An Introduction to Evolving Systems: Adaptive Key Component Control with Persistent Disturbance Rejection

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***Abstract*— This paper presents an introduction to Evolving Systems, which are autonomously controlled subsystems which self-assemble into a new Evolved System with a higher purpose. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process. This is the concept of Adaptive Key Component Control which operates through one specific component to maintain stability during the evolution. In addition this control must overcome persistent disturbances that occur while the evolution is in progress. We present theoretical results for the successful operation of Adaptive Key Component control in the presence of such disturbances and an illustrative example.**

Keywords- adaptive control; aerospace systems.

I. INTRODUCTION

Evolving Systems [1]-[2] are autonomously controlled subsystems which self-assemble into a new Evolved System with a higher purpose. Evolving Systems of aerospace structures often require additional control when assembling to maintain stability during the entire evolution process [3]-[5]. An adaptive key component controller has been shown to restore stability in Evolving Systems that would otherwise lose stability during evolution [6]. The adaptive key component controller uses a direct adaptation control law to restore stability to the Evolving System through a subset of the input and output ports on one key component of the Evolving System. Much of the detail of Evolving Systems appears in the chapter [8]. In this paper, we will deal with the situation where persistent disturbances can appear in some components and must be mitigated by the adaptive key component controller. Such disturbances will often be attendant in actively controlled rendezvous and docking.

The control laws used by the adaptive key component controller to restore stability in an Evolving System are guaranteed to have bounded gains and asymptotic tracking if the Evolved System is almost strictly dissipative. Hence, it is desirable to know when the dissipativity traits of the subsystem components, including the key component, are inherited in an Evolving System. We present results describing when an Evolving System will inherit the almost strict passivity traits of its subsystem components. Then we will present an adaptive key component controller that restores asymptotic stability with bounded adaptive gains and mitigates the effect of persistent disturbances during evolution.

II. MATHEMATICAL FORMULATION OF EVOLVING SYSTEMS

A mathematical formulation of a nonlinear time-invariant Evolving System is given here. Consider a system of L components of individually, actively controlled subsystems which can be described by the following equations for the i^{th} component:

$$\begin{cases} \dot{x}_i = f_i(x_i, u_i) \\ y_i = g_i(x_i, u_i) \end{cases} \tag{1}$$

where $i = 1,2,\ldots L$. The i^{th} component has a Performance Cost Function J_i and a Lyapunov Function V_i. These are the building blocks of the Evolving System. When these individual components are joined to form an Evolved System, the new entity becomes:

$$\begin{cases} \dot{x} = f(x,u) \\ y = g(x,u) \end{cases} \tag{2}$$

with $x = [x_1 ... x_L]^T$, $y = [y_1 ... y_L]^T$, Performance Cost Function J, and Lyapunov Function V.. The i^{th} component in the above Evolved System is given by:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} \varepsilon_{ij} f_{ij}(x_i, x_j, u_j); \quad 0 \le \varepsilon_{ij} \le 1 \tag{3}$$

where $f_{ij}(x_i, x_j, u_j)$ represents the interconnections between the i^{th} and j^{th} components. Note that when $\varepsilon_{ij} = 0$, the system is in component form and when $\varepsilon_{ij} = 1$, the

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system is fully evolved. As the system evolves, or joins together, the ε_{ij} 's evolve from 0 to 1.

The components of the Evolving System are actively controlled by means of local control. Local control means dependence only on local state or local output information, i.e., $u_i = h_i(x_i)$ or $u_i = h_i(y_i)$. In general, the local controller on the i^{th} component would have the form:

$$\begin{cases} u_i = h_i(y_i, z_i) \\ \dot{z}_i = l_i(y_i, z_i) \end{cases} \tag{4}$$

where z_i is the dynamical part of the control law. Local control will be used to keep the components stable and meet the individual component performance requirements, J_i.

Once the system is fully evolved, the i^{th} component in the fully evolved system $\varepsilon_{ij} = 1$, becomes:

$$\dot{x}_i = f_i(x_i, u_i) + \sum_{j=1}^{L} f_{ij}(x_i, x_j, u_j) \tag{5}$$

A state space version of the i^{th} individual component of an Evolving System where the components are connected through the states can be represented as:

$$\begin{cases} \dot{x}_i = A_i(x_i) + B_i(x_i)u_i \\ + \sum_{j=1}^{L} \varepsilon_{ij} A_{ij}(x_i, x_j);\ x_i(0) \equiv x_{0_i} \\ y_i = C_i(x_i) \end{cases} \tag{6}$$

where $i = 1, 2, \ldots L$, $x_i \equiv [x_1^i \ldots x_{n_i}^i]^{\mathrm{T}}$ is the component state vector, $u_i \equiv [u_1^i \ldots u_{m_i}^i]^{\mathrm{T}}$ is the control input vector, $y_i \equiv [y_1^i \ldots y_{p_i}^i]^{\mathrm{T}}$ is the sensor output vector, $(A_i(x_i), B_i(x_i), C_i(x_i))$ are vector fields of dimension $n_i \text{ x } n_i$, $n_i \text{ x } m_i$, and $p_i \text{ x } n_i$, respectively, and the connection forces between components are represented in the $n_i \text{ x } n_j$ connection matrix, $A_{ij}(x_i, x_j) \text{ with } \varepsilon_{ji} = \varepsilon_{ij}$. The state space representation of the Evolved System then becomes:

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases} \tag{7}$$

which can also be written as $(A(x), B(x), C(x))$.

III. INHERITANCE OF SUBSYSTEM TRAITS IN EVOLVING SYSTEMS

We say a subsystem trait, such as stability, is inherited when the Evolved System retains the characteristic of the trait from the subsystem. Previous papers have examined the inheritance of stability and shown that stability is not a generally inherited trait [3]-[5] and [8]. Inheritance of almost strict passivity of subsystems is desirable in Evolving Systems that use an adaptive key component controller to restore stability.

In previous papers, [5]-[6], a key component controller has been proposed to restore stability to Evolving Systems which would otherwise lose stability during evolution. The design approach used by the key component controller is for the control and sensing of the components to remain local and unaltered except in the case of one key component which has additional local control added to stabilize the system during evolution. The key component controller operates solely through a single set of input-output ports on the key component, see Figure 1.

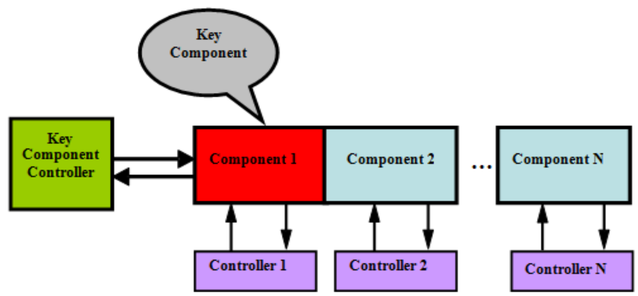

Figure 1. Key component controller.

Only the key component of the Evolving System needs modification to restore the inheritance of stability. A clear advantage of the key component design is that components can be reused in many different configurations of Evolving Systems without the need for component redesign. The reuse of components which are space-qualified, or at least previously designed and unit tested, could reduce the overall system development and testing time and should result in a higher quality system with potentially significant cost savings and risk mitigation.

In many aerospace environments and applications, the parameters of a system are poorly known and difficult to obtain. Adaptive key component controllers, which make use of a direct adaptation control law, are a good design choice for restoring stability in Evolving Systems where access to precisely known parametric values is limited. The sufficient condition for an Evolving System with an adaptive key component controller to be guaranteed to have bounded gains and to have asymptotic output tracking is that the system be almost strictly dissipative. So, we are interested in the conditions under which the inheritance of almost strict dissipativity can be guaranteed in Evolving Systems.

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IV. INHERITANCE OF ALMOST STRICT DISSIPATIVITY IN EVELOVING SYSTEMS

Inheritance of almost strict dissipativity of subsystems is desirable in Evolving Systems that use an adaptive key component controller to restore stability.

Consider a ***Nonlinear System*** of the form:

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ y = C(x) \end{cases}$$

We say this system is ***Strictly Dissipative*** when

$$\exists V(x) > 0 \, \forall x \neq 0$$

such that the Lie derivatives satisfy:

$$\begin{cases} L_A V \equiv \nabla V A(x) \leq -S(x) \ \forall x \\ L_B V \equiv \nabla V B(x) = C^T(x); \nabla V \equiv \text{gradient } V \end{cases} \quad (8)$$

The function $V(x(t))$ is called the ***Storage Function*** for (7), and the above says that the storage rate is always less than the external power. This can be seen from

$$\begin{aligned} \dot{V} &\equiv \nabla V[A(x) + B(x)u] \\ &\leq -S(x) + C^T(x)u \\ &= -S(x) + \langle y, u \rangle \end{aligned} \quad (9)$$

Taking $u \equiv 0$, it is easy to see that (9) implies (8a) but not necessarily (8b); so (8) implies (9) but not conversely. They are only equivalent if (8a) is an equality. (When equality holds in (8) and (9), the property is known as **Strict Passivity**.)

We will say a system (u, y) is **Almost Strictly Dissipative (ASD)** when there is some output feedback, $u = G_* y + u_r$, so that the following is strictly dissipative:

$$\begin{cases} \dot{x} = A_C(x) + B(x)u_r \\ A_C(x) \equiv A(x) + B(x)G_* C(x) \\ y = C(x) \end{cases} \quad (10)$$

Now if each component is ASD, then we have

$$\begin{cases} \nabla V_i [A_i(x_i) + B_i(x_i) G_i C_i(x_i)] \leq -S_i(x_i) \\ \qquad + \sum_{j=1}^{L} \varepsilon_{ij} \nabla V_i A_{ij}(x_j, u_j) \\ \nabla V_i B_i(x_i) = C_i^T(x_i); \nabla V_i \equiv gradient V_i \end{cases} \quad (11)$$

Due to the interconnection terms, (11) is not necessarily Strictly Dissipative. However, in some circumstances, the interconnection terms have a special form and ASD is inherited when the system evolves.

Suppose we have a pair of subsystems of the form:

$$\begin{cases} \dot{x}_i = A_i(x_i) + \varepsilon B_i(x_i) u_i + B_i^A(x_i) u_i^A \\ y_i = C_i(x_i) \\ y_i^A = C_i^A(x_i) \end{cases} \quad (12)$$

where $i = 1, 2$ and both subsystems $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix} \right)$ and $\left(\begin{bmatrix} u_2 \\ u_2^A \end{bmatrix}, \begin{bmatrix} y_2 \\ y_2^A \end{bmatrix} \right)$ have storage functions V_i. We have the following result:

Theorem 1: If the subsystems (u_1^A, y_1^A) and (u_2^A, y_2^A) are ASD and

$$\nabla V_i B_i(x_i) = C_i^T(x_i); \, i = 1, 2 \quad (13)$$

then the resulting feedback connection, $y_1 = u_2$ and $u_1 = -y_2$, will leave the composite system $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} \right)$ almost strictly passive.

Proof: See Appendix.

In [3]-[4], it was shown that the physical connection of components is equivalent to the feedback connection of the admittance of one to the impedance of the other. Consequently, if (u_1, y_1) and (u_2, y_2) are in Admittance/Impedance form, then Theorem 1 shows that ASD is an inherited property for Nonlinear Evolving Systems.

V. MATHEMATICAL FORMULATION OF ADAPTIVE KEY COMPONENT CONTROLLER WITH PERSISTENT DISTURBANCE MITIGATION

Our <u>Key Component</u> is chosen to be <u>Component #1</u> and will be modeled by the following ***Nonlinear System with an External Persistent Disturbance***:

$$\begin{cases} \dot{x}_1 = A_1(x_1) + \varepsilon B_1(x_1) u_1 + B_1^A(x_1) u_1^A + \Gamma_1(x_1) u_D \\ y_1 = C_1(x_1) \\ y_1^A = C_1^A(x_1) \end{cases} \quad (14)$$

All vector fields in (14) will have the appropriate compatible dimensions and be smooth in their arguments with a single equilibrium point at 0 in a neighborhood U.

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The persistent disturbance input vector $u_D(t)$ is N$_D$-dimensional and will be thought to come from the following Disturbance Generator:

$$\begin{cases} u_D = \Theta z_D \\ \dot{z}_D = F z_D ; z_D(0) = z_0 \end{cases}$$

where the disturbance state $z_D(t)$ is N$_D$-dimensional. Such descriptions of persistent disturbances were first used in [9] to describe signals of known form but unknown amplitude. For example, step disturbances yield $\Theta = 1$ and $\mathrm{F} = 0$ while sinusoidal disturbances can be described by

$$\begin{cases} \Theta = [1 \quad 0] \\ F = \begin{bmatrix} 0 & 1 \\ -\omega_D^2 & 0 \end{bmatrix} \end{cases}$$

where the frequency ω_D is known but the amplitudes are not.

We will assume that the Disturbance Generator parameter F is known. In many cases this is not a severe restriction, e.g. step disturbances. It turns out that it is better to rewrite the above in the following equivalent form:

$$\begin{cases} u_D = \Theta z_D \\ z_D = L\varphi_D \end{cases} \qquad (15)$$

where ϕ_D is a vector composed of the known basis functions for the solutions of $z_D(t)$ and (L, Θ) need not be known. This can be seen from the following:

$$\begin{aligned} z_D(t) &= e^{Ft} z_D(0) \\ &= \left[\varphi_1(t), \varphi_2(t), ..., \varphi_{N_D}(t) \right] z_D(0) \cdot \\ &= \sum_{i=1}^{N_D} z_D^i \varphi_i(t) = L\varphi_D \end{aligned}$$

Note that L is directly related to F via its columns but not to θ . Some rearrangement of the entries in the columns of F is needed to create ϕ_D . A simple example of the above is given by the following:

$$\begin{cases} \dot{x} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_D \\ u_D = a \sin(\omega_D t + b) \\ \quad = a_1 \sin(\omega_D t) + a_2 \cos(\omega_D t) \end{cases} \qquad (16)$$

Assume $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix} \right)$ is ASD. Also let the ***Matching Condition:***

$$R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1)) \qquad (17)$$

which says $\exists H_* \ni B_1^A(x_1) H_* = \Gamma_1(x_1)$.

Component #2 will represent all the rest of the evolving system and will be assumed to be strictly dissipative by choice of local controllers:

$$\begin{cases} \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) u_2 \\ y_2 = C_2(x_2) \end{cases} \qquad (18)$$

The Components are in Admittance-Impedance form so when they are joined $u_1 = -y_2$ and $u_2 = y_1$.

The Adaptive Key Component Controller with Disturbance Mitigation works through the control input-output ports (u_1^A, y_1^A) of Component #1:

$$\begin{cases} u_1^A = G_e y_1^A + G_D \phi_D \\ \dot{G}_e = -y_1^A (y_1^A)^T \gamma_e ; \gamma_e > 0 \\ \dot{G}_D = -y_1^A (\phi_D)^T \gamma_D ; \gamma_D > 0 \end{cases} \qquad (19)$$

This produces $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t \to \infty]{} 0$ with bounded adaptive gains (G_e, G_D) as the following convergence theorem shows:

Theorem 2: Assume that V_1 and V_2 are positive $\forall x \neq 0$ and radially unbounded, and $(A(x), B(x), C(x))$ are continuous functions of x and $S(x)$, above, is positive $\forall x \neq 0$ and has continuous partial derivatives in x. Furthermore, assume:

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1) The conditions of Theo.1 are satisfied; so that $\left(\begin{bmatrix} u_1 \\ u_1^A \end{bmatrix}, \begin{bmatrix} y_1 \\ y_1^A \end{bmatrix}\right)$ is **Almost Strictly Dissipative (ASD)**
2) The Matching Condition: $R(\Gamma_1(x_1)) \subseteq R(B_1^A(x_1))$
3) ϕ_D is bounded (or F has only simple imaginary poles and no right half-plane poles)

Then the adaptive Controller (6) produces

$$x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t\to\infty]{} 0$$

with bounded adaptive gains (G_e, G_D) when Component 1 is joined with Component 2 into an Evolved System and the outputs $y_i = C_i(x_i) \xrightarrow[t\to\infty]{} 0$.

Proof: See Appendix.

It should be noted that the above results might only hold on a neighborhood $N_i(0,r_i) \equiv \{x_i / \|x_i\| < r_i\}$. However, then the stability in Theo. 2 is only locally asymptotic to the origin.

VI. ILLUSTRATIVE EXAMPLE

Example 1, which follows, is a two component linear flexible structure Evolving System. The components of Example 1 are stable when they are unconnected components, but the Evolving System fails to inherit the stability of the components. This example will be used to demonstrate the inheritance and lack of inheritance of almost strict dissipativity in Evolving Systems.

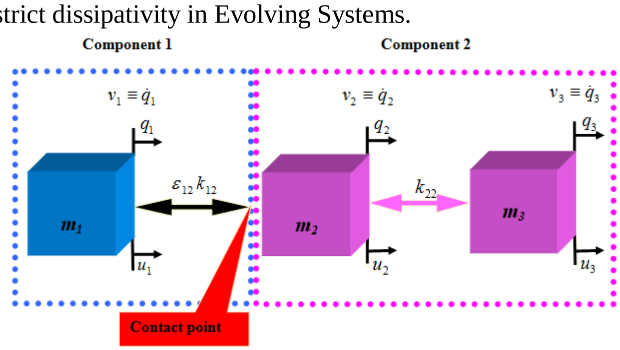

Figure 2. Example 1: A two component flexible structure Evolving System.

The dynamical equations for the components of Example 1 are:

$$\text{component 1: } \begin{cases} m_1\ddot{q}_1 = u_1 - \varepsilon_{12}k_{12}(q_1 - q_2) \\ y_1 = [q_1, \dot{q}_1]^{\mathrm{T}} \end{cases}$$
$$\text{component 2: } \begin{cases} m_2\ddot{q}_2 = u_2 - \varepsilon_{12}k_{12}(q_2 - q_1) \\ -k_{22}(q_2 - q_3) \\ m_3\ddot{q}_3 = u_3 - k_{22}(q_3 - q_2) \\ y_2 = [q_2, \dot{q}_2]^{\mathrm{T}} \\ y_3 = [q_3, \dot{q}_3]^{\mathrm{T}} \end{cases} \quad (20)$$

with $m_1 = 30$, $m_2 = 1$, $m_3 = 1$, $k_{12} = 4$, and $k_{22} = 1$. Example 1 has the following controllers:

$$\begin{cases} u_1 = -(0.9s + 0.1)q_1 \\ u_2 = -\left(\dfrac{0.1}{s} + 0.2s + 0.5\right)q_2 \\ u_3 = -(0.6s + 1)q_3 \end{cases} \quad (21)$$

When two components join to form an Evolved System, at their point of contact, their velocities are equal and the forces exerted are equal and opposite. If the two components are given by (f_1, v_1) and (f_2, v_2), then the contact dynamics of the Evolved System can be represented by:

$$\begin{cases} f_1 = -f_2 \\ v_1 \equiv \dot{q}_1 = v_2 \equiv \dot{q}_2 \end{cases} \quad (22)$$

This connection can be modeled as the admittance of one component connected in feedback with the impedance of the other component [1]-[3]. When we use this idea of the joining of two components of an Evolving System as the feedback connection of their admittance and impedance, we can apply Theorem 1 from above to determine whether almost strict dissipativty is inherited by the Evolved System.

The subsystem components from Example 1 are stable in closed-loop form when they are unconnected, i.e., $\varepsilon_{12} = 0$. When $\varepsilon_{12} = 1$, the system is fully evolved and it has a closed-loop eigenvalue at 0.17, resulting in an unstable Evolved System.

A Simulink model was created to implement an adaptive key component controller for Example 1 as described in the previous section. Simulations were run in which the connection parameter, ε_{12}, ranged from 0 to 1, allowing the system to go from unconnected components to a fully Evolved System. The key component controller was able to maintain system stability during the entire evolution process

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when it used the input-output ports on mass 1 of component 1, see Figure 3. When component 1 was the key component, $\left(\overline{A},\overline{B},\overline{C}\right)$ is ASPR.

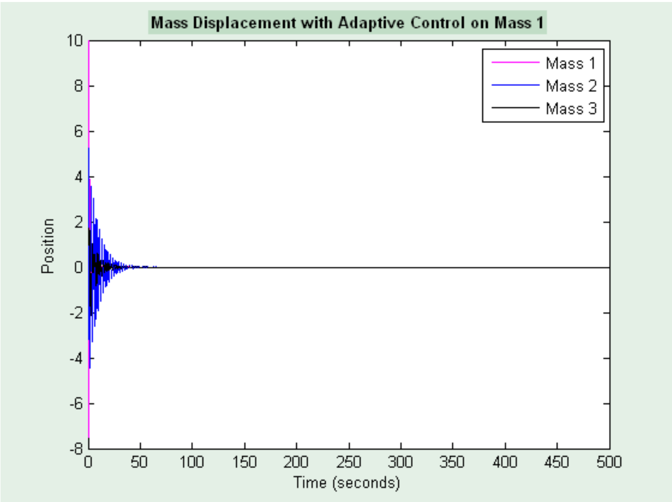

Figure 3. Adaptive key component controller on mass 1.

When the key component controller was located on component 2 and used the input-output ports on mass 3, stability was not maintained, see Figure 4. The adaptive key component controller was not able to restore stability on mass 3 because that system was not ASPR, i.e., it had nonminimum phase zeros at 0.00515±0.2009i.

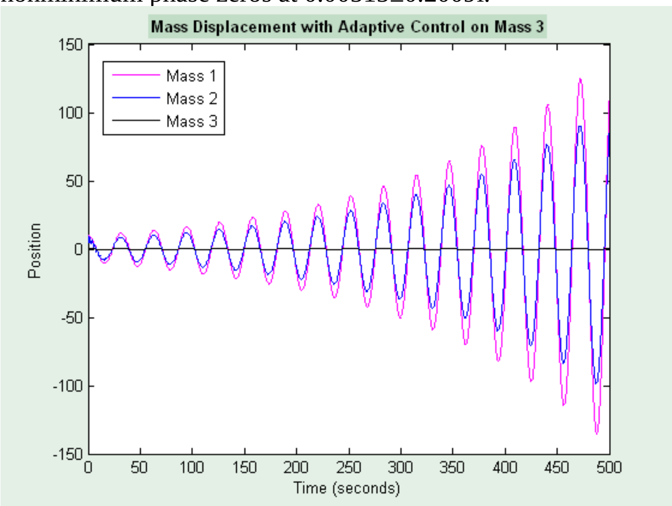

Figure 4. Adaptive key component controller on mass 3.

VII. CONCLUSION

We have presented a result (Theorem 1) describing when an Evolving System will inherit the almost strict dissipativity traits of its subsystem components. An example was given of successful inheritance of almost strict dissipativity and failed inheritance of almost strict dissipativity. This result allows a control system designer to determine a sufficient condition for an Evolving System to use an adaptive key component controller to restore stability. We also presented a convergence result (Theorem 2) for an adaptive key component controller to restore stability during evolution and mitigate persistent disturbances.

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Appendix

Proof of Theorem 1:

Let (u_i^A, y_i^A) be ASD. From (9) and (11),

$\exists G_i^*$ such that

$$\begin{cases} \nabla V_i A_i^C(x_i) \equiv \nabla V_i [A_i(x_i) + B_i^A(x_i) G_i^* C_i^A(x_i)] \\ \qquad \le -S_i(x_i) + \varepsilon_{ij} \nabla V_i A_{ij}(x_i, u_i, u_i^A) \\ \nabla V_i B_i^A(x_i) = C_i^A(x_i)^T \end{cases} \quad \text{(A.1)}$$

If we connect (u_1, y_1) in feedback with (u_2, y_2), then $y_1 = u_2$ and $u_1 = -y_2$ and, use (12) and (13), then we have $\nabla V_1 A_{12}(x_1, u_1, u_1^A) = \nabla V_1 B_1(x_1) u_1 = C_1^T(x_1)[-y_2] = -y_1^T y_2$ and similarly, $\nabla V_2 A_{21}(x_2, u_2, u_2^A) = y_2^T y_1$.

Let $x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ and, from (12),

$$\begin{cases} \dot{x} = A(x) + B(x)u \\ \quad = \begin{bmatrix} A_1^C(x_1) + \varepsilon_{12} A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon_{21} A_{21}(x_1) \end{bmatrix} \\ \quad + \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \\ y = \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} = C(x) = \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix} \end{cases} \quad \text{(A.2)}$$

with $V = V_1 + V_2$, using (13) and $\varepsilon_{ji} = \varepsilon_{ij} \equiv \varepsilon$ from (3),

$$\begin{aligned} \nabla V A(x) &= [\nabla V_1 \quad \nabla V_2] \begin{bmatrix} A_1^C(x_1) + \varepsilon A_{12}(x_2) \\ A_2^C(x_2) + \varepsilon A_{21}(x_1) \end{bmatrix} \\ &= \nabla V_1 A_1(x_1) + \varepsilon(-y_1^T y_2) + \nabla V_2 A_2(x_2) + \varepsilon(y_2^T y_1) \\ &\le -[S_1(x_1) + S_2(x_2)] + \varepsilon(-y_1^T y_2) + \varepsilon(y_2^T y_1) \\ &= -S(x) \end{aligned}$$

and

$$\begin{aligned} \nabla V B(x) &= [\nabla V_1 \ \nabla V_2] \begin{bmatrix} B_1^A(x_1) & 0 \\ 0 & B_2^A(x_2) \end{bmatrix} \\ &= \begin{bmatrix} C_1^A(x_1) \\ C_2^A(x_2) \end{bmatrix}^T = C^T(x) \end{aligned}$$

Therefore $\left(u_A \equiv \begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix}, y_A \equiv \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} \right)$ is ASD with output feedback $\begin{bmatrix} u_1^A \\ u_2^A \end{bmatrix} \equiv \begin{bmatrix} G_1^* & 0 \\ 0 & G_2^* \end{bmatrix} \begin{bmatrix} y_1^A \\ y_2^A \end{bmatrix} + \begin{bmatrix} u_1^{Ar} \\ u_2^{Ar} \end{bmatrix}$ as desired. #

Proof of Theorem 2:

Since the physical connection of Component 1 to Component 2 is equivalent to the feedback connection

$$u_1 = -y_2 \text{ and } u_2 = y_2,$$

By Theo.1 we have that the closed-loop system (u_1^A, y_1^A) below is ASD:

$$\begin{cases} \dot{x}_1 = A_1(x_1) - \varepsilon B_1(x_1) C_2(x_2) + B_1^A(x_1) u_1^A \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) C_1(x_1); 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \quad \text{(A.3)}$$

Rewrite (19),

$$\begin{cases} u_1^A = G_e y_1^A + G_D \phi_D = G_e^* y_1^A + G_D^* \phi_D + \underbrace{\Delta G \eta}_{w}; \\ \Delta G \equiv G - G^* = [\Delta G_e \quad \Delta G_D]; \eta \equiv \begin{bmatrix} y_1^A \\ \phi_D \end{bmatrix} \\ \Delta \dot{G} = \dot{G} = -y_1^A (y_1^A)^T \gamma; \gamma \equiv \begin{bmatrix} \gamma_e & 0 \\ 0 & \gamma_D \end{bmatrix} > 0 \end{cases} \quad \text{(A.4)}$$

Combining (A.3) and (A.4) yields:

$$\begin{cases} \dot{x}_1 = A_1^C(x_1) - \varepsilon B_1(x_1) C_2(x_2) \\ +[B_1^A(x_1) G_D^* + \Gamma_1(x_1) \theta L] \varphi_D + B_1^A(x_1) w \\ \qquad \text{with } w \equiv \Delta G \eta \\ \ \text{and } G_D^* \equiv -H_* \theta L \text{ from(19)} \\ \qquad \text{and } A_1^C(x_1) \equiv A_i(x_i) + B_i^A(x_i) G_i^* C_i^A(x_i) \\ \dot{x}_2 = A_2(x_2) + \varepsilon B_2(x_2) C_1(x_1); \ 0 \le \varepsilon \le 1 \\ y_1^A = C_1^A(x_1) \end{cases} \quad \text{(A.5)}$$

Let $V = V_1 + V_2$ and we have:

$$\dot{V} = -S(x) + \left\langle y_1^A, w \right\rangle \quad \text{(A.6)}$$

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Form $V_G \equiv \dfrac{1}{2} tr(\Delta G \gamma^{-1} \Delta G^T)$ and obtain from (A.3):

$$\begin{aligned} \dot{V}_G &\equiv tr(\Delta \dot{G} \gamma^{-1} \Delta G^T) \\ &= -tr(y_1^A (y_1^A)^T \Delta G^T) \\ &= -tr(y_1^A (w)^T) \\ &= -\langle y_1^A, w \rangle \end{aligned} \tag{A.7}$$

Define: $V(x, \Delta G) \equiv V(x) + V_G(\Delta G)$ and, from (A.5) and (26), we have:

$$\begin{aligned} \dot{V}(x, \Delta G) &\equiv \dot{V}(x) + \dot{V}_G(\Delta G) \\ &= -S(x) + \langle y_1^A, w \rangle - \langle y_1^A, w \rangle \\ &= -S(x) \le 0 \end{aligned} \tag{A.8}$$

This guarantees that all trajectories $(x, \Delta G)$ are bounded. If $\dot{V}(x, \Delta G)$ is uniformly continuous or $\ddot{V}(x, \Delta G)$ is bounded, then Barbalat's Lemma [10] yields:

$$S(x) \xrightarrow[t \to \infty]{} 0,$$

and the positivity and continuity of $S(x)$ imply that

$$x \equiv \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \xrightarrow[t \to \infty]{} 0.$$

Consider

$$\begin{aligned} \ddot{V}(x, \Delta G) &= -\dot{S}(x) \\ &\le \left| \dot{S}(x) \right| \\ &= \left| \frac{\partial S(x)}{\partial x} \dot{x} \right| \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left\| \dot{x} \right\| \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\left\| A(x) \right\| + \left\| B(x) \right\| \left\| w_1^A \right\| \right] \\ &\le \left\| \frac{\partial S(x)}{\partial x} \right\| \left[\left\| A(x) \right\| + \left\| B(x) \right\| \left\| \Delta G \right\| \left\| C_1^A(x_1) \right\| \right] \end{aligned}$$

which is bounded because $(x, \Delta G)$ is bounded, $S(x)$ has continuous partial derivatives and $\left(A(x), B(x), C(x)\right)$ are continuous, and a continuous function of bounded $x(t)$ is also bounded in *t*.

So, $y_i = C_i(x_i) \xrightarrow[t \to \infty]{} 0$ because $C_i(x_i)$ is continuous.#

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