Moment Generating Function Based Calculation of Average Bit Error Probability in an α-μ Fading Environment with Selection Diversity Receiver

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***Abstract*—In this paper, a wireless system in the presence of α-μ fading is analyzed. Also, there is Co-Channel Interference (CCI), i.e., crosstalk from two different radio transmitters using the same frequency channel. The CCI has the same distribution as the fading in the observed environment. To mitigate these adverse effects, a diversity receiver with Selection Combining (SC) is used. For such wireless system configuration, we calculated the Average Bit Error Probability (ABEP) based on the Moment Generating Function (MGF). Analytical results are presented graphically in order to highlight the influence of fading and CCI parameters.**

Keywords- α-μ fading; Average Bit Error Probability (ABEP); Co-Channel Interference (CCI); Moment Generating Function (MGF); Selection Combining (SC).

I. INTRODUCTION

One of the most critical disruptions to signal propagation in wireless channels is fading. A fading channel is a wireless communication channel that experiences fading [1]. Fading is modeled as a random variable with a certain statistical distribution. It is very important that this distribution describes the conditions in the wireless channel as closely as possible.

Recently, a lot of work has been done on the research of different distributions that satisfy these conditions. Thus, the α-μ distribution is introduced as a small-scale fading model. This model includes the nonlinearity of the propagation medium since the assumption of a homogeneous diffuse scattering field is only an approximation [2]. Actually, this is generalized Gamma distribution that encompasses some other distributions. Among them are: Gamma, with Erlang as its discrete versions and central Chi-squared, then Nakagami-*m* with its discrete version- Chi distribution, and also Weibull, Rayleigh, exponential, and One-sided Gaussian distributions. Therefore, α-μ distribution is suitable for a comprehensive analysis of the performance of wireless systems in the presence of the listed types of fading by reducing to special cases for certain values of parameters α and μ.

In addition to [2], there are not many papers in the literature considering this type of fading, despite its convenience. Nevertheless, we will mention some of them [3]-[8]. The authors in [3] evaluated the Moment Generating Function (MGF) for the Probability Density Function (PDF) that describes the α-μ wireless fading channel. The derived expression for MGF was utilized in obtaining the Bit Error Rate (BER) for different modulation techniques over this channel. An expression for the outage probability was also derived in the closed form. Both expressions can be reduced as special cases to those earlier obtained in the literature for known fading channel distributions such as Rayleigh, Nakagami-*m*, and Weibull.

The same authors worked with this fading even further, and in [4] they derived expressions for the amount of fading and the average channel capacity for this channel model. They confirmed once again the generality of this fading model as they reduced the expressions for the obtained performance metrics to other expressions for other channel models as special cases.

In [5], the authors proposed a novel MGF for α-μ fading distribution valid for all values of parameter α, as modification of the MGF from [3], where the MGF was valid only for non-integer values of α. Then, the closed-form expressions of BER for different modulation techniques such as Binary Phase-Shift Keying (BPSK), Binary Frequency Shift Keying (BFSK), Differential Quadrature PSF (DQPSK), Binary Differential PSK (BDPSK), and *M*-ary PSK (MPSK) over α-μ fading channels are determined.

In [6], the exact PDF of the square ratio of two multivariate exponentially correlated α-μ distributed variables is derived. Then, the expressions in the closed form are determined for the Cumulative Distribution Function (CDF) and PDF of the maximal and minimal square ratio of two multivariate exponentially correlated variables. These formulae are the base for Signal-to-Interference Ratio (SIR) based analysis of Selection Combining (SC) receiver through communication systems.

An enriched α − μ distribution which may act as fading model is analyzed in [7]. The complex α-μ fading channel is observed in [8] with an Orthogonal Frequency-Division Multiplexing (OFDM) application.

The Co-Channel Interference (CCI) can occur in wireless systems beside fading. The most common reasons causing CCI are: bad weather condition and bad frequency planning. Its influence must be studied along with the influence of fading. In this paper, we performed a MGF-based calculation of the Average Bit Error Probability (ABEP) in an α-μ fading and CCI environment when SC diversity receiver was used to mitigate the influences of these disturbances. As far as we know, the derivation the MGF for the defined scenario has not been reported in the open literature.

Here, we choose discuss MGFs since, beside other reasons, they are useful in analysis of sums of Random Variables (RVs). Namely, the MGF of RV gives all moments of this RV, which is a fact that gives the name to the moment generating function. Second, the MGF (if it exists) uniquely determines the distribution. This means, if two RVs have the same MGF, then they must have the same distribution. Thus, if the MGF of an RV is found, its distribution is determined.

Our paper consists of four sections. After the introduction, in Section II, the SIR-based performance analysis at the output of SC receiver in the presence of α-μ fading and CCI is presented. In Section III, some graphs highlighting the parameters influence are plotted. At the end, a conclusion part is given. Finally, we conclude the paper in Section IV.

II. PERFORMANCE ANALYSIS BASED ON THE SIR AT THE OUTPUT OF THE RECEIVER

In this section, we will derive the performance of a wireless system in the presence of α-μ fading and CCI. To mitigate the effects of fading and CCI, a SC diversity receiver with L branches, shown in Figure 1, is used. The SC receiver transmits to the user the signal from the input antenna whose value is the highest.

The input signals are x_i, i=1, 2, …, L; $L \geq 2$. The output signal is x. The CCI envelopes at the input are y_i, i=1, 2, …, L with output value y. We will derive the Signal-to-CCI ratio (SIR)-based system performance.

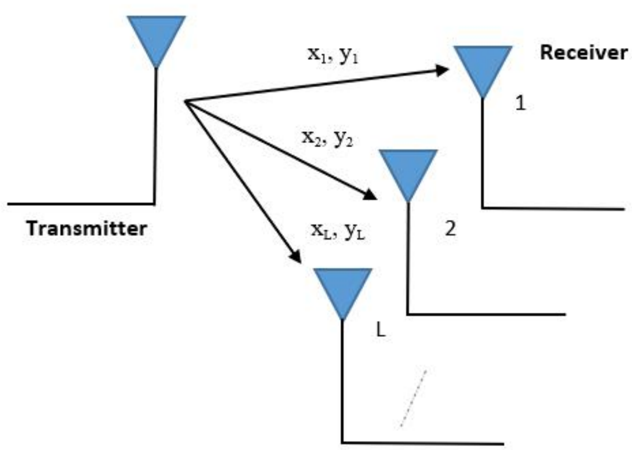

Figure 1. Model of a selection combining diversity receiver.

So, the input ratios of the useful signals and the CCIs are z_i.=x_i/ y_i, and the output SIR is denoted by z.

A. The PDF of the Output SIR

As mentioned, the transmitted signal has the α-μ distribution [2]:

$$p_{x_i}\left(x_i\right)=\frac{\alpha\mu_1^{\mu_1}x_i^{\alpha\mu_1-1}}{\Omega_i^{\mu_1}\Gamma\left(\mu_1\right)}\mathrm{e}^{-\mu_1\frac{x_i^{\alpha}}{\Omega_i}}, \tag{1}$$

where parameter α characterizes the nonlinearity of the propagation environment; parameter μ_j shows the number of clusters in that environment, where j=1 for the signal, and j=2 for the CCI; Ω_i, i=1, 2, …, L, represents the mean values of the input signals powers, and $\Gamma(\cdot)$ marks the Gamma function.

The CCI is also under α-μ distribution:

$$p_{y_i}\left(y_i\right)=\frac{\alpha\mu_2^{\mu_2}y_i^{\alpha\mu_2-1}}{s_i^{\mu_2}\Gamma\left(\mu_2\right)}\mathrm{e}^{-\mu_2\frac{y_i^{\alpha}}{s_i}}, \tag{2}$$

where s_i are the average powers of the CCI.

The PDFs of the SIRs z_i are mathematically given as [9]:

$$p_{z_i}\left(z_i\right)=\int_0^{\infty}y_i p_{x_i}\left(z_i y_i\right)p_{y_i}\left(y_i\right)dy_i, \tag{3}$$

Substituting (1) and (2) into (3), we can obtain the PDFs for SIRs as:

$$p_{z_i}\left(z_i\right)=\frac{\alpha\mu_1^{\mu_1}\mu_2^{\mu_2}z_i^{\alpha\mu_1-1}\Omega_i^{\mu_2}s_i^{\mu_1}\Gamma\left(\mu_1+\mu_2\right)}{\Gamma\left(\mu_2\right)\Gamma\left(\mu_1\right)\left(\Omega_i\mu_2+\mu_1 s_i z_i^{\alpha}\right)^{\mu_1+\mu_2}}. \tag{4}$$

Let us derive the expression for CDF of z_i from the definition given in [9]:

$$F_{z_i}\left(z_i\right)=\int_0^{z_i}p_{z_i}\left(t\right)dt. \tag{5}$$

Further, we can obtain the CDF of the SIR z_i after substituting (4) into (5):

$$F_{z_i}\left(z_i\right)=\frac{\alpha\left(\mu_1 s_i\right)^{\mu_1}\left(\mu_2\Omega_i\right)^{\mu_2}}{\Gamma\left(\mu_2\right)}\cdot\frac{\Gamma\left(\mu_1+\mu_2\right)}{\Gamma\left(\mu_1\right)}\times$$
$$\times\int_0^{z_i}\frac{z_i^{\alpha\mu_1-1}}{\left(\Omega_i\mu_2+\mu_1 s_i z_i^{\alpha}\right)^{\mu_1+\mu_2}}dt. \tag{6}$$

The integral appearing in (6) will be solved using Beta function [10], as presented below:

$$\int_0^{\lambda}\frac{x^m}{\left(a+bx^n\right)^p}dx=\frac{a^{-p}}{n}\left(\frac{a}{b}\right)^{\frac{m+1}{n}}B_z\left(\frac{m+1}{n},p-\frac{m+1}{n}\right)$$
$$z=\frac{b\lambda^n}{a+b\lambda^n}, a>0, b>0, n>0, 0<\frac{m+1}{n}<p. \tag{7}$$

Now, the CDF of z_i is obtained in the form:

$$F_{z_i}(z_i)=\frac{\Gamma(\mu_1+\mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)}B_{\frac{\mu_1 s_i z_i^\alpha}{\Omega_i\mu_2+\mu_1 s_i z_i^\alpha}}(\mu_1,\mu_2)\ . \tag{8}$$

The incomplete Beta function from (6) will be represented by [10, eq.8.391]:

$$B_x(p,q)=\int_0^x t^{p-1}(1-t)^{q-1}dt=\frac{x^p}{p}\,{}_2F_1(p,1-q;p+1;x)=$$

$$=\frac{x^p}{p}\,{}_2F_1(a,b;c;z)=\frac{x^p}{p}\sum_{j=0}^{\infty}\frac{a_j b_j}{c_j}\frac{z^j}{j!}\ , \tag{9}$$

where ${}_2F_1$ is the hyper geometric function of the second order. By some substitutions, the CDF can be written in the following form:

$$F_{z_i}(z_i)=\frac{\Gamma(\mu_1+\mu_2)}{\mu_1\Gamma(\mu_2)\Gamma(\mu_1)}\sum_{j=0}^{+\infty}\frac{(\mu_1)_j}{j!}\cdot$$

$$\cdot\frac{(1-\mu_2)_j}{(\mu_1+1)_j}\left(\frac{\mu_1 s_i z_i^\alpha}{\Omega_i\mu_2+\mu_1 s_i z_i^\alpha}\right)^{j+\mu_1}. \tag{10}$$

In SC, the received signal from the antenna that experiences the highest SIR (i.e., the strongest signal from L received signals) is chosen to be processed in the receiver (see Figure 1). Thus, the SIR z at the output of SC receiver with L branches is the maximum SIR of all the received signals:

$$z=\max(z_1,z_2,...,z_L). \tag{11}$$

The PDF of the SIR z from SC receiver output is calculated using the formula [11]:

$$p_{z_i}(z)=Lp_{z_i}(z_i)\left(F_{z_i}(z_i)\right)^{L-1}. \tag{12}$$

By substitutions of (4) and (10) into (12), the PDF of the output SIR z becomes:

$$p_{z_i}(z)=\frac{L\alpha\mu_2^{\mu_2}}{\mu_1^{L-\mu_1-1}}\cdot\frac{z_i^{\alpha\mu_1-1}\Omega_i^{\mu_2}s_i^{\mu_1}}{\left(\Omega_i\mu_2+\mu_1 s_i z_i^\alpha\right)^{\mu_1+\mu_2}}\left(\frac{\Gamma(\mu_1+\mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)}\right)^L\times$$

$$\times\left(\sum_{j=0}^{+\infty}\frac{(\mu_1)_j(1-\mu_2)_j}{(\mu_1+1)_j\,j!}\left(\frac{\mu_1 s_i z_i^\alpha}{\Omega_i\mu_2+\mu_1 s_i z_i^\alpha}\right)^{j+\mu_1}\right)^{L-1}. \tag{13}$$

B. Moment Generating Function

The complicated PDF expression often limits the evaluation of performance measures of generalized fading channel models. The MGF is an important statistical function for each distribution. In the theory of probability and statistics, the MGF of a real RV is an alternate feature of its PDF.

The main formula for derivation the MGF is [12, eq. (6)]:

$$M_z(h)=\overline{e^{zh}}=\int_0^\infty \mathrm{e}^{-zh}p_{z_i}(z)dz \tag{14}$$

By applying (4) into formula (14) for MGF, we obtain for our case:

$$M_z(h)=\frac{L\alpha\mu_2^{\mu_2}\Omega_i^{\mu_2}}{\mu_1^{L-1}\mu_1^{\mu_2}s_i^{\mu_2}}\cdot\left(\frac{\Gamma(\mu_1+\mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)}\right)^L\times$$

$$\times\left(\sum_{j=0}^{+\infty}\frac{(\mu_1)_j(1-\mu_2)_j}{(\mu_1+1)_j\,j!}\right)^{L-1}\times$$

$$\times\int_0^\infty\frac{z_i^{2\frac{\alpha jL-\alpha j+\alpha\mu_1 L}{2}-1}\mathrm{e}^{-hz}}{\left(\left(\sqrt{\frac{\mu_2\Omega_i}{\mu_1 s_i}}\right)^2+\left(z_i^{\frac{\alpha}{2}}\right)^2\right)^{1-(j-jL-\mu_1 L-\mu_2+1)}}dz\ . \tag{15}$$

By using [10; 3.389]:

$$\int_0^\infty\frac{x^{2v-1}e^{-\mu x}}{\left(u^2+x^2\right)^{1-q}}dx=\frac{u^{2v+2q-2}}{2\sqrt{\pi}\,\Gamma(1-q)}G_{13}^{31}\left(\frac{\mu^2u^2}{4}\middle|\begin{matrix}1-v\\1-q-v,0,\frac{1}{2}\end{matrix}\right), \tag{16}$$

where G[·] is the Meijer's G-function [10; 9.301], and by replacing form of (16) into (15), the MGF for output SIR z becomes:

$$M_z(h)=\frac{L\alpha}{2\sqrt{\pi}\mu_1^{L-1}}\left(\frac{\Gamma(\mu_1+\mu_2)}{\Gamma(\mu_2)\Gamma(\mu_1)}\right)^L\left(\frac{\mu_2\Omega_i}{\mu_1 s_i}\right)^{\frac{\mu_1 L(\alpha-2)}{2}}\cdot$$

$$\times\left(\sum_{j=0}^{+\infty}\frac{(\mu_1)_j(1-\mu_2)_j}{(\mu_1+1)_j\,j!}\left(\frac{\mu_2\Omega_i}{\mu_1 s_i}\right)^{\frac{j(\alpha-2)}{2}}\right)^{L-1}\times$$

$$\times\frac{1}{\Gamma(jL-j+\mu_1 L+\mu_2)}\times$$

$$\times G_{13}^{31}\left(\frac{h^2\mu_2\Omega_i}{4\mu_1 s_i}\middle|\begin{matrix}1-\left(\frac{\alpha jL-\alpha j+\alpha\mu_1 L}{2}\right)\\\left(\frac{(2-\alpha)(jL-j+\mu_1 L)+2\mu_2}{2}\right),0,\frac{1}{2}\end{matrix}\right). \tag{17}$$

C. Average Bit Error Probability

The ABEP is among the performance that best describe the nature of the system's behavior, and that is why it is most often used to describe that behavior. So, determining the ABEP in the simplest possible way is of great importance. In reality, the difficulty in evaluating the ABEP is that the conditional BEP is a nonlinear function of the SNR or SIR. The nonlinearity is a consequence of the modulation/detection scheme. This is the reason for considering the MGF-based approach to determine ABEP.

Using the MGF-based approach, the ABEP for two modulations will be determined very efficiently, without numerical integrations. By utilizing the expression for MGF from (17), the ABEP for non-coherent BFSK and BDPSK modulations will be [1]:

$$P_{be}(\Omega_0)=0.5M_z(0.5)\,,\quad \text{for BFSK,} \tag{18}$$

$$P_{be}(\Omega_0)=0.5M_z(1)\,,\quad \text{for BDPSK.} \tag{19}$$

III. PERFORMANCE RESULTS

To examine the influence of fading and CCI severity on the ABEP, numerically obtained results are illustrated in the next two subsections. In the first subsection, the case of BFSK modulation is observed, and in the second one, the case of BDPSK modulation is shown. We used the programs Origin and Mathematica to plot graphs.

A. *Binary Frequency Shift Keying Modulation*

In Figures 2 and 3, we present the curves for ABEP versus SIR *w*= Ω/*s* at the output of the SC receiver with *L* branches, in the case when BFSK modulation was used. In these figures, we changed the values for one group of parameters, and kept the values for the other parameters.

So, in Figure 2, the ABEP is presented for BFSK modulation and dual branch SC receiver (*L*=2), with μ_2=1, while parameters α and μ_1 are changing. One can see from Figure 2 that the ABEP grows with an increasing in parameter α. This means that system performance gets worse. When the parameter μ_1 increases, the ABEP decreases and the system has better performance.

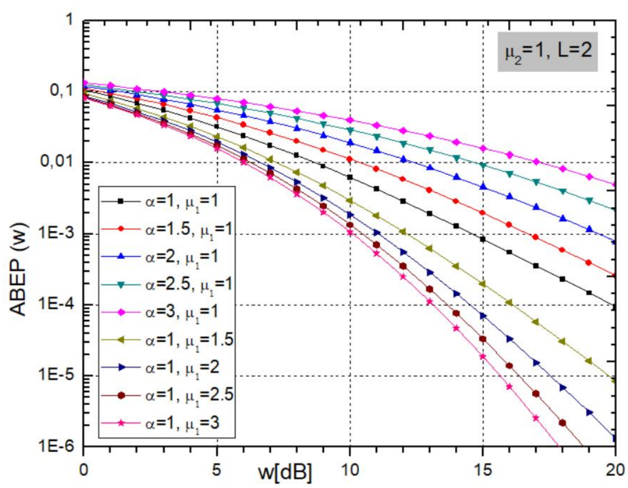

Figure 2. ABEP versus SIR for BFSK modulation when parameters α and μ_1 are changing.

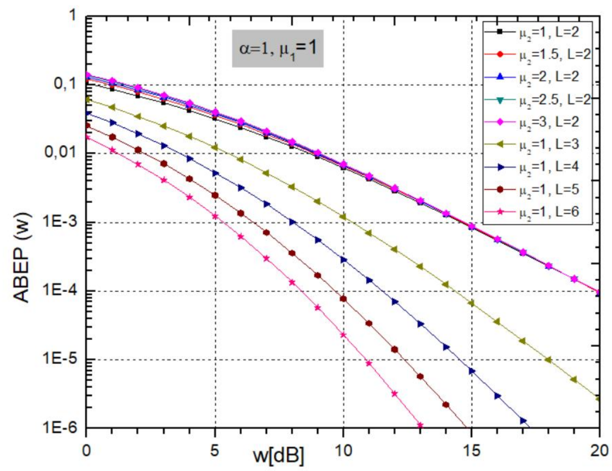

Figure 3. ABEP versus SIR for BFSK modulation with variable parameters μ_2 and *L*.

In Figure 3, the ABEP is plotted versus SIR for BFSK modulation with variable parameters μ_2 and *L*. In this figure, the parameters whose values are retained are: α=1, and μ_1=1, while parameters *L* and μ_2 are changing. From Figure 3, we can see that the increase in the parameter μ_2 has no effect on the ABEP, while with the increase in the number of branches *L*, the ABEP decreases significantly and the system has better performance.

B. *Binary Differential Phase-Shift Keying Modulation*

In Figures 4 and 5, the curves for ABEP versus SIR at the output of SC receiver with *L* branches are presented for BDPSK modulation. Figure 4 shows graphs for dual branch SC receiver (*L*=2). In this figure, μ_2=1 and parameters α and μ_1 took several values.

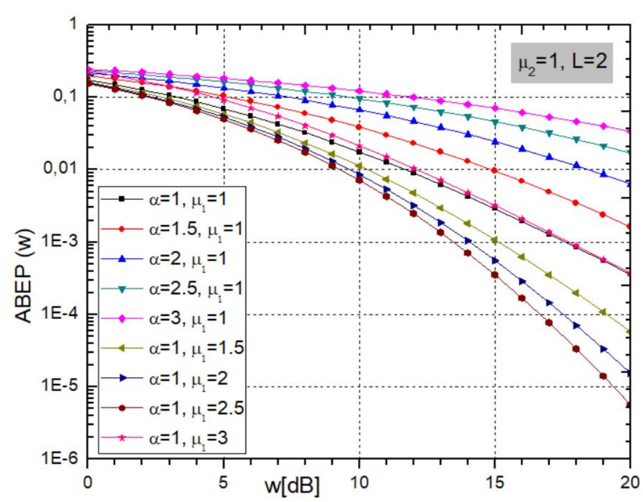

Figure 4. ABEP versus SIR for BDPSK modulation when parameters α and μ_1 are changing.

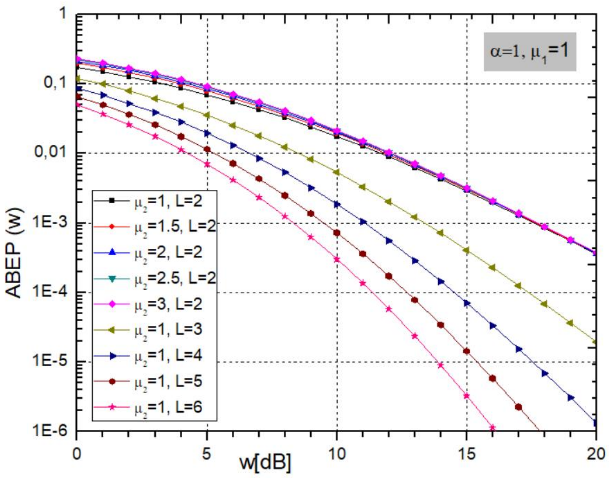

Figure 5. ABEP versus SIR for BDPSK modulation and variable parameters μ_2 and L.

It is visible from this figure that the ABEP grows when the parameter α increases. This spoils the system performance. When the parameter μ_1 increases, the ABEP decreases, improving the system performance.

In Figure 5, the ABEP is presented versus SIR for the BDPSK modulation with parameters μ_2 and L that are changing. In this figure, the parameters whose values are retained are: α=1, μ_1=1. From Figure 5, one can conclude that the increase in the parameter μ_2 is without significant impact on the ABEP, while with the increase in the number of branches L, the ABEP drops significantly. This leads to an improvement of the system performance and is in accordance with theoretical assumptions.

Comparing the results from these two figures, we can conclude that the system has better performance with smaller ABEP in the case of using BDPSK than BFSK modulation.

IV. CONCLUSION

In this work, a wireless system in an α-μ fading and CCI environment is analyzed. To improve the system performance by mitigating these harmful effects of CCI interference, SC diversity receiver is employed. The MGF-based ABEP is obtained for BFSK and BDPSK modulation types. The analytical results are presented graphically and the influence of parameters of fading and CCI is highlighted. Based on them, we concluded that in α-μ environments, it is more advantageous to use BDPSK than BFSK modulation.

One of the main contributions of our paper is that it can be used to determine the ABEP of wireless systems in the presence of other types of fading, namely for Rayleigh, Nakagami-*m*, Weibull, and One-sided Gaussian, in the presence of CCI, by setting certain values for parameters α and μ.

In our future work, we will consider correlated α-μ channels, since correlation between the faded channels also affects the PDF of SIR at the output of the SC or some other diversity system.

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