A Data-Reuse Approach for the RLS-DCD Algorithm

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***Abstract*—The mitigation of input signal correlation is one of the main advantages associated with the Recursive Least-Squares (RLS) algorithms. This paper proposes a low-complexity RLS adaptive algorithm based on the Dichotomous Coordinate Descent (DCD) iterations, with a Data-Reuse (DR) approach. In this way, the corresponding convergence speeds in tracking and low signal-to-noise scenarios are improved, with overall attractive costs in terms of chip areas for hardware implementations.**

***Index Terms*—adaptive algorithms; Data-Reuse (DR); low Signal-to-Noise Ratio; Recursive Least-Squares (RLS); Dichotomous Coordinate Descent (DCD); tracking.**

I. INTRODUCTION

In recent times, considerable research efforts were concentrated on improving convergence rates and tracking capabilities for practical implementations of adaptive systems. The Recursive Least-Squares (RLS) family of adaptive algorithms is a good way to accomplish this goal, unfortunately, with the cost of an increased arithmetic workload and numerical stability issues. The current industry workhorse is the Least-Mean-Square (LMS) algorithm [1], which has poor results when working with correlated input signals. To overcome these impediments, two versions of the RLS method have been previously introduced: the RLS adaptive algorithm combined with the dichotomous coordinate descent iterations (RLS-DCD) [2], [3], respectively the RLS adaptive algorithm based on the data-reuse approach (DR-RLS) [4].

The RLS-DCD has been designed to match the performance of classical RLS versions, and also to avoid the necessity of handling prohibitive amounts of arithmetic operations (usually, proportional to the square of the filter's length or even more complex) associated with the computation of the inverse correlation matrix. The usage of the DCD iterations exchanges the correlation matrix inversion with a solution based only on additions and bit-shifts, corresponding to an auxiliary system of equations, which exploits the statistical properties of the input signal [2]. Thus, hardware costs become appealing for practical applications.

The DR-RLS adaptive algorithm improves performance of the RLS method in tracking scenarios [4]. Along with the forgetting factor, the DR parameter can be used to compromise between convergence speeds and filter accuracy at steady-state, with just a minimal increase in terms of complexity (proportional to the filter's length).

In this paper, we analyze a new version of the RLS adaptive algorithm, based on the combination between the DCD iterations and the DR principles, and we study the corresponding performances in scenarios with tracking, respectively low Signal-to-Noise Ratio (SNR) conditions. Section II introduces the system model, which is employed in Section III to describe the proposed algorithm. Simulation results are discussed in Section IV, and conclusions are drawn in Section V.

II. SYSTEM MODEL

Starting with the estimated impulse response $\widehat{\mathbf{g}}(n)$ (of length L), we define the $a\ priori$ error signal as:

$$e(n) = d(n) - \widehat{y}(n) = d(n) - \widehat{\mathbf{g}}^T(n-1)\mathbf{x}(n), \quad (1)$$

where $\widehat{y}(n)$ is the output signal estimate obtained using the adaptive filter coefficients, $\mathbf{x}(n)$ is the $L\times 1$ input signal vector, and $d(n)$ represents the desired (or *reference*) signal [1].

The minimization of the cost function [1], [5], with respect to $\widehat{\mathbf{g}}(n)$, leads to the set of normal equations

$$\mathbf{R}(n)\widehat{\mathbf{g}}(n) = \mathbf{p}(n) = \lambda\mathbf{p}(n-1) + \mathbf{x}(n)d(n), \quad (2)$$

where λ is the forgetting factor, $\mathbf{R}(n)$ expresses the $L \times L$ correlation matrix, and $\mathbf{p}(n)$ represents the cross-correlation vector between $\mathbf{x}(n)$ and the reference signal $d(n)$.

When working with the RLS-DCD algorithm, we write the *residual vector* using the solution provided at previous time index of the filter:

$$\mathbf{r}(n-1) = \mathbf{p}(n-1) - \mathbf{R}(n-1)\widehat{\mathbf{g}}(n-1). \quad (3)$$

By using the DCD method, we aim to reduce the complexity of updating the filter coefficients through the estimation of the *solution* vector $\Delta\mathbf{g}(n)$, and adding it to the previous filter set of coefficients, such that $\widehat{\mathbf{g}}(n) = \widehat{\mathbf{g}}(n-1) + \Delta\widehat{\mathbf{g}}(n)$ [2], [3].

Consequently, the RLS-DCD solves the auxiliary set of normal equations:

$$\mathbf{R}(n)\Delta\mathbf{g}(n) = \mathbf{p}(n) - \mathbf{R}(n)\widehat{\mathbf{g}}(n-1) \overset{not}{=} \mathbf{p}_0(n), \quad (4)$$

and the computation of a direct solution is avoided [3].

After some calculations, the residual vector can be expressed as:

$$\mathbf{r}(n) = \mathbf{p}_0(n) - \mathbf{R}(n)\Delta\widehat{\mathbf{g}}(n). \quad (5)$$

Finally, we obtain:

$$\mathbf{p}_0(n) = \lambda\mathbf{r}(n-1) + e(n)\mathbf{x}(n) \overset{not}{=} \lambda\mathbf{r}(n-1) + \mathbf{r}_{e,x}(n). \quad (6)$$

The DCD method has two empiric roles for (4): it estimates the solution vector $\Delta\widehat{\mathbf{g}}(n)$ and it updates the residual values associated with the vectors $\mathbf{p}_0(n)$, respectively $\mathbf{r}(n)$.

III. DR-RLS-DCD Algorithm

We propose to apply the DR method [4] over the RLS-DCD adaptive algorithm. The goal is to employ N_{it} updates for $\widehat{\mathbf{g}}(n-1)$ in order to obtain $\widehat{\mathbf{g}}(n)$. Firstly, the error signal can be written in a recursive way [4]:

$$e_k(n) = \begin{cases} d(n) - \widehat{\mathbf{g}}^T(n-1)\mathbf{x}(n) = e_0(n), & k = 0 \\ e_{k-1}(n) - \Delta\widehat{\mathbf{g}}_{k-1}^T(n)\mathbf{x}(n) & k \geq 1 \end{cases} \quad (7)$$

where $k = 0 \ldots N_{it} - 1$ represents the current step. It is obvious that for $N_{it} = 1$ the algorithm is equivalent to the RLS-DCD approach.

Since we have used the DR method to update the error signal, we need to adapt the update of the residual vector in a similar manner [4]:

$$\mathbf{r}_k(n) = \begin{cases} \lambda\mathbf{r}_{N_{it}-1}(n-1) + \mathbf{r}_{e,x,0}(n), & k = 0 \\ \mathbf{r}_{k-1}(n) + \mathbf{r}_{e,x,k}(n), & k \geq 1, \end{cases} \quad (8)$$

where $\mathbf{r}_{e,x,k}(n) = e_k(n)\mathbf{x}(n)$.

Considering the worst case scenario, the newly introduced DR-RLS-DCD method supposes more $(N_{it} - 1)N_u L$ additions, with respect to the RLS-DCD approach. However, in real applications, since $N_u \ll L$ and $N_{it} \ll L$, the global complexity of this new adaptive algorithm is still proportional to the filter's length multiplied by a small factor. The performances of the algorithm, in terms of tracking capabilities/convergence rate, are improving with the increase of the DR control parameter N_{it}. The sequential behavior of the DR-RLS-DCD adaptive algorithm is presented in Table I.

IV. Simulation Results

We used as an input signal a Gaussian noise with the length of 280000 samples and SNR = 20 dB filtered through an autoregressive AR(1) model with the pole 0.9. The unknown system was chosen to be the forth impulse response from the G.168 ITU-T Recommendation [6], with the length $L = 128$.

We performed a simulation by combining two types of scenarios: tracking and temporary low SNR conditions. In the first part of the scenario, abrupt changes in the unknown system were triggered by changing the sign of the corresponding impulse response coefficients at the time index 60000. In the low SNR part of the scenario, the additive noise was experimentally changed from SNR = 20dB to SNR = -20dB, for a duration of 5000 samples, starting with time index 180000. The performance has been measured with the normalized misalignment [3]. The forgetting factor is the same in all circumstances, $\lambda = 1 - 1/(KL)$, with $K = 128$. The DCD parameters were set to $N_u = 4$, $M_b = 16$, and $H = 1$. The simulations results are illustrated in Figure 1.

TABLE I: DR-RLS-DCD ADAPTIVE ALGORITHM

Step no.	Step action
Init.	Set $\widehat{\mathbf{g}}(0) = \mathbf{0}_{L\times 1}; \mathbf{r}(0) = \mathbf{0}_{L\times 1}$ $\mathbf{R}(0) = \delta\mathbf{I}_L$, with $\delta > 0$
	For $n = 1, 2, \ldots,$ number of iterations :
A	Update vector $\mathbf{x}(n)$ and matrix $\mathbf{R}(n)$
B	For $k = 1, 2, \ldots, N_{it} - 1$:
1	Compute $e_k(n)$ using (7)
2	Compute $\mathbf{p}_k(n)$ using (8)
3	$\mathbf{R}(n)\Delta\mathbf{g}_k(n) = \mathbf{p}_k(n) \xrightarrow{\text{DCD}} \Delta\widehat{\mathbf{g}}_k(n), \mathbf{r}_k(n)$
C	$\widehat{\mathbf{g}}_k(n) = \widehat{\mathbf{g}}_k(n-1) + \Delta\widehat{\mathbf{g}}_k(n)$

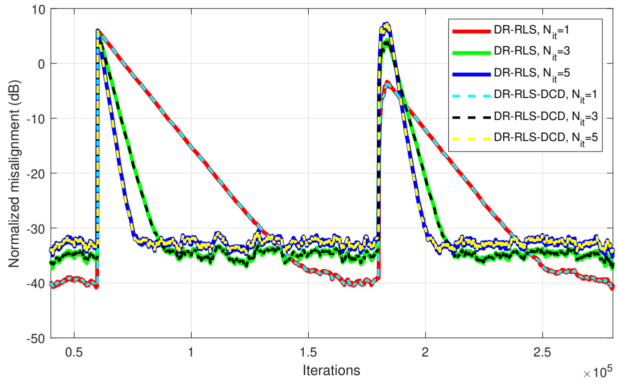

Figure 1. Performance of the DR-RLS and DR-RLS-DCD algorithms for different values of N_{it}. The unknown system changes at time index 60001, and the SNR is decreased for 5000 iterations, starting with index 180001.

V. Conclusions

The algorithm introduced in this paper is an efficient combination between the exponentially weighted RLS algorithm based on the DCD method [2], [3], enhanced by a data-reuse approach. The DR-RLS-DCD adaptive algorithm has proven to offer a useful compromise between tracking capabilities and estimation accuracy. The trade-off is controlled from the DR part of the algorithm through the number of iterations N_{it}.

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