

# Multi-modal Optimization using a Simple Artificial Immune Algorithm

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**Abstract**— Evolutionary Algorithms have an inherent parallelism that should enable them to locate several optima of a multi-modal function. However, in practice they are found to converge to a single (global) optimum. This has led to the research in the design of highly specialized evolutionary algorithms to obtain the maximum number of global and local optima of multi-modal functions. However, this is an over-kill, since in most cases the management needs no more than a handful of optima to make decisions. We demonstrate that the ordinary CLONALG algorithm, without any special modification to handle multi-modal optimization, is powerful enough to obtain several global and local optima to support the decision-making process.

**Keywords** - evolutionary computation; multi-modal optimization; artificial immune algorithm;

## I. INTRODUCTION

In real-world optimization problems, sometimes we are not satisfied with only one optimal solution. The demand for multiple solutions is more prominent when there exist several near optimal answers to a problem. Evolutionary Algorithms (EA) are widely used for function optimization. The EAs have an inherent parallelism that should enable them to locate several optima of a multimodal function. However, in practice they are found to converge to a single (global) optimum. The Genetic Algorithm (GA), in particular, is found to converge to a single solution when attempting to optimize a multimodal function [10].

The inability of EAs to handle Multi Modal Optimization (MMO) has led to an extensive research in the design of new algorithms. Extended EAs are devised to locate all the global optima and as many local optima as possible. However, at a practical level, the effort and the cost of designing such high-caliber and computationally expensive EAs do not seem to be justified, since the management cannot possibly refer to *all* the global and the local optima in making important managerial decisions. Knowledge of *a handful of optima* in the multi-modal problem is sufficient to make quick and speedy decisions.

In this paper, we demonstrate that an implementation of the simple CLONALG algorithm meets this end. This algorithm need not be stretched to locate all the optima of a multi-modal function. It has an inherent mechanism to locate some of the global and local optima, which presents a sufficiently comprehensive scenario to aid the managerial decision making process.

The Artificial Immune System (AIS) algorithm is inspired by the biological immune system [5,21]. The biological immune system is made up of primarily two types of cells - B cells which are produced in the bone marrow and T cells which are produced in the thymus. The pathogens like bacteria and viruses invading the body are called antigens. Both the antigen and the receptors on the surface of the B cells have three-dimensional structures. The affinity between the structure of the receptors and that of the antigen is a measure of the complementarities between the two. When an antigen invades the body, the immune system generates antibodies to diminish the antigen. Initially, the invaded antigen is recognized by a few of the B cells with high affinity for the antigen. Stimulated by the helper T cells, these high affinity B cells proliferate by cloning. This process is called clonal selection principle. The new cloned cells undergo a high rate of somatic mutations called hyper-mutation. The mutations undergone by the clones are inversely proportional to their affinity to the antigen. The highest affinity antibodies experience the lowest mutation rates, whereas the lowest affinity antibodies have the highest mutation rates. The high affinity B cells and their clones proliferate and differentiate into plasma cells. Finally, the plasma cells generate a large number of antibodies to neutralize and eliminate the antigens.

After the cloning and hyper-mutation stage, the immune system must return to its normal condition, eliminating the extra cells. However, some cells remain circulating throughout the body as memory cells. When the immune system is later attacked by the same type of antigen (or a similar one), these memory cells are activated, presenting a better and more efficient secondary response.

Among the various mechanisms in the biological immune system that are explored as AISs, negative selection [12], immune network model [6] and clonal selection [7] are the most discussed models. The CLONALG algorithm based on the above clonal selection principle is also used in optimization [7-8]. In this study, we show that the CLONALG algorithm routinely locates several global and local optima of a multi-modal function.

This paper is organized as follows: Section II presents a review of the literature on multi-modal optimization functions. Section III explains the CLONALG algorithm in detail and introduces the multi-modal test functions. The experimental results are presented in section IV. The paper ends with a brief conclusion in section V.

## II. LITERATURE REVIEWED

EAs are either extended or hybridized with other optimization techniques to solve the MMO problems. In addition, new algorithms are also designed. This section presents a review of the algorithms found in literature.

### A. Extended Algorithms

The standard Genetic Algorithm (GA) is extended towards multi-modal function optimization by introducing a niche-preserving technique [10]. The technique deals with finding and preserving multiple stable niches of the solution space possibly around multiple solutions so as to prevent convergence to a single solution [11]. Niching methods maintain diversity in the population and permit the EA to find many optima in parallel. Clearing [18], Crowding [9,16], Clustering[19], Sharing [3, 11], Restricted Selection [13], Species [19] and Conserving [14] are some of the notable niching techniques employed to extend EAs to find solutions to the multi-modal optimization problems.

The Particle Swarm Optimization (PSO) is a simple, but efficient algorithm based on the swarm intelligence metaphor. NichePSO is an extended form of PSO designed to handle multi-modal optimization. The Species-based PSO (SPSO) implements proximity-based speciation and creates turbulent regions around the already found solutions to prevent unnecessary function evaluations [2]. The Bottleneck Assigned Binary Any System is inspired by the traffic organization in real ants under crowded conditions [24].

### B. Hybrid Algorithms

Memetic algorithm which introduces local search techniques in PSO is developed by Wang et al. [23]. The PSO disperses the solutions in diverse sub-regions, where an adaptive local search takes place to locate the optima. The Niche Hybrid Algorithm (NHGA) is a hybrid form of the Nelder-Mead's Simplex Method and GA and is used in the multi-modal optimization of vehicle suspension system [1]. An agent-based hybrid algorithm is found in [15] and a Differential Evolution hybrid algorithm is found in [20].

### C. New Algorithms

The Artificial Immune Network Algorithm for multi-modal optimization (opt-aiNet) is a novel algorithm [12]. One of the salient features of this algorithm is the increase in population at every iteration. The increasing population is an indication that the problem has many local optima which the algorithm finds efficiently. Estimation of distribution algorithms (EDAs) are a new set of algorithms used in MMO. Unlike most EAs, EDAs do not make use of variation operators (e.g., crossover and/or mutation) in the combination step. Instead, EDAs generate the offspring population at each iteration by learning and subsequent simulation of a joint probability distribution for the individuals selected [17]. The ensemble of niching algorithms (ENA) approach uses several niching methods in parallel in order to preserve diversity of the populations and to benefit from the best method [22]. Other miscellaneous evolutionary approaches, including the Multi-objective

Optimization (MO) algorithms are found in the comprehensive survey on MMO by Das et al. [4].

## III. AIS FOR MMO ALGORITHMS

In this section, we define the objective functions and the AIS clonal selection algorithm that is used in finding a set of global and local optima in these MMO benchmark test functions.

### A. AIS Algorithm

The AIS clonal algorithm described below consists of the following steps:

#### *Generation of antibody population*

A population consisting of N antibodies (Abs) is randomly generated. Each antibody represents a feasible solution to the optimization problem. The Abs in our application are represented as binary bit strings or as real numbers.

#### *Objective function evaluation*

The multimodal test functions (Equations 1 ~ 4) are evaluated numerically for each of the antibodies.

#### *Affinity Calculation*

The affinity (or the fitness) of each individual antibody is evaluated based on the value of the objective functions mentioned in this sub-section.

#### *Clone Selection*

A certain percentage of the antibodies with greater affinities are selected from the population. These are then cloned to produce additional antibodies.

#### *Affinity Proportional Mutation*

The clones produced in the above step are subjected to mutations in proportion to their affinities.

#### *Memory Renewal*

The antibodies with relatively lower affinities (i.e., with higher values of the objective function) are eliminated. The selected clones are introduced into the antibody population as the immune memory cells. The above steps are iterated for M number of cycles. The Ab with the highest affinity (maximum values) found in all the iterations is the optimal solution.

### B. Test Functions

We have chosen the following benchmark multi-modal maximization functions to demonstrate the applicability of the simple CLONALG algorithm: Rastrigin (1), Schaffer (2), Multi-function (3) and Roots function (4).

$$f(x, y) = 10 * n + \sum_{i=1}^n (x_i^2 - 10 * \cos(2\pi x_i)) \quad (1)$$

$$-5.12 \leq x_i \leq 5.12, i = 1, 2, \dots, n.$$

$$f(x, y) = 0.5 + \frac{\sin^2(\sqrt{x^2 + y^2} - 0.5)}{(1 + 0.001(x^2 + y^2))} \quad (2)$$

$$(x, y) \in [-10, 10]$$

$$f(x, y) = x * \sin(4\pi x) - y * \sin(4\pi y + \pi) + 1 \quad (3)$$

$$(x, y) \in [-1, 2]$$

$$f(z) = \frac{1}{1 + |z^6 - 1|} \quad (4)$$

$$z \in \mathbf{C}, z = x + iy, (x, y) \in [-2, 2]$$

#### IV. EXPERIMENTAL RESULTS

The different parameters of the algorithms used to produce the experimental results are shown in Table 1. The table shows a modest number of populations evolving through a modest number of generations (iterations). Since our aim is to demonstrate that the CLONALG algorithm produces a good number of optima under ordinary performance conditions, we have tried not to vary the (default) parameters used in conventional test runs.

TABLE I. EXECUTION PARAMETERS OF ALGORITHMS

Algorithm	Parameter		
	Population (N)	Total generations (Ngen)	Mutation Probability
PSO	40	200	-
GA	100	200	0.01
AIS (binary)	100	200	0.01
AIS (real)	100	200	0.01

The test results are plotted as 3D graphs (Figure 1 to Figure 4). In our experiments, PSO is coded as real-valued, while GA is coded as binary. However, in the case of the CLONALG algorithm, binary as well as real-valued algorithm is implemented. Both the implementations of CLONALG find a good deal of the global and local optima of the multi-modal test functions as explained below.

*a) Rastrigin function:* PSO and GA converge to a single global optimum (shown as black dots) (Figure 1 a, b). As to which optimum these algorithms converge to, depends on the randomly generated initial populations. But the binary as well as the real version of the CLONALG algorithm locate at least one of the global maxima and a couple of the local maxima (Figure 1 c, d).

*b) Schaffer's function:* All the particles in PSO rapidly converge to the central ridge of the function (Figure 2 a), while the GA is spreadout onto a few global optima. Some of the solutions are clearly sub-optimal. But the binary as

well as the real version of the CLONALG algorithm locate at least 2~3 central peaks of the function (Figure 2 c, d).

*c) Multi-function:* PSO easily converges to the global optimum (Figure 3 a), while the GA surrounds the global optimum and is spreadout in the search space (Figure 3 b).

*d) Roots function:* PSO locates two of the six peaks (Figure 4 a). On the other hand, GA locates just one of the peaks and is rather spread out (Figure 4 b). Both the versions of CLONALG successfully locate all the six peaks of the Roots function (Figure 4 c, d). In all the experiments, the simple CLONALG algorithm finds a number of global and local optima without using the MMO techniques.

#### V. CONCLUSION

In spite of their inherent parallelism, Evolutionary Algorithms are found to converge to a single global optimum when they attempt to optimize multi-modal functions. Extensive research has been done to design EAs to locate all the global optima and as many local optima as possible. However, the effort and the cost of designing such high-caliber and computationally expensive EAs do not seem to be justified, since the management cannot possibly refer to *all* the global and the local optima in making important managerial decisions. Knowledge of a handful of optima in the multi-modal problem is sufficient to make quick and speedy decisions. We demonstrated through a series of multi-modal test functions that an implementation of the simple CLONALG algorithm, without any special modification toward multi-modal optimization, meets this end.

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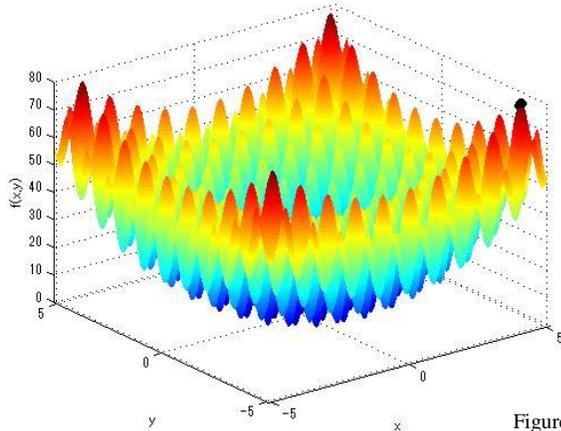
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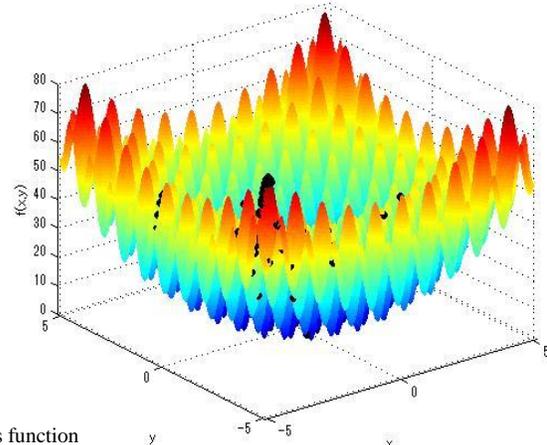
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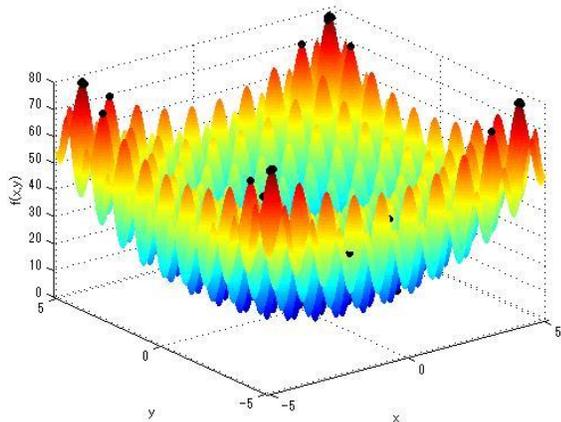
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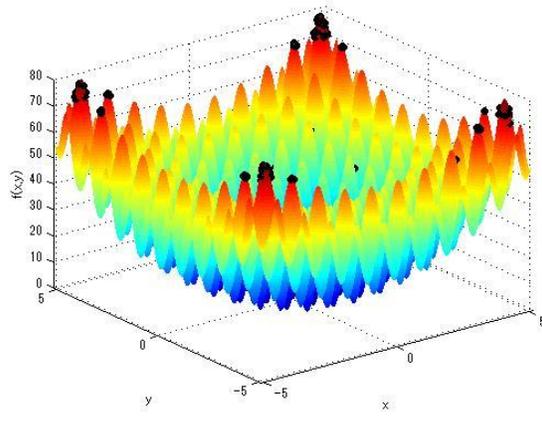
(a) PSO locates a single global peak



(b) GA locates a single global peak



(c) CLONALG (binary) locates all global and a few local optima



(d) CLONALG (real) locates all lobal and a few local optima

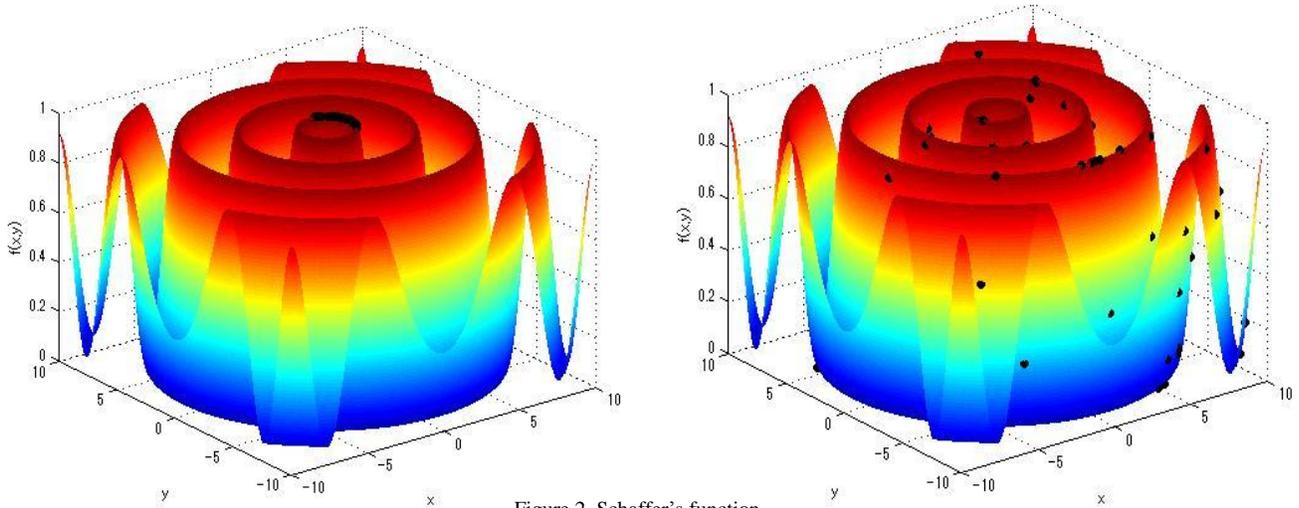
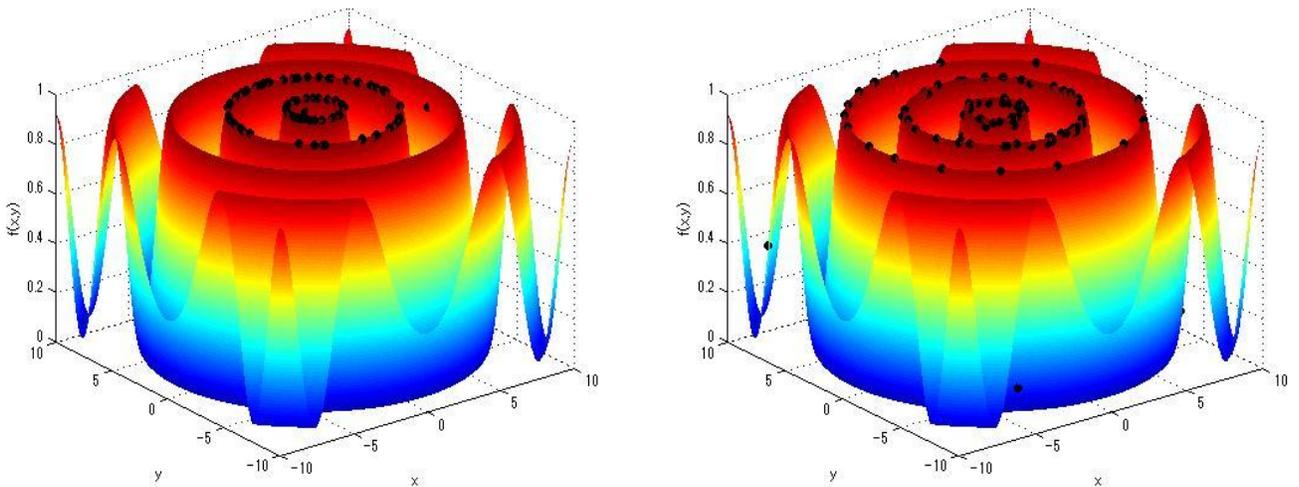


Figure 2. Schaffer's function

(a) PSO converges to the central peak

(b) GA is spread out with some sub-optimal solutions



(c) CLONALG (binary) locates 2 global maxima

(d) CLONALG (real) locates 3 global maxima

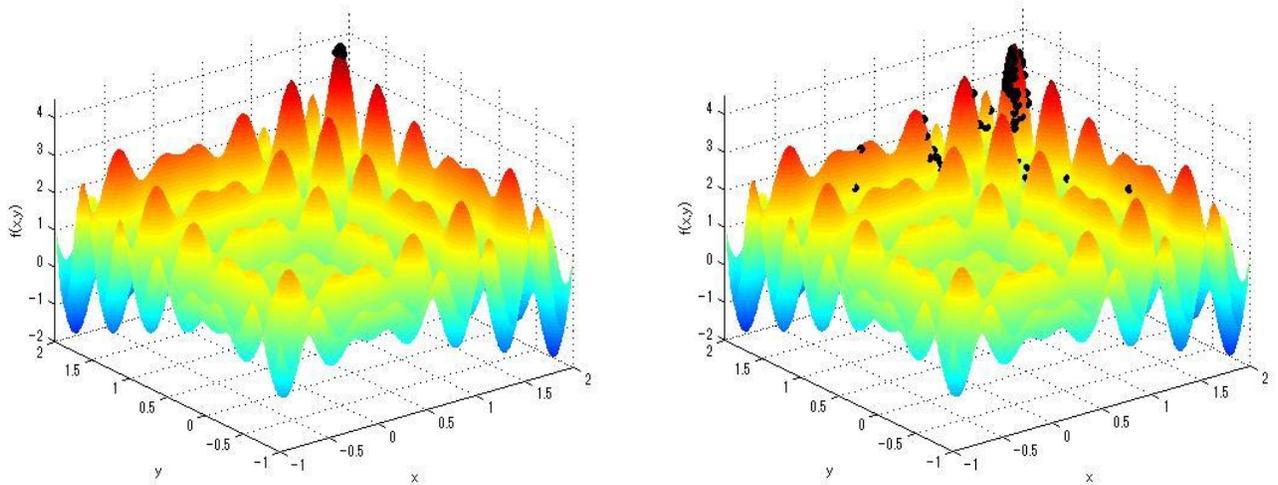
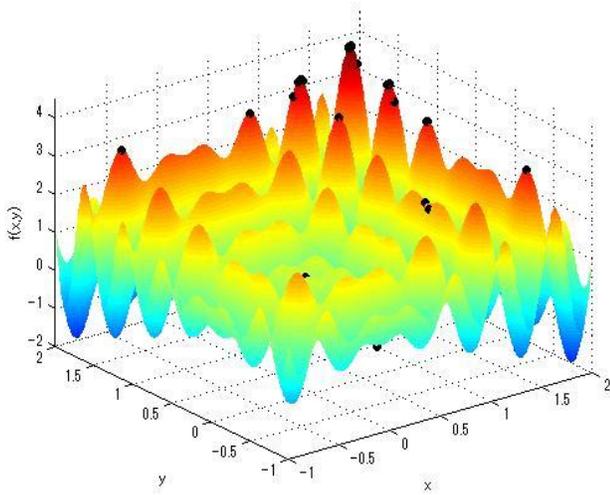


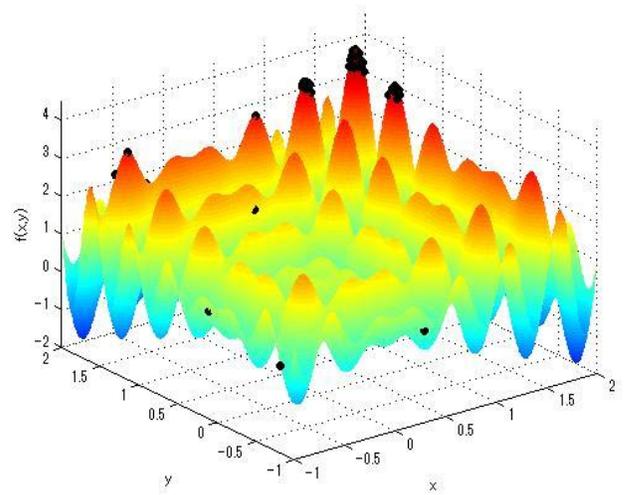
Figure 3. Multi-function

(a) PSO converges to the global optima

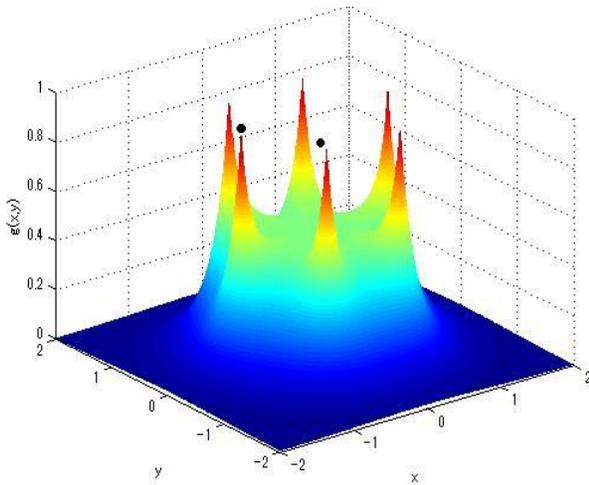
(b) GA surrounds the global optima and is spread out



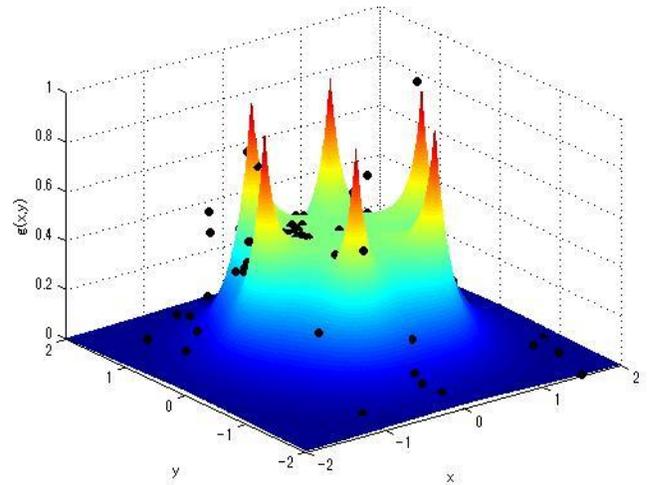
(c) CLONALG (binary) locates the global and some local optima



(d) CLONALG (real) locates the global and some local optima

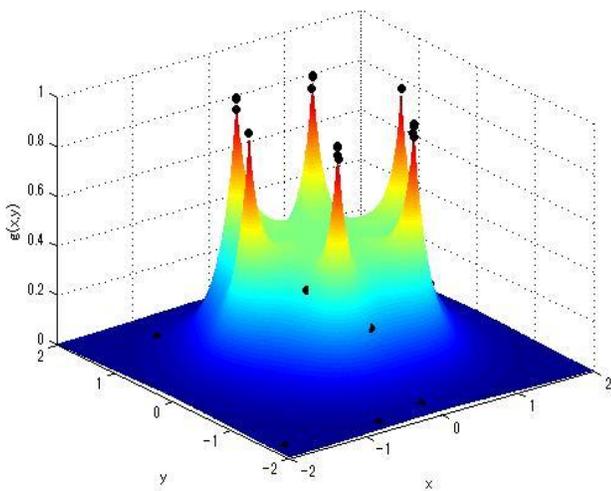


(a) PSO locates 2 global optima

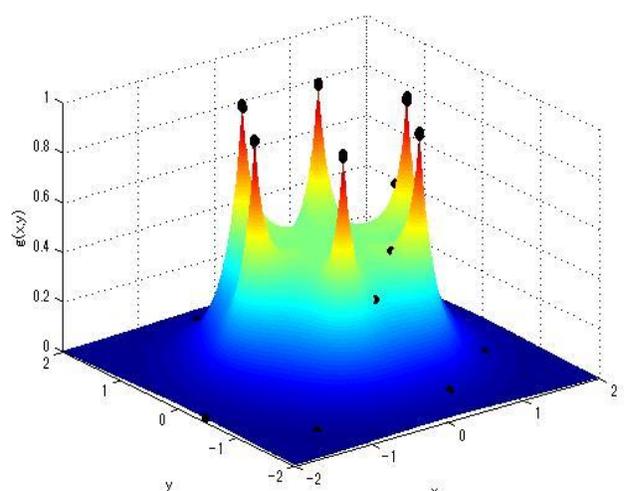


(b) GA locates 1 global optimum and is spread out

Figure 4. Roots function



(c) CLONALG (binary) locates all the global optima



(d) CLONALG (real) locates all the global optima