Adaptive Control for Persistent Disturbance Rejection in Linear Infinite Dimensional Systems

Mark J. Balas
Embry-Riddle Aeronautical University
Daytona Beach, FL, USA
email: balasm@erau.edu

Susan A. Frost
NASA Ames Research Center
Moffet Field, CA, USA
emai: susan.frost@nasa.gov

***Abstract*—Given a linear continuous-time infinite-dimensional plant on a Hilbert space and persistent disturbances of known waveform but unknown amplitude and phase, we show that there exists a stabilizing direct model reference adaptive control law with disturbance rejection and robustness properties. The plant is described by a closed, densely defined linear operator that generates a continuous semigroup of bounded operators on the Hilbert space of states. There is no state or disturbance estimation used in this adaptive approach. Our results are illustrated by adaptive control of general linear diffusion systems.**

Keywords- Hilbert space; persistent disturbances; general linear diffusion systems; adaptive control

I. INTRODUCTION

Many control systems are inherently infinite dimensional when they are described by partial differential equations. Currently there is renewed interest in the control of these kinds of systems especially in flexible aerospace structures and the quantum control field [1][2]. In this paper, we want to consider how to make a linear infinite-dimensional system regulate its output to zero in the presence of persistent disturbances.

In our previous work [3]-[6] we have accomplished direct model reference adaptive control and disturbance rejection with very low order adaptive gain laws for MIMO finite dimensional systems. When systems are subjected to an unknown internal delay, these systems are also infinite dimensional in nature. Direct adaptive control theory can be modified to handle this time delay situation for infinite dimensional spaces [7]. However, this approach does not handle the situation when partial differential equations (PDEs) describe the open loop system.

This paper addresses the effect of infinite dimensionality on the adaptive control approach of [4]-[6]. We will show that the adaptively controlled system is globally asymptotically stable using a new Barbalat-Lyapunov result. We apply this controller to linear PDEs with analytic semigroup generators and compact resolvent which model general linear diffusion systems.

II. ADAPTIVE REGULATION WITH DISTURBANCE REJECTION

Let X be an infinite dimensional separable Hilbert space with *inner product* (x, y) and corresponding norm $\|x\| \equiv \sqrt{(x,x)}$. Consider the Linear Infinite Dimensional Plant with *Persistent Disturbances*:

$$\begin{cases} \frac{\partial}{\partial t}x(t) = Ax(t) + Bu(t) + \Gamma\, u_D(t) \\ x(0) \equiv x_0 \in D(A) \subseteq X \\ Bu \equiv \sum_{i=1}^{m} b_i u_i \\ y(t) = Cx(t) + Eu_D(t) \\ y_i \equiv (c_i, x(t)), i = 1...m \end{cases} \tag{1}$$

where $x \in D(A)$ is the plant state, $b_i \in D(A)$ are actuator influence functions, $c_i \in D(A)$ are sensor influence functions, $u, y \in \Re^m$ are the control input and plant output m-vectors respectively, u_D is a disturbance with known basis functions ϕ_D. The persistent disturbances u_D will enter the plant through the state channels Γ and the output channels E.

In order to accomplish disturbance rejection in a direct adaptive scheme, we will make use of a definition, given in [4][7], for persistent disturbances:

Definition 2: *A disturbance vector* $u_D \in R^q$ *is said to be* ***persistent*** *if it it satisfies the* ***disturbance generator equations****:*

$$\begin{cases} u_D(t) = \theta z_D(t) \\ \dot{z}_D(t) = Fz_D(t) \end{cases} \text{or} \begin{cases} u_D(t) = \theta z_D(t) \\ z_D(t) = L\phi_D(t) \end{cases} \tag{2}$$

where F is a marginally stable matrix and $\phi_D(t)$ is a vector of known functions forming a basis for all the possible disturbances. This is known as “a disturbance with known waveform but unknown amplitudes”. We can easily show that an operator L exists to relate the persistent disturbances

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to a known basis vector $\phi_D(t)$, but the adaptive controller will not need to know the actual L.

The *objective of control* in this paper will be to cause the output *y(t)* of the plant to regulate asymptotically:

$$y \xrightarrow[t\to\infty]{} 0 \tag{3}$$

and this control objective will be accomplished by a *Direct Adaptive Control Law* of the form:

$$u = G_e y + G_D \phi_D \tag{4a}$$

The *direct adaptive controller* will have adaptive gains given by:

$$\begin{cases} \dot{G}_e = -yy^* \gamma_e ; \gamma_e > 0 \\ \dot{G}_D = -y\phi_D^* \gamma_D ; \gamma_D > 0 \end{cases} \tag{4b}$$

Note that the output feedback gains are directly adapted and no estimation or identification of plant information is used in the control law.

III. Ideal Trajectories

We define the *Ideal Trajectories* for (1) in the following way:

$$\begin{cases} x_* = S_1 z_D \\ u_* = S_2 z_D \end{cases} \text{with } z_D \in \Re^{N_D} \tag{5}$$

where the *ideal trajectory* $x_*(t)$ is generated by *the ideal control* $u_*(t)$ from

$$\begin{cases} \dfrac{\partial x_*}{\partial t} = Ax_* + Bu_* + \Gamma u_D \\ y_* = Cx_* + Eu_D = 0 \end{cases} \tag{6}$$

If such ideal trajectories exist, they will be linear combinations of disturbance state, and they will produce exact output tracking in a disturbance-free plant (8).

By substitution of (5) into (6), we obtain the *Model Matching Conditions:*

$$\begin{cases} AS_1 + BS_2 = S_1 F + \underbrace{H_1}_{\Gamma\theta} \\ CS_1 = H_2 = -E\theta \end{cases} \tag{7}$$

where $S_1 : \Re^{N_D} \to D(A) \subset X, S_2 : \Re^{N_D} \to \Re^M$.

Because (S_1, S_2) are both of finite rank, they are bounded linear operators on their respective domains.

A Special Case occurs when *E=0* and $Range(\Gamma) \subseteq Range(B)$. Then there exists S_2 such that $BS_2 + \Gamma\theta = 0$ and $S_1=0$. In this case the full system state x becomes disturbance-free, but in general we really only want to make the output y disturbance-free.

IV. Normal Form

We need two lemmae:

Lemma 1: If CB is nonsingular then $P_1 \equiv B(CB)^{-1}C$ is a (non-orthogonal) bounded projection onto the *range of B*, R(B), along the *null space of C*,N(C) with $P_2 \equiv I - P_1$ the complementary bounded projection, and $X = R(B) \oplus N(C)$, as well as $D(A) = R(B) \oplus [N(C) \cap D(A)]$.

Proof of Lemma 1: See [17].

Now for the above pair of projections $(P_1.P_2)$ we have

$$\begin{cases} \dfrac{\partial P_1 x}{\partial t} = P_1 \dfrac{\partial x}{\partial t} = (\underbrace{P_1AP_1}_{A_{11}})P_1x + (\underbrace{P_1AP_2}_{A_{12}})P_2x + (\underbrace{P_1B}_{B})u \\ \dfrac{\partial P_2 x}{\partial t} = P_2 \dfrac{\partial x}{\partial t} = (\underbrace{P_2AP_1}_{A_{21}})P_1x + (\underbrace{P_2AP_2}_{A_{22}})P_2x + (\underbrace{P_2B}_{=0})u \\ y = (\underbrace{CP_1}_{C})P_1x + (\underbrace{CP_2}_{=0})P_2x \end{cases}$$

which implies

$$\begin{cases} \dfrac{\partial P_1 x}{\partial t} = A_{11}P_1x + A_{12}P_2x + Bu \\ \dfrac{\partial P_2 x}{\partial t} = A_{21}P_1x + A_{22}P_2x \\ y = CP_1x = Cx \end{cases}$$

because $y = Cx = C(B(CB)^{-1}C)x = CP_1x$

and $P_1x = B(CB)^{-1}Cx = B(CB)^{-1}y$.

and $CP_2 = C - CB(CB)^{-1}C = 0$

and $P_2B = B - B(CB)^{-1}CB = 0$.

Lemma 2: If CB is nonsingular, then there exists and invertible, bounded linear operator

$$W \equiv \begin{bmatrix} C \\ W_2P_2 \end{bmatrix} : X \to \tilde{X} \equiv R(B)xl_2$$

such that

$$\bar{B} \equiv WB = \begin{bmatrix} CB \\ 0 \end{bmatrix}, \bar{C} \equiv CW^{-1} = \begin{bmatrix} I_m & 0 \end{bmatrix}, \text{and } \bar{A} \equiv WAW^{-1}.$$

This coordinate transformation puts (1) into *normal form:*

$$\begin{cases} \dot{y} = \bar{A}_{11}y + \bar{A}_{12}z_2 + CBu \\ \dfrac{\partial z_2}{\partial t} = \bar{A}_{21}y + \bar{A}_{22}z_2 \end{cases} \tag{8}$$

where the subsystem: $(\bar{A}_{22}, \bar{A}_{12}, \bar{A}_{21})$ is called the *zero dynamics* of (1) and

$$\bar{A}_{11} \equiv CA_{11}B(CB)^{-1} = CAB(CB)^{-1}; \bar{A}_{12} \equiv CAW_2^*;$$
$$\bar{A}_{21} \equiv W_2A_{21}B(CB)^{-1}; \bar{A}_{22} \equiv W_2A_{22}W_2^*$$

and $W_2 : X \to l_2$ by $W_2 x \equiv \begin{bmatrix} (\theta_1, P_2 x) \\ (\theta_2, P_2 x) \\ (\theta_3, P_2 x) \\ \cdots \end{bmatrix}$ is an isometry from $N(C)$ into l_2.

Proof of Lemma 2: See [17].

Now we can prove the following theorem about the *Existence of Ideal Trajectories*:

Theorem 1: Assume CB is nonsingular. Then

$$\sigma(F) = \sigma_p(F) \subset \rho(\bar{A}_{22})$$
$$\equiv \{\lambda \in C / (\lambda I - \bar{A}_{22})^{-1} : l_2 \to l_2 \text{ is a bounded linear operator}\}$$

(or $\sigma_p(F) \cap \sigma(\bar{A}_{22}) = \varphi$ where $\sigma(\bar{A}_{22}) \equiv [\rho(\bar{A}_{22})]^c$)

if and only if there exist unique bounded linear operator solutions (S_1, S_2) satisfying the Matching Conditions (7).

Proof: Define $\bar{S}_1 \equiv W^{-1}S_1 = \begin{bmatrix} \bar{S}_a \\ \bar{S}_b \end{bmatrix}$ and $\bar{H}_1 \equiv WH_1 = \begin{bmatrix} \bar{H}_a \\ \bar{H}_b \end{bmatrix}$. From (7), we obtain

$$\begin{cases} \bar{A}\bar{S}_1 + \bar{B}S_2 = \bar{S}_1 L_m + \bar{H}_1 \\ \bar{C}\bar{S}_1 = H_2 \end{cases}$$

where $(\bar{A}, \bar{B}, \bar{C})$ is the Normal Form (8). From this we obtain:

$$\begin{cases} \bar{S}_a = H_2 \\ S_2 = (CB)^{-1}[H_2 L_m + \bar{H}_a - (\bar{A}_{11}H_2 + \bar{A}_{12}\bar{S}_b)] \\ \bar{A}_{22}\bar{S}_b - \bar{S}_b F = \bar{H}_b - \bar{A}_{21}H_2 \end{cases}$$

We can rewrite the last of these equations as

$(\lambda I - \bar{A}_{22})\bar{S}_b - \bar{S}_b(\lambda I - F) = \bar{A}_{21}H_2 - \bar{H}_b \equiv \bar{H}$ for all complex λ. Now assume that F is simple and therefore provides a basis of eigenvectors $\{\phi_k\}_{k=1}^{N_D}$ for $\Re^{N_D}$. This is not essential but will make this part of the proof easier to understand. The proof can be re-done with generalized eigenvectors and the Jordan form. So we have

$$(\lambda_k I - \bar{A}_{22})\bar{S}_b\varphi_k - \bar{S}_b \underbrace{(\lambda_k I - F)\varphi_k}_{=0} = \bar{A}_{21}H_2 - \bar{H}_b \equiv \bar{H}$$

which implies

$\bar{S}_b\varphi_k = (\lambda_k I - \bar{A}_{22})^{-1}\bar{H}\varphi_k$ because $\lambda_k \in \sigma(F) \subset \rho(\bar{A}_{22})$

Thus we have

$$\bar{S}_b z = \sum_{k=1}^{L} \alpha_k (\lambda_k I - \bar{A}_{22})^{-1} \bar{H}\phi_k \forall z = \sum_{k=1}^{L} \alpha_k \phi_k \in \Re^L .$$

Since $\lambda_k \in \sigma(F) \subset \rho(\bar{A}_{22})$,

all $(\lambda_k I - \bar{A}_{22})^{-1}$ are bounded operators. Also $\bar{H} \equiv \bar{A}_{21}H_2 - \bar{H}_b$ is a bounded operator on $\Re^{N_D}$. Therefore $\bar{S}_b$ is a bounded linear operator, and this leads to S_1 also bounded linear.

If we look at the converse statement and let $\lambda_* \in \sigma(F) \cap \sigma(\bar{A}_{22}) = \phi$.

Then there exists $\varphi_* \neq 0$ such that

$$(\lambda_* I - \bar{A}_{22})\bar{S}_b\varphi_* - \bar{S}_b \underbrace{(\lambda_* I - F)\varphi_*}_{=0} = (\lambda_* I - \bar{A}_{22})\bar{S}_b\varphi_*$$
$$= \bar{H}.$$

In this case 3 things can happen when $\lambda_* \in \sigma(\bar{A}_{22})$: $(\lambda_* I - \bar{A}_{22})$ can fail to be 1-1 so multiple solutions of $\bar{S}_b$ will exist, $R(\lambda_* I - \bar{A}_{22})$ can fail to be all of X so no solutions $\bar{S}_b$ may occur, or $(\lambda_* I - \bar{A}_{22})^{-1}$ can fail to be a bounded operator so solutions $\bar{S}_b$ may be unbounded. In all cases these 3 alternatives lead to a lack of unique bounded operator solutions for S_1.

And the proof of Theo. 1 is complete.

It is possible to relate the point spectrum $\sigma_p(\bar{A}_{22}) \equiv \{\lambda / \lambda I - \bar{A}_{22} \text{ not 1-1}\}$ to the set Z of *transmission (or blocking) zeros* of (A, B, C).

Similar to the finite-dimensional case [16], we can see that

$$Z \equiv \left\{ \begin{array}{l} \lambda / V(\lambda) \equiv \begin{bmatrix} \lambda I - A & B \\ C & 0 \end{bmatrix} : \\ D(A)x\Re^m \to Xx\Re^m \text{ linear operator is not 1-1} \end{array} \right\}$$

Lemma 3: $Z = \sigma_p(\bar{A}_{22}) \equiv \{\lambda / \lambda I - \bar{A}_{22} \text{ is not 1-1}\}$ is called the *point spectrum* of $\bar{A}_{22}$. *So the transmission zeros of the infinite-dimensional open-loop plant* (A, B, C) *are the eigenvalues of its zero dynamics* $(\bar{A}_{22}, \bar{A}_{12}, \bar{A}_{21})$.

Proof of Lemma 3:
From

$$\bar{V}(\lambda)=\begin{bmatrix}\lambda I-\bar{A} & \bar{B}\\ \bar{C} & 0\end{bmatrix}$$

$$=\begin{bmatrix}W^{-1} & 0\\ 0 & I\end{bmatrix}\underbrace{\begin{bmatrix}\lambda I-A & B\\ C & 0\end{bmatrix}}_{V(\lambda)}\begin{bmatrix}W & 0\\ 0 & I\end{bmatrix}$$

we obtain $\begin{bmatrix}\lambda I-\bar{A} & \bar{B}\\ \bar{C} & 0\end{bmatrix}$ not 1-1 if and only if $\begin{bmatrix}\lambda I-A & B\\ C & 0\end{bmatrix}$ not 1-1.

But, using normal form from Lemma 2,

$$\bar{V}(\lambda)\equiv\begin{bmatrix}\lambda I-\bar{A} & \bar{B}\\ \bar{C} & 0\end{bmatrix}=\begin{bmatrix}\lambda I-\bar{A}_{11} & -\bar{A}_{12} & CB\\ -\bar{A}_{21} & \lambda I-\bar{A}_{22} & 0\\ I_m & 0 & 0\end{bmatrix}$$

And therefore

$$0=\bar{V}(\lambda)h=\bar{V}(\lambda)\begin{bmatrix}h_1\\ h_2\\ h_3\end{bmatrix}\text{ if and only if}$$

$$h_1=0;h_3=(CB)^{-1}\bar{A}_{12}h_2;(\lambda I-\bar{A}_{22})h_2=0.$$

So $h\neq 0$. if and only if $h_2\neq 0$ Therefore $\begin{bmatrix}sI-\bar{A} & \bar{B}\\ \bar{C} & 0\end{bmatrix}$ not 1-1 if and only if $\lambda\in\sigma_p\ (\bar{A}_{22})$.

This completes the proof of Lemma 3.

Using Lemma 3 and Theo. 1, we have the following *Internal Model Principle*:

Corollary 1: Assume *CB* is nonsingular and $\sigma(\bar{A}_{22})=\sigma_p(\bar{A}_{22})\ =\sigma_p(P_2AP_2)$ where $\bar{A}_{22}\equiv W_2^*P_2AP_2W_2$. There exist unique bounded linear operator solutions (S_1,S_2) satisfying the Matching Conditions (10) if and only if $\sigma(F)\cap Z=\varphi,$ i.e., no eigenvalues of *F* can be zeros of the open-loop plant *(A,B,C)*.

Note: $\lambda I-\bar{A}_{22}$ is not 1-1 if and only if there exists $x\neq 0$ such that $P_2x\neq 0$ and

$$0=(\lambda I-\bar{A}_{22})W_2P_2x$$
$$=(\lambda\underbrace{W_2W_2^*}_{I}-W_2PAP_2W_2^*)W_2P_2x$$
$$=[W_2(\lambda I-P_2AP_2)W_2^*]W_2P_2x$$

if and only if

$W_2(\lambda I-P_2AP_2)W_2^*$ is not 1-1 on *N(C).*

But W_2 is an isometry on *N(C).*

Therefore $\sigma_p(\bar{A}_{22})=\sigma_p(P_2AP_2)$.

V. STABILITY OF THE ERROR SYSTEM

The error system can be found from (1), (2) and (6): Define $e\equiv x-x_*$ and $\Delta u\equiv u-u_*$ this implies

$$\begin{cases}\dfrac{\partial e}{\partial t}=Ae+B\Delta u\\ y=y-0=\Delta y\equiv y-y_*=Ce\end{cases}\qquad(9)$$

Now we consider the definition of Strict Dissipativity for infinite-dimensional systems and the general form of the “adaptive error system” to prove stability. The main theorem of this section will later be utilized to assess the convergence and stability of the adaptive controller with disturbance rejection for linear diffusion systems.

Noting that there can be some ambiguity in the literature with the definition of strictly dissipative systems, we modify the suggestion of Wen in [8] for finite dimensional systems and expand it to include infinite dimensional systems.

Definition 1: *The triple* (A_c, *B, C*) *is said to be* ***Strictly Dissipative (SD)*** *if* A_c *is a densely defined ,closed operator on* $D(A_c)\subseteq X$ *a complex Hilbert space with inner product* (x,y) *and corresponding norm* $\|x\|\equiv\sqrt{(x,x)}$ *and generates a* C_0 *semigroup of bounded operators* $U(t)$ *,and* (B,C) *are bounded finite rank input/output operators with rank M where* $B:R^m\to X$ and $C:X\to R^m$ *. In addition there exist symmetric positive bounded operator P and Q* on *X such that*

$$0\leq p_{\min}\|e\|^2\leq(Pe,e)\leq p_{\max}\|e\|^2;0\leq q_{\min}\|e\|^2\leq(Qe,e)\leq q_{\max}\|e\|^2$$

i.e. P,Q are bounded and coercive, and

$$\begin{cases}\mathrm{Re}(PA_ce,e)\equiv\dfrac{1}{2}[(PA_ce,e)+\overline{(PA_ce,e)}]\\ =\dfrac{1}{2}[(PA_ce,e)+(e,PA_ce)]\\ =-(Qe,e)\leq-q_{\min}\|e\|^2;e\in D(A_c)\\ PB=C^*\end{cases}\qquad(10)$$

where W^* is the adjoint of the operator W.

We also say that (A, B, C) is *Almost Strictly Dissipative (ASD)* when there exists G_*mxm gain such that (A_c,*B*,*C*) is SD with $A_c\equiv A+BG_*C$. Note that if P=I in (5a), by the Lumer-Phillips Theorem [10], p 405, we would have $\|U_c(t)\|\leq e^{-\sigma t};t\geq 0\ ;\sigma\equiv q_{\min}>0$.

Henceforth, we will make the following set of assumptions:

Hypothesis 1: *Assume the following:*

i.) *There exists a gain,* G_e^* *such that the triple* $(A_C \equiv A + BG_e^*C, B, C)$ *is SD, i.e.* (A,B,C) is ASD;

ii.) *A is a densely defined ,closed operator on* $D(A) \subseteq X$ *and generates a* C_0 *semigroup of bounded operators* $U(t)$,

iii.) ϕ_D is bounded

From (5), we have $u_* = S_2 z_D$ and using (4a), we obtain:

$$\Delta u \equiv u - u_* = (G_e y + G_D \varphi_D) - (S_2 \underbrace{z_D}_{L\varphi_D}) = G_e^* y + \Delta G_e y + \Delta G_D \varphi_D = G_e^* e_y + \Delta G \eta \quad (11)$$

where

$$\Delta G \equiv G - G_*; G \equiv \begin{bmatrix} G_e & G_D \end{bmatrix}; G_* \equiv \begin{bmatrix} G_e^* & S_2 L \end{bmatrix};$$

$$G_D^* \equiv S_2 L; \text{and } \eta \equiv \begin{bmatrix} y \\ \varphi_D \end{bmatrix}$$

From (4), (9) and (11), the *Error System* becomes

$$\begin{cases} \dfrac{\partial e}{\partial t} = \underbrace{(A + BG_e^* C)}_{A_c} e + B\Delta G\eta = A_c e + B\rho; \\ e \in D(A); \rho \equiv \Delta G \eta \\ e_y = Ce \\ \Delta \dot{G} = \dot{G} - \dot{G}_* = \dot{G} = -e_y \eta^* \gamma \end{cases} \quad (12)$$

where $\gamma \equiv \begin{bmatrix} \gamma_e & 0 \\ 0 & \gamma_D \end{bmatrix} > 0$

Since B,C are finite rank operators, so is BG_e^*C. Therefore

$A_c \equiv A + BG_e^*C$ which has $D(A_c) = D(A)$ and generates a C_0 semigroup $U_c(t)$ because A does (see [9] Theo 2.1 p 497). Furthermore, by Theo 8.10 p 157 in [11], $x(t)$ remains in $D(A)$ and is differentiable there for all $t \geq 0$ .This is because $F(t) \equiv B\rho = B\Delta G\eta$ is continuously differentiable in $D(A)$.

We see that (12) is the *feedback interconnection* of an infinite-dimensional linear subsystem with $e \in D(A) \subseteq X$ and a finite-dimensional subsystem with $\Delta G \in \Re^{mxm}$. This can be written in the following form using $w \equiv \begin{bmatrix} e \\ \Delta G \end{bmatrix} \in D \equiv D(A) x \Re^{mxm} \subseteq \bar{X} \equiv X x \Re^{mxm}$:

$$\begin{cases} \dfrac{\partial w}{\partial t} = w_t = f(t,w) \equiv \begin{bmatrix} A_c e + B\rho(t) \\ -e_y \eta^* \gamma \end{bmatrix} \\ w(t_0) = w_0 \in D \text{ dense in } \bar{X} \equiv X x \Re^{mxm} \end{cases} \quad (13)$$

The inner product on $\bar{X} \equiv X x \Re^{mxm}$ can be defined as

$$(w_1, w_2) \equiv \left(\begin{bmatrix} x_1 \\ \Delta G_1 \end{bmatrix}, \begin{bmatrix} x_2 \\ \Delta G_2 \end{bmatrix} \right) \equiv (x_1, x_2) + tr(\Delta G_2 \Delta G_1^*)$$

which will make it a Hilbert space also.

Now we present a new version of Barbalat-Lyapunov for systems on an infinite dimensional Hilbert space:

Theorem 2 (Lyapunov-Barbalat): Let $w(t) = w(t, t_0, w_0) \in D$ and $V(t,w)$ satisfy:

$$\begin{cases} \alpha \|w\|^2 \leq V(t,w) \leq \beta \|w\|^2 \\ \dot{V}(t,w) \equiv \dfrac{\partial V(t,w)}{\partial t} + \dfrac{\partial V(t,w)}{\partial w} f(t,w) \leq -S(w) \leq 0 \end{cases}$$

for all $w \in D$. Then $w(t)$ is bounded in $\bar{X}$. Furthermore, if the following are true:

a) $S(w) \geq \mu \|\aleph w\|^2 \; \forall w \in D; \mu > 0;$ with $\aleph$ a bounded operator on $D \subseteq \bar{X} \equiv X x \Re^{mxm} \to X$ such that $(\aleph w)_t = \aleph w_t$

b) $\mathrm{Re}(\aleph w, \aleph f(t,w))$ is bounded on bounded sets of $w \in D$.

Then $\aleph w(t) \xrightarrow[t \to \infty]{} 0$.

Proof: See Appendix I in [17].

For this proof, we will need the following version of Barbalat's Lemma; see [15] pp210-211:

Lemma 4: We say *f(t)* is a *uniformly continuous* function on $(0,\infty)$ when for all $\varepsilon > 0$ there exists $\delta = \delta(\varepsilon) > 0$ such that $|f(t_2) - f(t_1)| < \varepsilon \;\; \forall |t_2 - t_1| < \delta$. If *f(t)* is a real, *uniformly continuous* function on $(0,\infty)$ with $\int_0^\infty f(t)dt < \infty$, then $f(t) \xrightarrow[t \to \infty]{} 0$.

Now we can prove the stability and convergence of the direct adaptively controlled error system (15):

Theorem 3: Under Hypothesis 1and $\mathrm{Re}(A_c e, e)$ bounded on bounded sets of $e \in D(A)$ we will have state and output tracking of the reference model: $e \xrightarrow[t \to \infty]{} 0$, and since C is a bounded linear operator: $e_y = y - y_m = Ce \xrightarrow[t \to \infty]{} 0$ with bounded adaptive gains

$$G \equiv \begin{bmatrix} G_e & G_m & G_u & G_D \end{bmatrix} = G_* + \Delta G$$

Proof: See Appendix II in [17].

VI. Application: Adaptive Control of Unstable Diffusion Equations

We will apply the above direct adaptive controller on the following single-input/single-output Cauchy problem which represents a *general linear diffusion problem*:

$$\begin{cases} \frac{\partial x}{\partial t} = Ax + b(u + u_D), x(0) \equiv x_0 \in D(A) \\ y = (c, x), \text{ with } b = c \in D(A) \end{cases} \quad (14)$$

where A has compact resolvent and generates and analytic C_0 semigroup.

From the compact resolvent property, we know that $\sigma(A) = \sigma_p(A)$ and by the analyticity requirement there will only be a finite number of unstable eigenvalues $\lambda_k \in \sigma_p(A)$.

Consequently, there exists G_* such that $A_c \equiv A + BG_*C$ satisfies

$\operatorname{Re} \lambda_k \leq -\mu < 0 \forall \lambda_k \in \sigma_p(A_c)$ which implies that

$$\operatorname{Re}(A_c x, x) \equiv \frac{1}{2}[(A_c x, x) + \overline{(A_c x, x)}] = \frac{1}{2}[(A_c x, x) + (x, A_c x)]$$
$$= -(Qx, x) \leq -\mu \|x\|^2 ; x \in D(A_c)$$

Also, since b=c we have $C^* = B$. Therefore we have that (A, B, C) is ASD with $P = I$.

From $\operatorname{Re}(A_c x, x) \leq -\mu \|x\|^2 \forall x \in D(A)$ we clearly have $\operatorname{Re}(A_c x, x)$ bounded on bounded sets of $x \in D(A)$.

For this application we will *assume the disturbances are sinusoidal with frequency 1 rad/sec* (but this is not a restriction as long as φ_D is bounded:

o

$$\begin{cases} u_D = \begin{bmatrix} 1 & 0 \end{bmatrix} z_D \\ \dot{z}_D = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} z_D \end{cases}$$

implies that

$$F = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}; \theta_D = \begin{bmatrix} 1 & 0 \end{bmatrix}; \varphi_D \equiv \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$$

implies that

$$u = G_e y + G_D \varphi_D \text{ with } \begin{cases} \dot{G}_e = -yy^* \gamma_e \\ \dot{G}_D = -y \varphi_D^* \gamma_D \end{cases}$$

So, since $B = \Gamma$, there is a gain $S_2 = -\theta$ such that $BS_2 + \Gamma\theta = B(-\theta + \theta) = 0$ which implies that $S_1 = 0$ and this is the special case of (7). Finally E=0 and the eigenvalues of F are $\pm j$ but the zeros of *(A,B,C)* are real; so the matching conditions are satisfied and ideal trajectories exist. Therefore we satisfy the hypothesis of Theo. 3 and we have, via the direct adaptive controller, state regulation $x \xrightarrow[t \to \infty]{} 0$ and output regulation $y \xrightarrow[t \to \infty]{} 0$ with bounded adaptive gains $G \equiv \begin{bmatrix} G_e & G_D \end{bmatrix}$ in the presence of sinusoidal persistent disturbances.

VII. Conclusions

In Theorem 1, we showed conditions under which ideal trajectories exist for a linear infinit-dimensional system to be capable of rejecting a persistent disturbance in the the output of the plant. In Theorem 3 we used an extension of Barbalat-Lyapunov result for linear dynamic systems on infinite-dimensional Hilbert spaces under the hypothesis of almost strict dissipativity for infinite dimensional systems, to show that direct adaptive control can regulate the state and the output of a linear infinite-dimensional system in the presence of persistent disturbances without using any kind of state or parameter estimation.. We applied these results to a general linear diffusion problem with sinusoidal disturbances using a single actuator and sensor and direct adaptive output feedback.

These results do not require deep knowledge of specific properties or parameters of the system to accomplish model tracking. And they do not require that the disturbance enter through the same channels as the control.

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