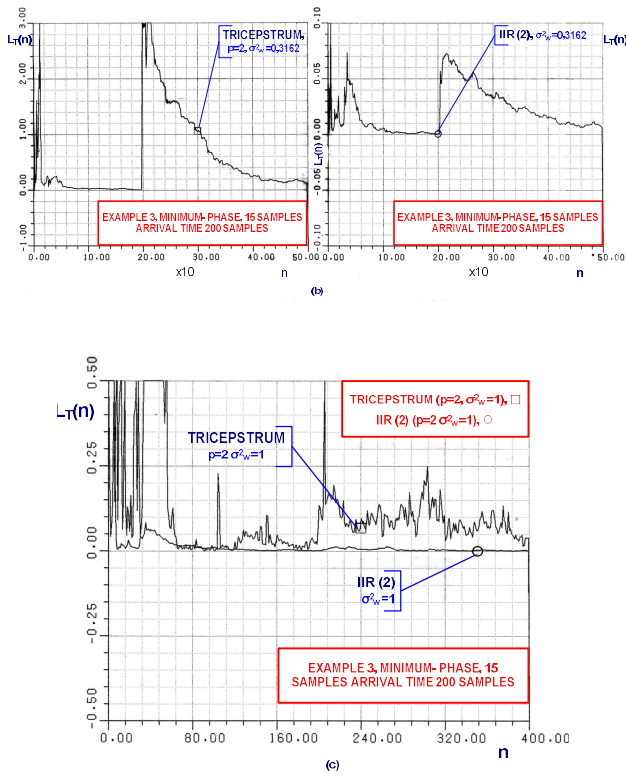


Figure 2. Additive White Gaussian Noise, minimum-phase signal, example 1, 15 samples, arrival time, 200 samples, L_T versus time $\lambda = 0.99$, (a) Tricepstrum, $p = 2$ and IIR(2), $\sigma_w^2 = 3.162 \times 10^{-1}$, (b) $\sigma_w^2 = 0.1$, (c) $\sigma_w^2 = 1$, $\lambda = 0.98$.

Test Case 2 (Mixed phase transients, unknown arrival times),
example 2,

$$F(z) = \frac{(1-0.5z)(z^2-(1.09754)z+(0.3012))}{z^2-(0.6098)z+0.3012} \quad (28.1)$$

example 3,

$$F(z) = \frac{(1-0.5z)(1-0.2z)(z^4-(2.1481)z^3+(1.8221)z^2-(0.7202)z+0.1108)}{z^4-(1.7092)z^3+(1.3395)z^2-(0.5555)z+0.1108} \quad (28.2)$$

For example 2, we use 15 sample signal and arrival time at 150 samples. The tricepstrum and IIR detection statistics versus time are shown in Figures 3a, 3b. The noise variance is $\sigma_w^2 = 0.1, 3.162 \times 10^{-2}$ and we choose $p = 2$ for the tricepstrum and order 6 for the IIR, $\lambda = 0.99$ for both. It is clear that because of the inability of the IIR model to catch the non-minimum phase character of the signal its performance becomes much worse. This becomes more

apparent if we compare Figures 3a and 3b, where the SNR values are 13.8 db and 12 db correspondingly.

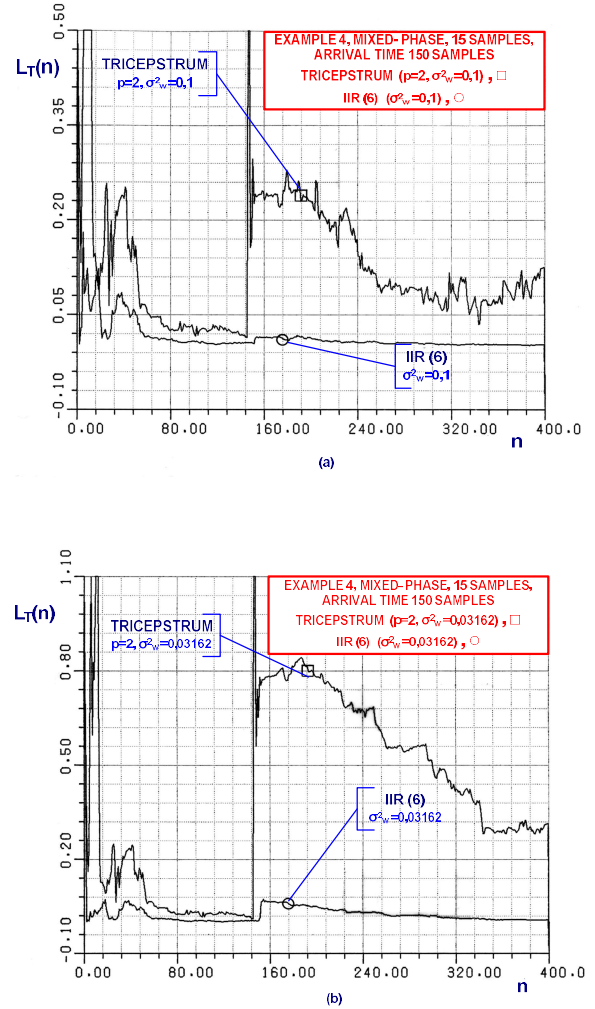


Figure 3. Additive White Gaussian Noise, mixed-phase signal, example 2, 15 samples, arrival time, 150 samples:(a) L_T versus time, $\lambda = 0.9$, Tricepstrum, $p = 2$ and IIR(6), $\sigma_w^2 = 0.1$, (b) L_T versus time, $\lambda = 0.99$, $\sigma_w^2 = 3.162 \times 10^{-2}$.

In example 3, we make the non-minimum phase character of the signal even stronger and we plot also the detection statistics for both methods in Figures 4a-4c, for 25 sample signal and corresponding noise variances, $\sigma_w^2 = 3.162 \times 10^{-1}, 0.1, 3.162 \times 10^{-2}, 10^{-2}$. The orders of the tricepstrum and IIR methods were 2 and 10 with $\lambda = 0.99$. The performance of the first improves and for the second becomes worse with respect to examples 2 and 1.

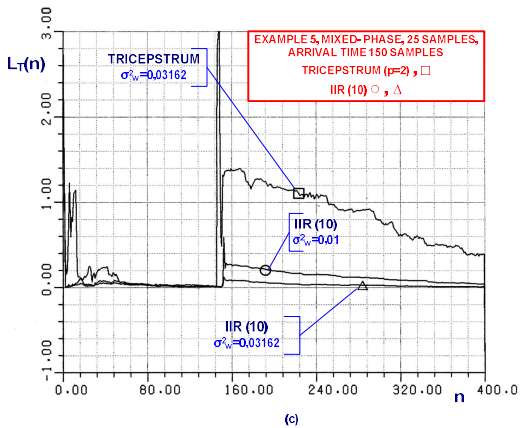
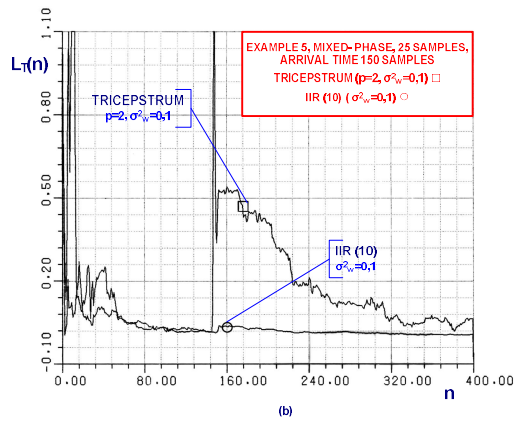
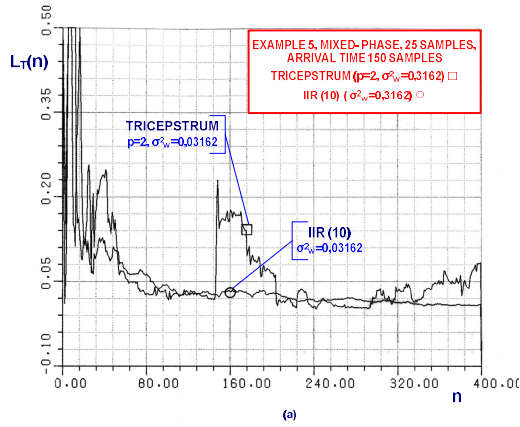


Figure 4. Additive White Gaussian Noise, mixed-phase signal, example 3, 25 samples, arrival time 150 samples: (a) L_T versus time, $\lambda = 0.9$, Tricepstrum, $p = 2$ and IIR(10), $\sigma_w^2 = 3.162 \times 10^{-1}$, (b) L_T versus time, $\lambda = 0.99$, $\sigma_w^2 = 0.1$, (c) L_T versus time, $\lambda = 0.99$, $\sigma_w^2 = 3.162 \times 10^{-2}$, 10^{-2} for IIR(10) and $\sigma_w^2 = 3.162 \times 10^{-2}$ for tricepstrum.

In Figure 5, we plot for $p = 3, \lambda = 0.99$ and $\sigma_w^2 = 3.162 \times 10^{-2}, 0.1$ the tricepstrum detection statistic for examples 2, 3 to demonstrate performance with increased order. Note that varying the order of the IIR method does not change the situation shown in Figures 2-4.

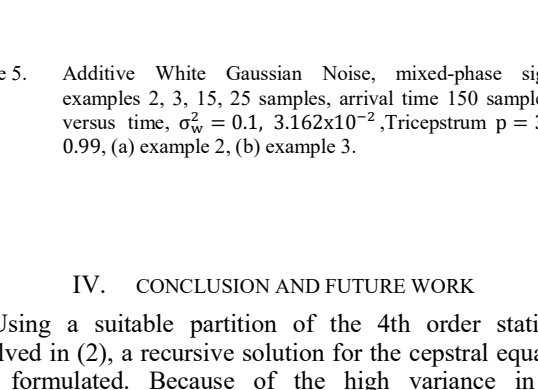
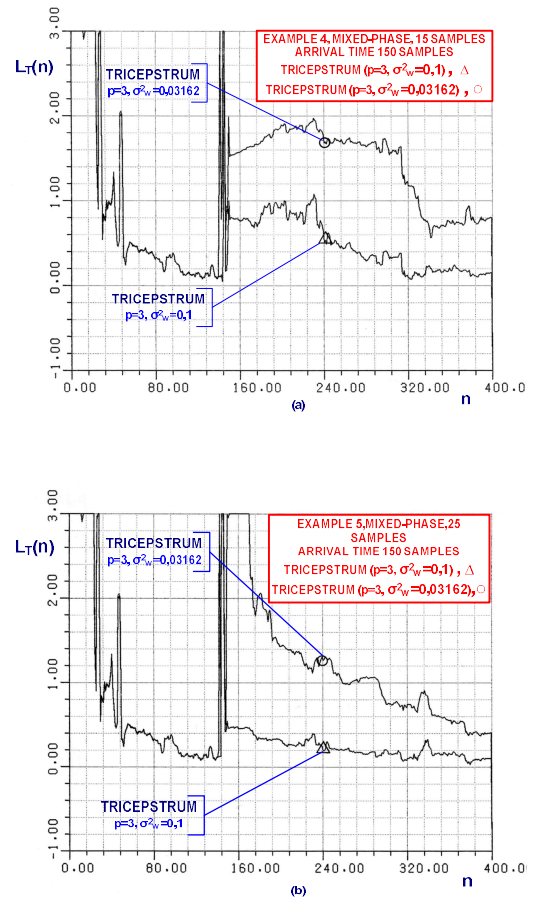


Figure 5. Additive White Gaussian Noise, mixed-phase signals, examples 2, 3, 15, 25 samples, arrival time 150 samples, L_T versus time, $\sigma_w^2 = 0.1, 3.162 \times 10^{-2}$, Tricepstrum $p = 3, \lambda = 0.99$, (a) example 2, (b) example 3.

IV. CONCLUSION AND FUTURE WORK

Using a suitable partition of the 4th order statistics involved in (2), a recursive solution for the cepstral equation was formulated. Because of the high variance in the estimation of the 4th order statistics, the recursive approach was based on orthogonal Q-R decompositions of the partitioned data matrices which consist the cepstral equation. By means of simulation examples, it was demonstrated that the proposed algorithm is capable to detect transients of unknown arrival times. Comparing this technique with a fast adaptive algorithm based on an IIR model for the signal, significant improvement in terms of signal detection capability was demonstrated. Future work could investigate the performance of the proposed algorithm in the presence of i.i.d. noise with probability density function which follows non Gaussian distribution either symmetric (for example non-skewed) or asymmetric.

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