Low Complexity Estimation of Frequency Offset for OFDM Systems

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***Abstract*—A frequency offset caused by the Doppler shift or the mismatch of the oscillators destroys orthogonality between the subcarriers in the orthogonal frequency division multiplexing (OFDM) signals. This paper proposes a frequency offset estimation scheme with low complexity for OFDM systems. Specifically, we use the coherence phase bandwidth to overcome the effect of timing offset and employ a threshold to estimate the frequency offset with low complexity. In addition, we also propose a timing offset estimation scheme as a next stage of the frequency offset estimation. Numerical results demonstrate that the proposed scheme can estimate frequency offset with lower computational complexity and does not require additional memory while maintaining the same level of estimation performance in terms of error variance.**

Keywords-OFDM; frequency offset estimation; threshold; coherence phase bandwidth; low complexity

I. INTRODUCTION

In orthogonal frequency division multiplexing (OFDM) modulation, the data is transmitted on the multiple orthogonal subcarriers. Due to its high spectral efficiency and immunity to multipath fading, OFDM has attracted much attention in the field of wireless communications [1], [2]. For example, OFDM has been adopted as the transmission method of many standards in wireless communications, including European digital video broadcasting (DVB) [3], IEEE 802.11a [4], IEEE 802.16 [5], and Long Term Evolution (LTE) [6]. However, the OFDM is very sensitive to the frequency offset caused by Doppler shift or the mismatch of the oscillators. To alleviate these problems, various techniques have been proposed [7]-[9]. Recently, estimation schemes for integer frequency offset have been proposed in [10] and [11]. However, the schemes in [10] and [11] are either sensitive to timing offset or complex to implement. †

In this paper, a novel frequency offset estimation scheme based on the coherence phase bandwidth (CPB) and a threshold is proposed for OFDM systems. The proposed scheme is robust to a symbol timing offset and has lower computational complexity when compared with Nogami's [10] and Bang's [11] schemes. We also propose a timing offset estimation scheme as the next stage of the frequency offset estimation.

The remainder of this paper is organized as follows. Section II introduces the related works on the frequency offset estimation. In Section III, an OFDM signal model is influenced by the frequency and timing offsets is described. The effect of timing offset on conventional schemes is shown in Section IV. In Section V, a low complexity frequency offset estimation scheme is proposed, and then, a timing offset estimator is proposed as the next step of the frequency offset estimation. Section VI presents numerical results, and finally, Section VII concludes this paper.

II. RELATED WORKS

To estimate the frequency offset, several schemes have been proposed in [7]-[11]. The frequency offset estimation schemes can be classified into two categories: fractional frequency offset estimation schemes [7]-[9] and integer frequency offset estimation schemes [10], [11]. In [7], the fractional frequency offset was estimated by using the cyclic prefix of an OFDM symbol. The scheme in [8] provides an improved estimation performance over the scheme in [7] by using a training symbol with identical halves, yet, the estimation range was as large as the sub-carrier spacing only. In [9], a training symbol with several repeated parts is employed to provide the extended estimation range. On the other hand, for integer frequency offset estimation, an estimation scheme was proposed in [10] using the crosscorrelation between the received and locally generated training symbols. However, the scheme in [10] is very sensitive to the timing offset, thus, in [11], an estimation scheme robust to the timing offset was proposed considering CPB in its estimation process. However, the scheme in [11] has the problem that the complexity rapidly increases, as the frequency offset estimation range increases.

III. SIGNAL MODEL

An OFDM symbol is generated by inverse fast Fourier transform (IFFT) as

$$s_n = \frac{1}{\sqrt{N}} \sum_{l=0}^{N-1} S_l e^{j2\pi nl/N}, \tag{1}$$

where S_l is a phase shift keying (PSK) or quadrature amplitude modulation (QAM) symbol in lth subcarrier and N is the size of IFFT.

In the presence of frequency and timing offsets, the received OFDM signal r_n can be expressed as

$$r_n = s_{n-n_0} e^{j2\pi f_0 (n-n_0)/N} + w_n, \tag{2}$$

where f_0 and n_0 represent the frequency offset normalized to the subcarrier spacing $1/N$ and the timing offset,

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respectively, and w_n is the zero-mean complex additive white Gaussian noise (AWGN).

A frequency offset f_0 can be divided into integer and fractional parts as

$$f_0 = \varepsilon + f_f, \quad (3)$$

where ε is the integer part of f_0 and $f_f \in [-0.5, 0.5)$ is the fractional part of f_0.

Since our focus in this paper is to estimate ε, we assume that the fractional part f_f is known and perfectly compensated. The received symbol is demodulated using FFT operation. The kth FFT output R_k can be expressed for $k = 0, 1, 2, \cdots, N-1$ as

$$R_k = S_{k-\varepsilon} e^{-j2\pi n_0 (k-\varepsilon)/N} + W_k, \quad (4)$$

where W_k is the FFT output of w_n.

IV. THE EFFECT OF TIMING OFFSET ON FREQUENCY OFFSET ESTIMATION

A. Nogami's Scheme

To estimates the integer frequency offset, Nogami's scheme calculates the correlation between the known training symbol and the cyclically shifted version of the received training symbol, and then, obtain the integer frequency offset estimate $\hat{\varepsilon}_{\text{Nogami}}$ as

$$\hat{\varepsilon}_{\text{Nogami}} = \arg\max_{d} \left\{ \left| \sum_{k=0}^{N-1} Z_k^* R_{(k+d)_N} \right| \right\}, \quad (5)$$

where Z_k is the transmitted training symbol, R_k is the received training symbol, d is the amount of cyclic shift, and $(\cdot)_N$ is the modulo-N operator. The scheme was proposed assuming perfect timing synchronization, thus, its estimation performance degrades in the presence of timing offset.

Fig. 1 shows the correlation values of Nogami's scheme when $d = \varepsilon$ and $N = 1024$ in the absence and presence of timing offset (i.e., $\tau = 0$ and $\tau = 0.5, 1,$ and 2). In the absence of timing offset, the correlation value increases as the integration range increases, on the other hand, in the presence of timing offset, the correlation value oscillates.

Fig. 2 shows Nogami's correlation function as a function of timing offset. From the figure, we can see that the correlation value becomes smaller as the absolute value of timing offset increases. Especially, when the timing offset is an integer, the correlation becomes extremely small. Therefore, Nogami's scheme could be very sensitive to the timing offset since the estimate is determined by the maximum value of the correlation function.

B. Bang's scheme

The correlation functions of Bang's scheme are calculated individually within each CPB block, and then, summated. Here, CPB can be expressed as

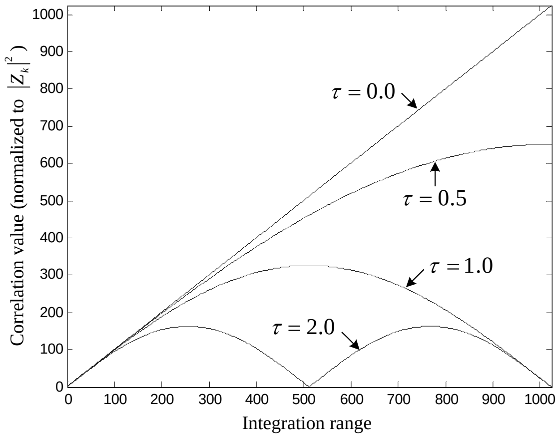

Figure 1. Correlation values of Nogami's scheme in the absence and presence of timing offset when $d = \varepsilon$ and $N = 1024$.

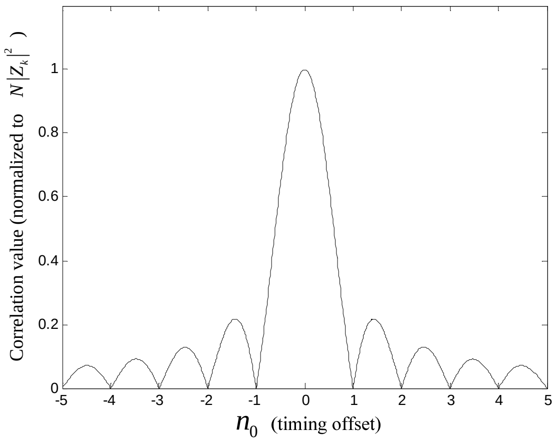

Figure 2. Correlation values of Nogami's scheme as a function of timing offset n_0 when $d = \varepsilon$.

$$\text{CPB} = \frac{1}{2n_0^t} N, \quad (6)$$

where n_0^t is a maximum tolerable timing offset value.

Bang's scheme estimates the integer frequency offset as

$$\hat{\varepsilon}_{\text{Bang}} = \arg\max_{d} \left\{ \sum_{m=0}^{K-1} \left| \sum_{k=0}^{\text{CPB}-1} Z_{k+m\text{CPB}}^* R_{(k+m\text{CPB}+\text{d})_N} \right| \right\}, \quad (7)$$

where $K = N/n_0^t$. As shown in Fig. 3, the correlation value of Bang's scheme increases monotonically as the value of integration range increases. Fig. 4 shows Bang's correlation function as a function of timing offset. From the figure, we can observe that the correlation function has no zero-crossing point unlike Nogami's correlation function. Thus, Bang's scheme would have the better estimation performance than Nogami's scheme in the presence of timing offset. However, Bang's scheme has high computational complexity since the correlation values are calculated for all possible values of d.

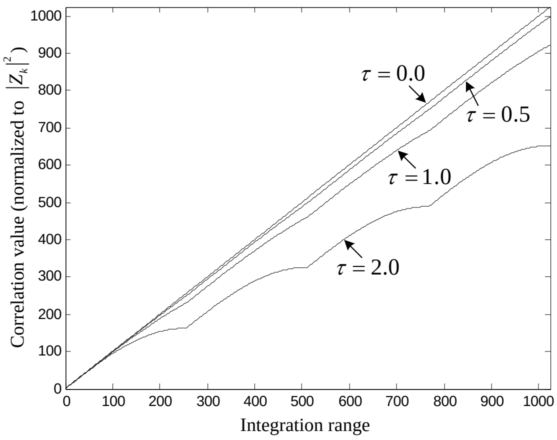

Figure 3. Correlation values of Bang's scheme in the absence and presence of timing offset when $d=\varepsilon$, $N=1024$, and $\text{CPB}=256$.

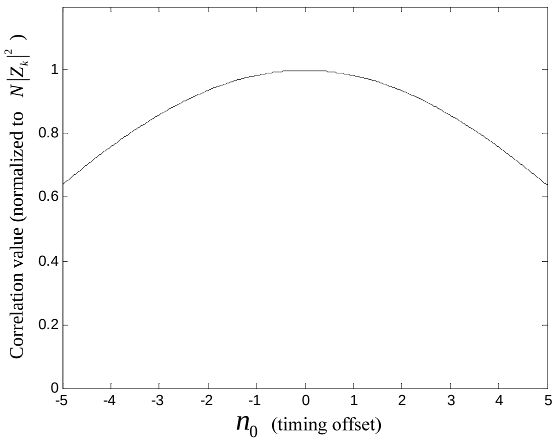

Figure 4. Correlation values of Bang's scheme as a function of timing offset n_0 when $d=\varepsilon$ and $\text{CPB}=N/10$.

V. PROPOSED SCHEME

A. Frequency offset estimation

When we calculate one CPB block in (7), correlation value C can be expressed as

$$C = \left| \sum_{k=0}^{\text{CPB}-1} Z_k^* R_{(k+d)_N} \right|. \quad (8)$$

Assuming $d=\varepsilon$ and ignoring AWGN components in (8), the correlation value C can be expressed as

$$C = |Z_k|^2 \left| \sum_{k=0}^{\text{CPB}-1} e^{-j2\pi n_0 k/N} \right|. \quad (9)$$

The value C in (9) is minimum when $n_0 = n_0^t$. Therefore, the minimum correlation value $C_{\min}$ can be expressed as

$$C_{\min} = |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right|. \quad (10)$$

We also express the minimum value of full correlation in a similar manner as

$$C_{\min}^{full} = 2n_0^t |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right|. \quad (11)$$

In this paper, we use a threshold η as a half of $C_{\min}^{full}$

$$\eta = n_0^t |Z_k|^2 \left| 1 - j\cot\left(\frac{\pi n_0^t}{N}\right) \right|. \quad (12)$$

The proposed scheme calculates the η and CPB as in (6) and (12). Then, the correlation value is acquired using the CPB as in (7). If the correlation value exceeds η, the d is decided to be the correct estimate of ε. Otherwise, the received signal is cyclically shifted by $d=d+1$ and the above procedure is repeated. The operation of the proposed scheme is described in Fig. 5. When the frequency offset range is sufficiently large, the proposed scheme will generally have about a half computational complexity compared with Bang's scheme. It also should be noted that the proposed scheme does not require additional memory for correlation values unlike Nogami's and Bang's schemes.

B. Timing offset estimation

If we assume that frequency offset is completely recovered, the received symbol can be expressed in frequency domain as

$$R_k = H_k Z_k e^{j2\pi n_0 k/N}, \quad (13)$$

where H_k is a frequency response of the channel. The argument of correlation between the transmitted and received training symbol can be expressed as

$$\Phi_k = \angle\left(Z_k^* R_k\right) = 2\pi n_0 \frac{k}{N} + \angle\left(H_k\right), \quad (14)$$

where $\angle$ denotes the argument of a complex number.

Let us assume that the channel is time invariant during one OFDM symbol duration, then the difference between Φ_k and Φ_{k-1} can be re-written as

$$\Delta_k = \Phi_k - \Phi_{k-1} = 2\pi n_0 \frac{1}{N}. \quad (15)$$

From (15), we can see that the effect of the channel (i.e., $\angle H_k$) is removed completely. Thus, the timing offset can be estimated as

$$\hat{n}_0 = \frac{N}{2\pi}\text{avg}\left(\Delta_k\right), \quad \text{for } k = 1, 2, \cdots, N-1, \quad (16)$$

where $|\Delta_k| \le 2\pi n_0^t/N$ and $\text{avg}(\cdot)$ denotes the average. The average value of Δ_k $(|\Delta_k| \le 2\pi n_0^t/N)$ is used to improve the estimation performance, since the allowed range of $\angle$ operator is $[-\pi, \pi)$. Fig. 6 shows the block diagram of the proposed timing offset estimation scheme.

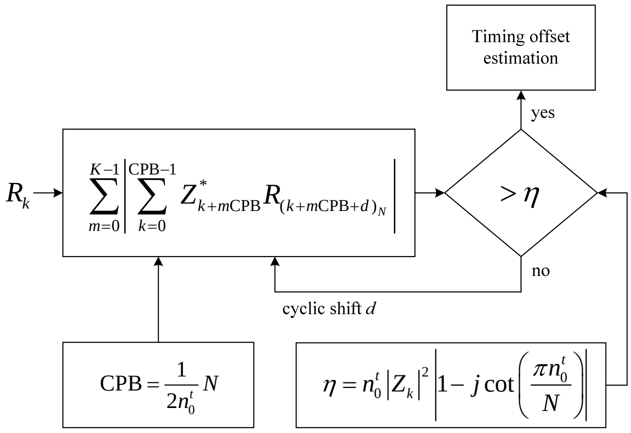

Figure 5. Block diagram of the proposed frequency offset estimation scheme.

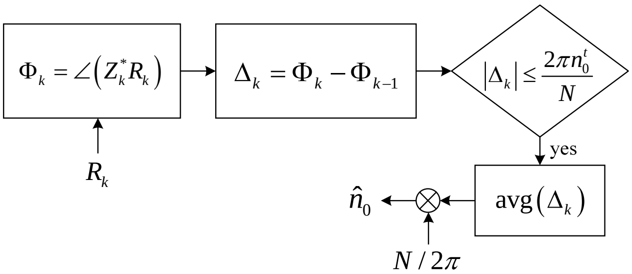

Figure 6. Block diagram of the proposed timing offset estimation scheme.

VI. NUMERICAL RESULTS

A. Frequency offset estimation

In this section, we first compare the computational complexity of the proposed and conventional schemes as shown in Table 1. We can see from Table 1, for $L \gg 1$, where L is the frequency offset range, the proposed scheme has approximately a half computational complexity compared with the Nogami's and Bang's schemes (computational complexity of proposed scheme is an averaged value).

In this paper, we use two channel models for performance comparisons: AWGN and multipath channel models. The signal-to-noise ratio (SNR) of AWGN channel model is 5 dB. The multipath channel model has four paths with 5, 10, and 15 sample delays from the first path, respectively, each path has 4, 8, and 12 dB attenuation in the amplitude and random phase jitter, and the SNR is 10 dB.

TABLE I. COMPUTATIONAL COMPLEXITY OF THE SCHEMES

Scheme	Number of complex multiplication	Number of comparison operation	Size of memory for correlation value
Nogami's scheme	LN	$L-1$	L
Bang's scheme	LN	$L-1$	L
Proposed scheme	$N(L+1)/2$	$(L+1)/2$	-

Tables 2 and 3 compare the accuracy of the proposed and conventional schemes over AWGN and multipath channels, respectively. The frequency offset used in this simulation is an integer value in [0, 500], $N=1024$, and $\mathrm{CPB}=N/32$. As we can see from tables, the proposed scheme exhibits significantly improved frequency offset estimation performance over Nogami's scheme in the presence of the timing offset. Specifically, Nogami's scheme can correctly estimate the integer frequency offset only when the value of the timing offset equals to zero, otherwise, the accuracy of the scheme severely decreases. On the other hand, Bang's scheme shows the same estimation performance as that of the proposed scheme, however, the computational complexity of Bang's scheme is almost twice the proposed scheme as shown in Table 1.

B. Timing offset estimation

Fig. 7 shows the error variance of the proposed scheme as a function of SNR over AWGN channel. The simulation performed 10,000 iterations at each SNR level. We can see that the error variance decreases as the value of SNR increases.

Fig. 8 shows the error variance as a function of SNR over multipath channel model. Since we assume that the channel is time-invariant during one symbol time, the error variance of proposed scheme on multipath channel is similar to that in Fig. 7.

VII. CONCLUSION AND FUTURE WORKS

In this paper, we have proposed a low complexity frequency offset estimation scheme using the CPB and employing a threshold. Moreover, we have proposed a timing offset estimation scheme as the next stage of the frequency offset estimation. From numerical results, we have shown that the proposed scheme exhibits much lower complexity than the conventional schemes while maintaining the same level of estimation performance.

The proposed scheme is designed to utilize a training symbol without assumption on its structure. However, we can further improve the performance of the scheme by considering the training symbol structure used in industrial applications, which is our future research topic. Moreover, since the scheme is derived in the absence of noise, we will evaluate the estimation performance of the scheme analytically in AWGN channel and numerically in practical channel environments.

TABLE II. ACCURACY COMPARISON OF THE FREQUENCY OFFSET ESTIMATION SCHEMES IN AWGN CHANNEL ENVIRONMENTS

Timing offset (samples)	Nogami's scheme	Bang's scheme	Proposed scheme
0	100	100	100
1	0	100	100
2	0	100	100
5	0	100	100

TABLE III. ACCURACY COMPARISON OF THE FREQUENCY OFFSET ESTIMATION SCHEMES IN MULTIPATH CHANNEL ENVIRONMENTS

Timing offset (samples)	Nogami's scheme	Bang's scheme	Proposed scheme
0	100	100	100
1	0	100	100
2	0	100	100
5	0	100	100

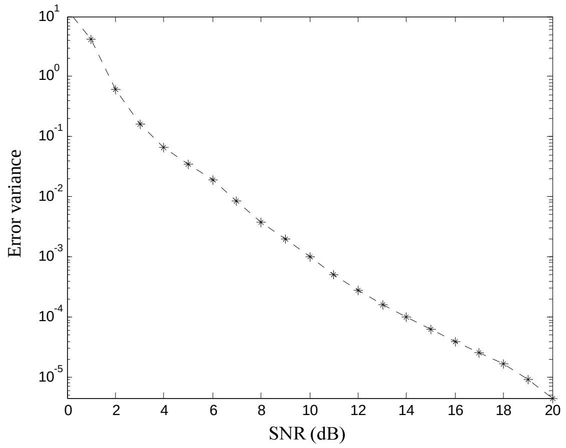

Figure 7. Error variance of the proposed timing offset estimation scheme in AWGN channel environments.

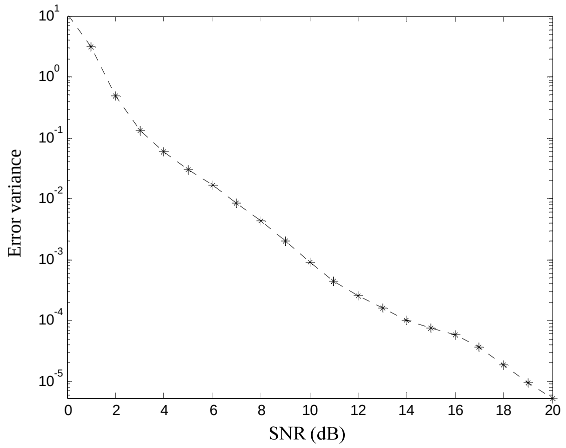

Figure 8. Error variance of the proposed timing offset estimation scheme in multipath channel environments.

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