

# Task-Space Torque Controller Based on Time-Delay Control

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**Abstract**—In this paper, a torque control method in task-space for redundant manipulators with friction is proposed. A previous simple control approach based on virtual spring damper hypothesis is used to generate human-like motions. The method is efficient in the system which is difficult to identify the exact dynamics, however, the controller has steady state errors. To eliminate the steady state error, the gravity and friction, which is the part of the system dynamics, are compensated. Although the gravity and friction are compensated, the error of the modelling remains in the system. Hence, to reduce the nonlinearity, unknown effects, and modelling errors of the system, a torque controller based on Time-Delay Control (TDC) that eliminates the friction and unknown effects, is used. The performance of the control method, in Cartesian space control, is experimented with the torque sensor based 3-joints robot manipulator.

**Keywords**—Task-Space; Virtual spring damper hypothesis; Time-Delay Control(TDC).

## I. INTRODUCTION

Recently, many robot research for redundant manipulators have been developed, and to control the robot precisely in task-space has been an issue, especially for industrial robots. The traditional control method is to compute the inverse kinematics of the system [1][2][3]. The control input is computed from the joint angle velocity, which is calculated from the given end-effector velocity. Another approach is to create the control input directly from the inertia matrix and Coriolis and centrifugal force, which is called the inverse dynamics approach [4]. However, these control methods are especially difficult to compute in redundant systems because of the calculation for pseudo-inverse of the Jacobian matrix. Therefore, a simple approach that does not need for any computation of the inverse kinematics nor dynamics which is proposed by Arimoto *et al.* is considered. This is a natural control method based on virtual spring-damper hypothesis [7]-[11], which offers human-like motions. In this paper, the natural control method based on virtual spring damper hypothesis is used for the task-space controller.

For precise control of the end-effector, the dynamic model of the system is required. However, non-linearity of the system makes it difficult to model and causes control problems. The non-linearity of the system is the friction from the harmonic drive and bearing, noise and flexibility of the sensor, and dynamic modelling error of the plant. To deal with these problems many researches have been proposed such as using an observer to estimate the disturbance [13][24][25], friction compensation method [12], Time-Delay Control (TDC) method that eliminates the uncertainties without using the system parameters [14][15][16][18][19], and impedance control [6].

This paper addresses a task-space torque control method to control the friction existing redundant manipulator accurately. To overcome the friction and uncertainties, the TDC method is applied. With the TDC method used, the torque controller estimates the non-linear friction, unknown effects, and dynamic errors and cancel them out without any parameter identification [14].

This paper is organized as follows. The dynamic model of the redundant manipulator and the friction model is handled in Section II, and the task-space torque controller is designed in Section III. Experiments are carried on to validate the proposed method in Section IV, and Section V concludes this paper.

## II. DYNAMIC MODEL OF THE REDUNDANT MANIPULATOR

### A. Dynamic Model

The dynamic model of the redundant manipulator is considered as [20]

$$M(q)\ddot{q} + c(q, \dot{q})\dot{q} + g(q) = \tau_o + \tau_{ext} \quad (1)$$

$$B\ddot{\theta} + \tau_o = \tau_m + \tau_f \quad (2)$$

$$\tau_o = k_s(\frac{\theta}{N} - q) + k_d(\frac{\dot{\theta}}{N} - \dot{q}) \quad (3)$$

where (1) is the model of the link, (2) is the model of the motor, and (3) is the joint torque of the manipulator.  $q$  is the link angle vector,  $\theta$  is motor angle vector,  $M(q)$  is the mass inertia matrix of the manipulator,  $c(q, \dot{q})$  Coriolis force,  $g(q)$  is the gravity,  $\tau_{ext}$  is the vector of the friction and external disturbance,  $\tau_o$  is the torque measured by the joint torque sensor,  $B$  is the motor inertia,  $\tau_f$  stands for the friction torque of the motor,  $k_s$ ,  $k_d$ ,  $N$ , and  $\tau_m$  depict is the joint stiffness, joint damping, gear ratio, and the motor input torque, respectively. Each joint torque is measured by the joint torque sensor installed in each joint of the manipulator.

### B. Friction Model

Friction, from the harmonic drives and bearings, causes control problems [18][21][22][23]. In velocity or position servo, the friction can be ignored by the appropriately chosen gain of the controller. However, with joint torque servo, friction lowers the performance of the system, as shown by Hur *et al.* [18]. To improve the control performance and make a margin of gain the friction should be compensated with a appropriate model. The friction is identified with a simple experiment with the concept that the friction depends on the

velocity and torque of the joints [19]. Based on the Coulomb Viscous friction model [22] the 3 joint manipulator friction model is estimated as

$$\tau_f = \hat{\tau}_c \tanh(\alpha\dot{\theta}) + \hat{\tau}_v \tanh(\alpha\dot{\theta}) \sqrt{|\dot{\theta}|} \quad (4)$$

where  $\alpha$  is the slope of the ramp,  $\hat{\tau}_c$  denotes the Coulomb friction parameter, and  $\hat{\tau}_v$  represents the Viscous friction parameter. With the estimated friction model of the 3-joints robot manipulator the friction is compensated by adding to the control input. From the experiment, the friction is estimated; however, because of the Coulomb friction compensator the system starts to chatter [18]. To eliminate the static friction while ensuring the stability of the system a stiction feed-forward method is applied. The static friction is expressed as

$$\tau_{st} = \begin{cases} +\tau_s & \text{for } |\dot{q}| \geq \dot{q}_b, \beta_e > \beta_{eb} \\ -\tau_s & \text{for } |\dot{q}| \geq \dot{q}_b, \beta_e < -\beta_{eb} \\ 0 & \text{others} \end{cases} \quad (5)$$

where  $\tau_{st}$  is the static feed-forward parameter,  $\tau_s$  stand for the static friction coefficient,  $\dot{q}_b$  is the velocity boundary,  $\beta_e$  is the error of the control target,  $\beta_{eb}$  symbolizes the control error boundary. The boundary of the joint velocity and the control error is decided from amplitude of the sensor noise at steady state. The final friction model of the system is as follow.

$$\tau_f = \hat{\tau}_c \tanh(\alpha\dot{\theta}) + \hat{\tau}_v \tanh(\alpha\dot{\theta}) \sqrt{|\dot{\theta}|} + \tau_{st}. \quad (6)$$

### III. TASK-SPACE CONTROLLER

To control the redundant manipulator a traditional method is an inverse dynamics approach when the dynamic equation is as (1).

$$\tau = M(q)J^+(q)(\ddot{X}_c - \dot{J}(q, \dot{q})\dot{q}) + C(q, \dot{q}) + g(q) \quad (7)$$

where  $J^+$  is the pseudo inverse matrix, and  $X$  denotes the Cartesian position vector of the task-space

$$X = \begin{bmatrix} p_x \\ p_y \\ p_z \\ \phi_x \\ \phi_y \\ \phi_z \end{bmatrix}, \quad (8)$$

and  $\ddot{X}_c$  is the task-space command acceleration and is expressed as

$$\ddot{X}_c = \ddot{X}_d + K_v(\dot{X}_d - \dot{X}) + K_p(X_d - X) \quad (9)$$

where  $X_d$ ,  $\dot{X}_d$ ,  $\ddot{X}_d$  are the desired Cartesian position, velocity, and acceleration, and  $K_v$ ,  $K_p$  are the gain matrices. In an ideal condition, the controller follows the error dynamics

$$\ddot{e} + K_v\dot{e} + K_p e = 0 \quad (10)$$

where  $e = X_d - X$ , the Cartesian position error; however, to use this method the inverse jacobian matrix needed and makes the system complicated. To simplify the controller a natural control method based on virtual spring damper hypothesis is used.

#### A. Virtual Spring Damper Hypothesis

In this paper, the manipulation controller for the redundant manipulator to obtain a human-like motion is based on the virtual spring damper hypothesis which is suggested in [5]. This is a simple control approach which does not need calculation of the pseudo-inverse of the jacobian matrix or the dynamics of the system. With the Cartesian position error  $e$  the virtual spring potential energy  $U$  is as [17]

$$U = \frac{1}{2} e^T K e \quad (11)$$

where  $K$  is the stiffness coefficients matrix of the end-effector. The potential energy  $U$  is derivative in time  $\frac{dU}{dt}$  to obtain the torque and joint velocity. When  $\dot{X} = J(q)\dot{q}$

$$\frac{dU}{dt} = e^T K \dot{e} = -e^T K J(q)\dot{q} = -\tau^T \dot{q} \quad (12)$$

From (12), the torque is

$$\tau = -J^T(q) K e. \quad (13)$$

With the torque (13), the virtual spring hypothesis is expressed as

$$\tau = -C\dot{q} - J^T(q) K e \quad (14)$$

where  $C$  represent the damping coefficients matrix of the joint. For human like motions, the joint damper,  $-C\dot{q}$ , occur over-damping problem and to show similar movements a virtual damper,  $K_v \dot{X}$  is added with the virtual spring and is extended to virtual spring damper hypothesis.

$$\tau = -C\dot{q} - J^T(q)(K_v \dot{X} + K e) \quad (15)$$

where  $K_v$  denotes the damping coefficient matrix. To improve the controller performance, a friction and gravity compensator is considered with the virtual spring damper controller and is as

$$\tau = -C\dot{q} - J^T(q)(K_v \dot{X} + K e) + \tau_f + g(q). \quad (16)$$

#### B. Torque Controller Based on Time Delay Control

As shown in [19], the friction can be estimated by using the concept that the friction is related with velocity and torque, yet it is difficult to identify the non-linear phenomena and unknown effects. To eliminate the friction, non-linear and unknown effects without additional experiments, a torque controller based on TDC method is proposed [15]. The non-linear and linear time invariant system is defined to consider the TDC method.

$$\dot{x} = f(x, t) + B(x, t)u + d(t) \quad (17)$$

$$\dot{x}_m = A_m x_m + B_m r \quad (18)$$

where  $x$  denotes the state vector,  $t$  is the time,  $f(x, t)$  the full dynamics of the robot which includes the non-linear and unknown effects,  $B(x, t)$  control distribution,  $u$  is the control input, and  $d(t)$  is the external disturbance.  $x_m$  represents the state vector of the reference model,  $A_m$  system matrix,  $B_m$  is the command distribution matrix, and  $r$  is the command vector. The linear time invariant system (18) is a system without friction or disturbance. The non-linear systems control input  $u$  is defined by the error dynamics,  $\dot{e} = A_m e$ , to be controlled as the linear time invariant system where  $e = x_m - x$ . By (17) and (18) the error dynamics is as

$$\dot{e} = A_m e + \{-f(x, t) - B(x, t)u - d(t) + A_m x_m + B_m r\}. \quad (19)$$

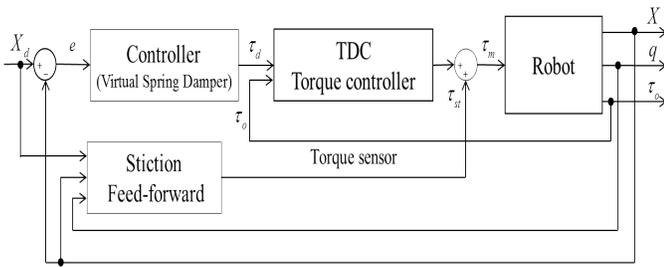


Figure 1. Block diagram of Task-Space Controller with TDC based Torque Controller

When the non-linear term is as

$$-f(x, t) - B(x, t)u - d(t) + A_m x_m + B_m r = 0$$

the system is stable; hence, the control input is derived as

$$u = B^+ [-f(x, t) - d(t) + A_m x_m + B_m r] \quad (20)$$

where  $B^+$  is the pseudo-inverse of  $B$ , which is  $B^+ = (B^T B)^{-1} B^T$ .

The assumption of the TDC method is that the present time value is the same after a very short time  $\delta$  have passed. It is expressed as

$$f(x, t) + d(t) \cong f(x, t - \delta) + d(t - \delta) \quad (21)$$

From the assumption the non-linear effects are estimated.

$$f(x, t) + d(t) \cong \dot{x}(t - \delta) - B(x, t - \delta)u(t - \delta) + d(t - \delta). \quad (22)$$

Substituting (22) into the control input  $u$  the TDC control input is as follow:

$$u(t) = u(t - \delta) + B^+ [-\dot{x}(t - \delta) + A_m x + B_m r]. \quad (23)$$

For the torque controller of this paper based on TDC control law, control input,  $u$ , is defined as

$$\tau_m(t) = \tau_m(t - \delta) + \hat{M} [-\ddot{x}_d(t - \delta) + \ddot{x}_d(t) + k_p \tau_e + k_v \dot{\tau}_e]. \quad (24)$$

where  $\tau_d$  stand for the desired torque,  $\hat{M}$  denotes a constant diagonal matrix followed by the stability analysis [16],  $\tau_e$  is the torque error  $\tau_e = \tau_d - \tau_o$ . Although the TDC is a controller that eliminates the non-linearity effects there is a limitation on canceling the static friction. Therefore, the static friction compensator is applied to improve the control performance. The overall task-space controller is described in Figure 1.

#### IV. EXPERIMENTS

The redundant manipulator that is used in this paper is a 3-joints robot arm equipped with a joint torque sensor at each joint. Each parameters of the link mass, length, the position of the center of mass, mass moments of inertia are shown in Table 1. With the proposed approach, the performance controlling the manipulator in task-space by eliminating the friction and unknown effects is tested in this section. The actuators are controlled by the motor controller from ELMO, and the harmonic drives are directly connected to the actuators with a 101:1 gear ratio.

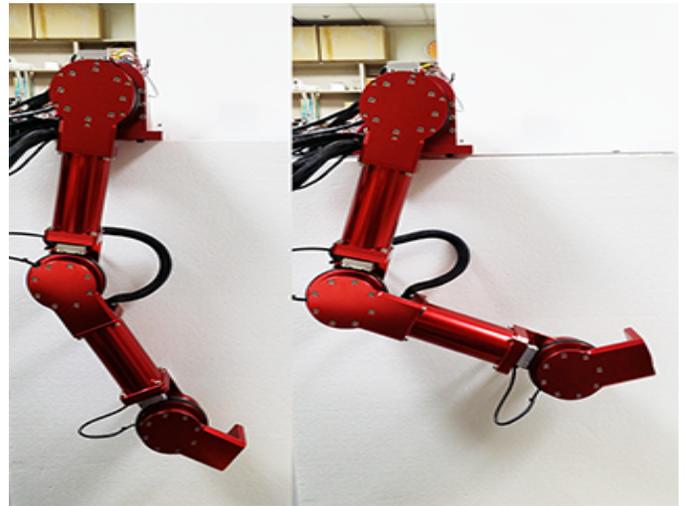

 Figure 2. Initial Position,  $X = [0.150, -0.550]^T$ , and Target Point,  $X = [0.300, -0.400]^T$ , of the 3-Joint robot arm

TABLE I. 3-JOINT ROBOT ARM PARAMETERS

Parameters	Value	Unit
$m_1$	2.3292	kg
$m_2$	2.2589	kg
$m_3$	2.0013	kg
$l_1$	0.300	m
$l_2$	0.300	m
$l_3$	0.146	m
$l_{c1}$	0.13552	m
$l_{c2}$	0.14023	m
$l_{c3}$	0.06857	m
$I_1$	0.041629	kgm <sup>2</sup>
$I_2$	0.039832	kgm <sup>2</sup>
$I_3$	0.0082305	kgm <sup>2</sup>

##### A. Task-Space Controller without Friction Compensator

The task-space controller, which is based on virtual spring damper, is experimented. In this experiment, the control input is as

$$\tau = -C\dot{q} - J^T(q)(K_v \dot{X} + K_e) + g(q). \quad (25)$$

To verify the controller performance the experiment starts at  $X = [0.150, -0.550]^T$ , point 1, and moves to  $X = [0.300, -0.400]^T$ , point 2, and comes back to point 1 as Figure 2. Figure 3 is the experiment result of the controller with the 3-joint manipulator. From the end-effector position of Figure 3, the red line is the desired position of the  $x$ -axis, and the blue line is the  $x$ -axis position of the end-effector. The magenta line represents the desired position of the  $y$ -axis, and the green line is the  $y$ -axis position of the end-effector. In the end-effector error graph, the red line,  $x_{err}$ , is the  $x$ -axis position error, and the blue line,  $y_{err}$  is the  $y$ -axis position error. The errors are calculated between the desired and current position of each axis. From the result of the task-space controller without friction compensation, it converges to the desired position with approximately  $x$ -axis 0.01[m], and  $y$ -axis 0.01[m] error. The error comes from the modelling error, non-linear friction, and the unknown effects, therefore, the next experiment include the friction of each joint and is compensated. Next, the task-space controller with the friction compensator is tested.

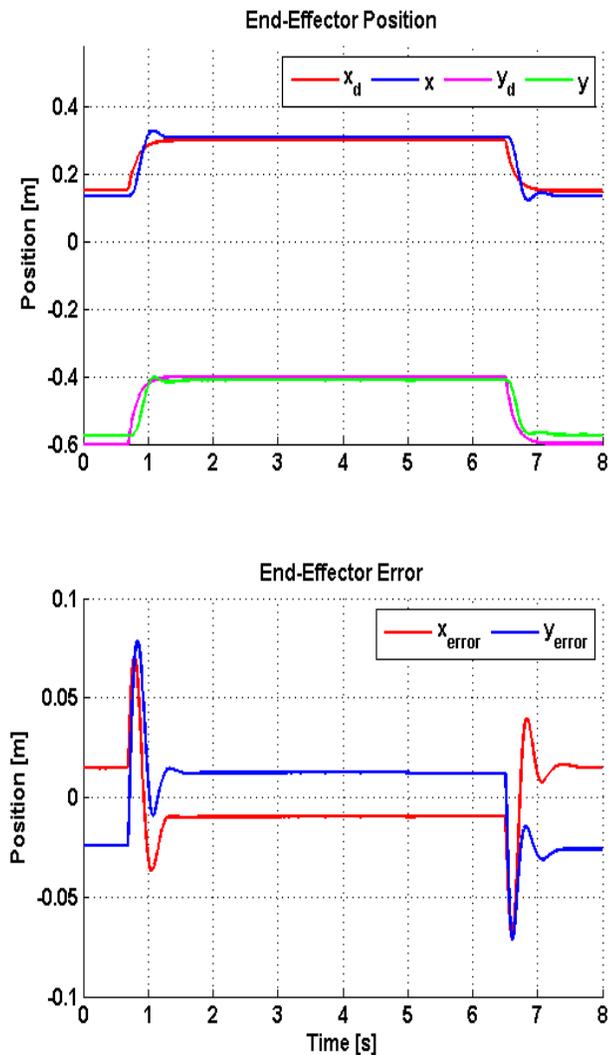


Figure 3. Control Result with Virtual Spring Damper Hypothesis

### B. Task-Space Controller with Friction Compensator

Without considering friction as the early experiment, from the experimental result friction degrades the performance of the system. Therefore, by implementing the friction compensator to the task-space controller, better performance of the controller is achieved. The control input of this experiment is as

$$\tau = -C\dot{q} - J^T(q)(K_v\dot{X} + Ke) + \tau_f + g(q). \quad (26)$$

Figure 4 shows the result of the controller with the friction compensator and the lines denotes the same as Figure 3. By applying the friction model, the position error decreases to approximately  $x$ -axis  $0.001[m]$ , and  $y$ -axis  $0.004[m]$ . With the friction compensator an appropriate performance of the controller is obtained. From the result of Figure 4, it appears that the controller performance depends on how the friction and the system is modeled; however, these models are difficult

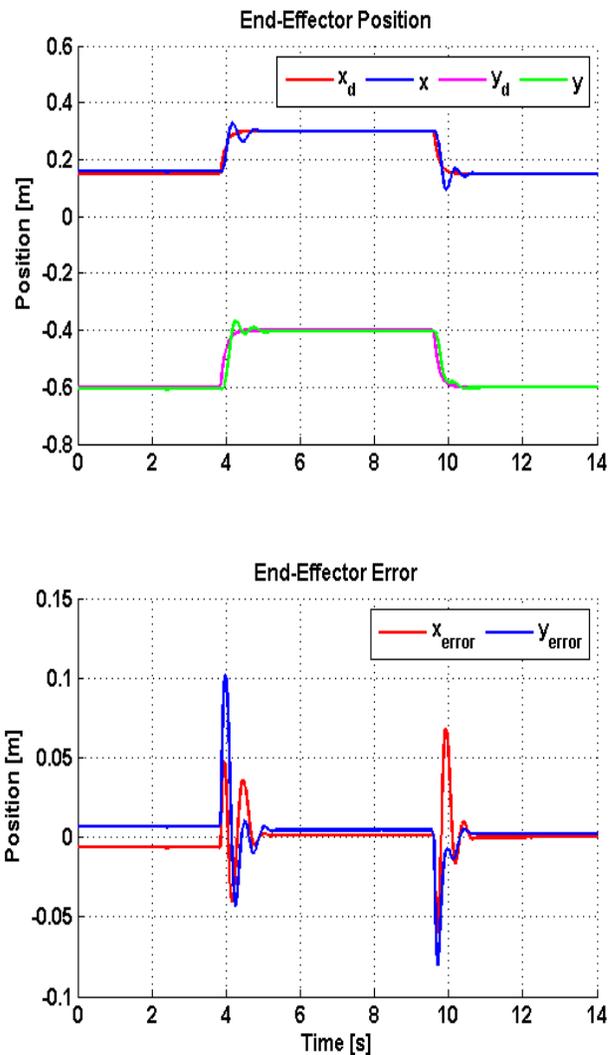


Figure 4. Control Result with Virtual Spring Damper Hypothesis and Friction Compensation

to identify. A TDC control method, as explained at the early section, is used to treat the difficulties in the next experiment.

### C. Task-Space Torque Controller Based on TDC

To eliminate the non-linear friction, unknown effects, and modelling error a TDC torque controller is used instead using the estimated friction model. This method needs no additional experiments for identifying the friction nor system dynamic parameters and is very adaptive in friction existing systems. The torque control input in task-space control is designed as

$$\begin{aligned} \tau_d &= -J^T(q)(K_v\dot{X} + Ke) \\ \tau_m(t) &= \tau_m(t - \delta) \\ &\quad + \hat{M}[-\ddot{\tau}_d(t - \delta) + \ddot{\tau}_d(t) + k_p\tau_e + k_v\dot{\tau}_e] - C\dot{q}. \end{aligned} \quad (27)$$

From the result in Figure 5, the lines have the same meaning as in Figure 2. The result shows that the controller

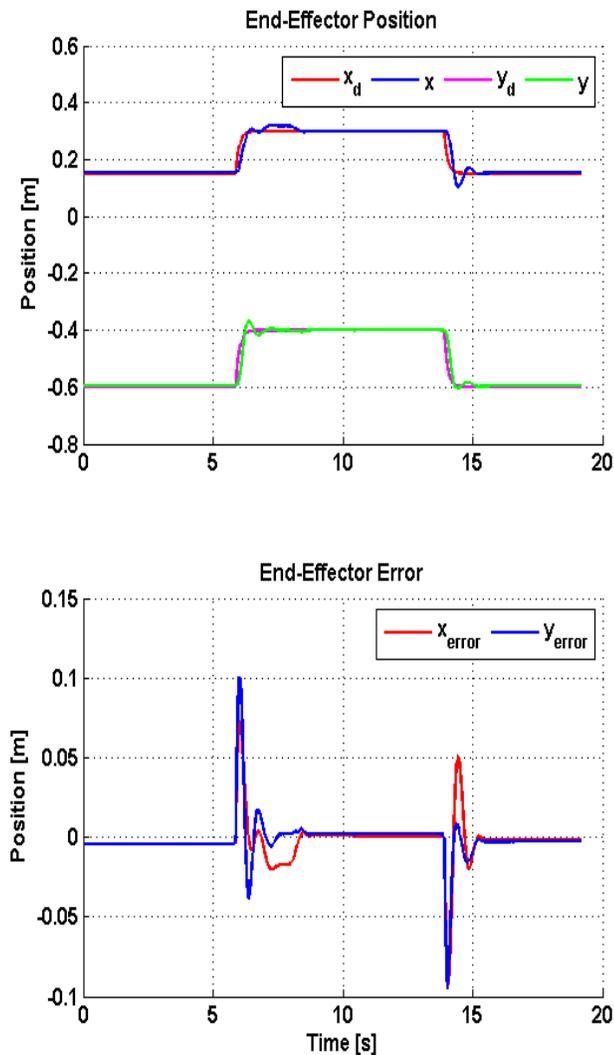


Figure 5. Control Result with Task-Space Torque Controller Based on TDC

offers a more accurate result. It decreases the  $x$ -axis error to  $0.00005[m]$ , and  $y$ -axis to  $0.004[m]$ .

## V. CONCLUSION

In this paper a task-space control method, which is based on virtual spring damper hypothesis, that can eliminate the non-linear friction, and modelling errors was proposed. Advantage of the Time-Delay Control (TDC) method, that does not need the friction and dynamic model, the proposed control method is adaptive in friction existing systems. To prove the controller performance the 3-joint manipulator is used. From the experimental results, the task-space controller has improved its performance in help of the TDC method.

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