C^2 Spline Quasi-Interpolation To Downscale A Digital Elevation Model

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***Abstract*— Digital Elevation Models (DEMs) are one of the products delivered by most of the national and regional cartographic agencies of the states. They are discrete representations of a territory and are of undoubted practical importance. Algorithms based on available discrete data make it possible to estimate terrain-related features and perform on DEMs operations of interest. Resampling is one of them (particularly downscaling). Traditional algorithms compute slope and curvature from discrete samples. In this paper a low computational cost spline-based procedure to construct a C^2 continuous surface fitting the data is proposed which will allow to compute slope and curvature. To assess the downscaling quality of the quasi-interpolation-based algorithm, it is proposed two analysis: a) an horizontal displacement computation based on particle image velocimetry and b) a visual analysis for the height error pattern using a threshold parameter**

Keywords-Powell-Sabin triangulation; Bernstein-Bézier representation; data quality; approximating splines; DEM; resampling.

I. INTRODUCTION

Digital Elevation Models (DEMs) are used as the basis for multiple projects concerning Civil Engineering, hydrology, geology and land-use planning. DEM resolution (cell size) depends on the project objective: a geological study will require less resolution than the DEM used to compute the watershed sinking on a small structure crossing a road. Many times there is not a DEM with the appropriate resolution and it is necessary to resample it at smaller or larger scales: an example for the first case occurs when a higher resolution has been used to improve urban flood zones in the absence of denser models [1]; the second occurs when assessing the altimetric accuracy between of a lower resolution DEM versus a higher resolution reference DEM [2]. It is interesting knowing the resampling process quality in the final product: Leon Tan et al. [3] analyze the influence of resampling on the streamflows derived from a DEM. Nevertheless, in most cases, the error introduced by the resampling from a higher resolution to a lower one was left unanalyzed, as indicated in [4]. There are traditional resampling methods but they are not based on continuous functions; they are also weak when computing variables that involve derivatives such as curvature and slope. The quasi-interpolant algorithm we propose solves this problem because the base function is of C^2-continuity.

In Section II we explain the C^2 quasi-interpolant algorithm. The method to assess the algorithm is based on the horizontal displacement computation: a DEM product (DEMpro) is downscaled and assessed versus a DEM reference (DEMref) using the particle image velocimetry technique (Figure 1). A pattern visual analysis is also carried out (Figure 2). Both methods are included in section II. And section III contains the conclusions.

II. METHODOLOGY

The resampling and evaluation algorithm proposal will be made assuming that the pixel size of the DEM_{ref} is XxX m and will be denoted DEM_{refXxX}. The pipeline will be as follow:

1. Upscaling DEM_{refXxX} (Fig. 1): the DEM_{refXxX} is upscaled using the nearest neighbour algorithm to a larger cell size which is called $DEM_{respYxY}$.
2. An approximant surface ($S_{respYxY}$) is fitted to the $DEM_{respYxY}$ by the algorithm Aapx, based on C^2 quasi-interpolation having a good approximation order (Figure 1).
3. Downscaling (Figure 1): Since the definition domain $S_{respYxY}$ is the same as DEM_{refXxX}, $S_{respYxY}$ can be evaluated at the same points that the reference DEM_{refXxX} matrix, from which a homologous

$DEM_{derXxX\_from\_YxY}$ of the same size as DEM_{refXxX} is obtained.
4. The assessment is carried out comparing the DEM_{refXxX} and $DEM_{derXxX\_from\_YxY}$.
5. The A_{apx} planimetric quality is assessed using the [5] method based in particle image velocimetry approach.
6. The altimetric quality is assessed computing the height discrepancy (dh_i) in absolute value and representing in red on a shadow map the cells where dh_i > threshold (Figure 2).

The algorithm A_{apx} constructs, with low computational cost, a spline surface $S_{respYxY}$ that fits the data based on quasi-interpolation in the Bernstein basis. Suppose that the DEM of the terrain under study has an associated decomposition into squares of length h, whose vertices are $v_{i,j}=(ih,jh)$, $0 \leq i \leq n$, $0 \leq j \leq m$. Let us define a triangulation by decomposing the square of opposite vertices $v_{i,j}$ and $v_{i+1,j+1}$ into macro-triangles $T_{i,j,1} = < v_{i,j}, v_{i+1,j}, v_{i+1,j+1} >$ and $T_{i,j,2} = < v_{i,j}, v_{i+1,j+1}, v_{i,j+1} >$. The approximating spline will be defined on the subtriangulation obtained by refining $T_{i,j,1}$ and $T_{i,j,2}$ as follows: a Powell-Sabin 6-splits results if the vertices of each triangle are joined with the midpoints of the corresponding opposite sides, intersecting at the barycenter [7]. Each macro-triangle is decomposed into 6 micro-triangles. The restriction of the spline surface to each micro-triangle will be a polynomial of total degree less than or equal to three. Therefore, it can be written as a linear combination of the corresponding ten cubic Bernstein polynomials associated with that micro-triangle. The Bernstein-Bézier coefficients [6] of the constraints must be calculated to get a C^2-continuous surface in such that way that, if the data came from a cubic surface, the approximating spline obtained is itself (exactness on the space of polynomials of degree up most three). Those coefficients will be calculated as linear combinations of DEM values at neighboring points. Their coefficients will provide masks to be computed to ensure C^2continuity and the required exactness.

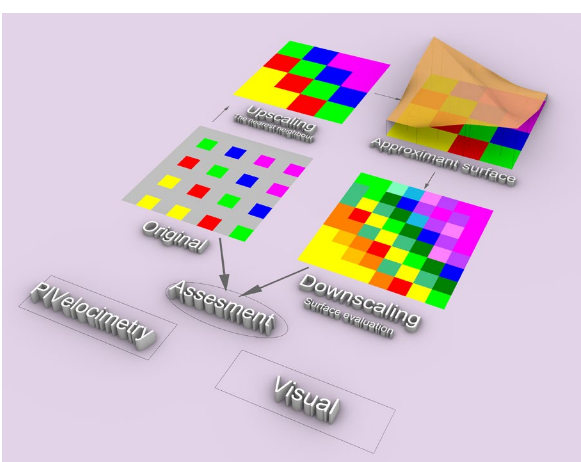

Figure 1: Proccess flowchart.

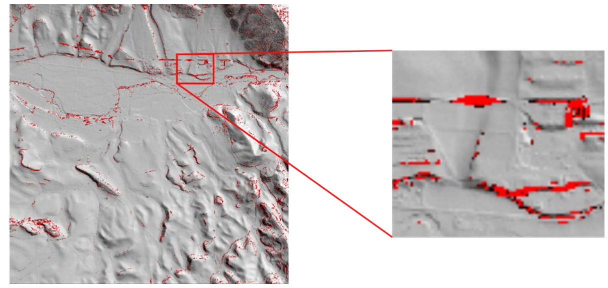

Figure 2: Absolute height discrepancy larger than a predefined threshold.

III. Conclusions

In this work, it is proposed a new algorithm A_{apx} to get a DEM_{proYxY}, motivated by the necessity to get a downscaled version ($DEM_{derXxX\_from\_YxY}$) The horizontal and vertical accuracy have been assessed comparing it versus a DEM_{refXxX}. It is proposed as well a pipeline to assess the planimetric and altimetric quality. The planimetric quality assessment is innovative because very few cases use the particle image velocimetry approach; It is proposed a graphical threshold approach to inform about the altimetric error which detects pattern where the higher errors occur, e.g., road limits (Figure 2). In future work we will implement the particle image velocimetry algorithm.

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