A Method for Calculating Shape Similarity among Trajectory of Moving Object Based on Statistical Correlation of Angular Deflection Vectors

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***Abstract*—In recent years, mobile electronic devices, specially smartphones, are gaining more attention in people's daily life. These devices provide many features that often process information gathered by one of their several built-in sensors. Among them, one of the most popular is the Global Positioning System (GPS) receiver. This sensor allows the systematic and chronological collection of location information that represents the trajectory of the moving object that carries it. Trajectories are considered valuable sources of information for analysing and understanding of moving objects' behavior. There is a considerable number of researches that analyse objects' trajectories. Among them, the identification of trajectories' similarity is one where researches are currently being developed. The similarity of trajectories may indicate common behaviors inside groups of individuals that can be useful in various application areas. However, how to measure the trajectories' similarity even though they are in different directions and far from each other? Trying to solve this problem, this paper proposes a method to calculate similarity among trajectories applying statistical correlation over their vectors of angular deflections. Preliminary results indicate that the method can identify similarity in shape among trajectories. However, when their shapes are too complex, it can not reach suitable results.**

***Keywords*—Moving Objects; Trajectory; Similarity; Correlation Statistics.**

I. INTRODUCTION

In the last years, mobile electronic devices have gained an increasing importance in peoples daily life. Among all the types of existing mobile devices, smartphones are the most widespread. Their use is pushed by the amount of functions that they provide. These features are performed by applications that often process information collected by one of its several built-in sensors. Concerning the sensors that they have, one of the most popular is the Global Positioning System (GPS) [1]. This sensor allows individuals that carry mobile devices to record their movement in time. The chronological collection of location information can represent the trajectory of an object [2]. Trajectories are considered valuable sources of information for analysing and understanding the behavior of moving objects. An increasing volume of trajectories makes it possible to find patterns inside the movement of an individual or group of individuals. Research areas such as bio-monitoring, logistics, navigation systems, car and pedestrians traffic planning are examples of trajectory analysis applications.

Commonly, a trajectory (T) is represented by one id (T_{id}) and a set of points composed of x, y and t values, where x and y are geographic coordinates, and t is a timestamp [3], [4]. Additionally, a trajectory and its points can be enriched with information like: speed, direction, acceleration, etc. [5], [6]. In order to gather such information, it is necessary to use other sensors instead of only GPS. But in the case of smart phones, it is not a big deal considering its number of embedded sensors.

The moving objects' trajectories data is a vast source for data analysis and its analysis can bring direct contribution to people's life. There is a considerable number of researches that analyse objects' trajectories in order to find answers for phenomena observed into the cities' environment. Among them, the identification of trajectories' similarity is one where researches are currently being developed. Works like those carried out by [7]-[14] present approaches for identifying similarity considering some trajectory aspects.

The similarity of trajectories' shape may indicate common behaviors inside groups of individuals that can be useful in various application areas like identifying drivers behaviour, cargo stolen, dangerous places and so forth.

Each proposed approach presents a specific set of metrics to identify similarity among trajectories. However, according to Pelekis et al. [12], trajectories can be considered similar if some of the following aspects are true: (i) fully or partially overlapping in space; (ii) similar shapes in different places; (iii) same start and/or end position; (iv) partially or fully synchronous movement behavior; (v) or totally separated in time, but with similar dynamic behavior (speed, acceleration, etc.). This work focuses on the aspect (ii) considered by [12], where the similarity of shape is used as criterion for defining trajectories similarity. However, shape is just one of the aspects that must be tackled when dealing with trajectory similarity as a whole. It must be combined with time and length characteristics, as well as with extra semantic aspects, when possible.

In the literature, there is a considerable number of works that propose different metrics to calculate the shape similarity among trajectories. Examples include works conducted by [9], [13] who present solutions to identify the degree of similarity for sets of nearby trajectories. Another example is the work of [15], which presents a compendium of distance metrics used to evaluate similarity. Another research [16] proposes a qualitative trajectory calculus (QTC^c), to find distances and directions of trajectories.

Despite their important contribution, none of the related works address the problem of identifying shape similarity between trajectories that are far from each other or that have different moving direction. For example, trajectories that are in different cities or even in different countries and from objects moving from West to East and objects moving from South to North. There are several situations where this proposal can be useful. The dissemination of diseases could be represented by a trajectory. In this case, the behaviour of the similarity trajectories considering the shape of trajectories, regardless of orientation, could be a very important knowledge in order to take actions to interrupt the dissemination of the diseases. Another possible context of the use of similarity, regardless of orientation, is the observation of behaviour of a set of individuals of an animal species. The similarity of trajectories independent of orientation could be an indicator of an occurrence of a particular event in that group. In this case, the same event could be identified in different regions, regardless of distance and orientation. A damaged ship also is an example; the movement of a ship in this situation has some characteristics that could be identified in other trajectories. In this case the shape of the trajectory is more important than its orientation.

Therefore, this paper tackles the following problem: How to measure the trajectories' shape similarity even though they are in different directions and far from each other? Trying to solve this problem, this paper proposes a method to calculate the shape similarity among trajectories applying statistical correlation over their angular deflections.

The measurement of geometry angles can be encompassed into the topography area, where azimuth (AZ) and deflection (DF) angles are considered the basic measures to calculate segment orientation. Considering that a trajectory is a geometry formed by several points, after calculating the deflection of each one of its segments, this will result in a sequence of $n - 1$ deflection angles, where n is the number of points in the trajectory. Having the sequence of angular deflections of both compared trajectories, it is possible to reduce the problem of calculating shape similarity of two trajectories by calculating the statistical correlation coefficient of both deflection sequences. Statistical correlation analysis is a discipline that aims to measure the coefficient of relationship or association between two variables [17].

Therefore, this work addresses the hypothesis that it is possible to identify the shapes similarity between two trajectories calculating the statistical correlation coefficient over the sequences of segment deflection (DFV) from both trajectories.

The paper is structured as follow: Section II reviews the related work. Section III lists definitions concepts found in the literature. Section IV lists the steps of the proposed method. Section V reports the experiments. Section VI presents the findings and results of the experiments. Finally, Section VII provides a view of future works.

II. RELATED WORKS

Shape similarity between trajectories is a field with many challenges and several works have proposed solutions to this problem, as mentioned in section I. In this section, we detail some approaches found in the literature.

Vlachos et al. [7] propose the longest common subsequence (LCSS). Its main idea is to match two sequences by allowing them to stretch, without rearranging the sequence of the elements, and allowing some elements to be unmatched. This method has reached great effectiveness in the presence of noise. However, it does not penalize unmatched sub-sequences, given no information of how to separate the unmatched subsequences. In addition, its original concept does not consider the direction, and may fail to separate two trajectories near in space with very different directional behavior.

Chen et al. [8] propose the Edit Distance on Real Sequences (EDR) function. This function is based on the Edit Distance Function which has been used to quantify the similarity between two strings. Given two strings, the Edit Distance function calculates the minimum number of insertions, deletions and replacements needed in order for both to become identical. Like LCSS, this function also assumes that the trajectories have the same length and sampling rate.

Van de Weghe et al. [16] propose the Qualitative Trajectory Calculus (QTC_c), that is a qualitative approach to represent two vectors by means of a 4-tuple representing the orientation of both vectors with respect to each other. The relative movement of two objects are represented by a four-component label, where the first two components describe the tendency of distance changing of an object to the current position of another object, and the other two components describe the relative orientation of the object movements with respect to the reference line that connects them. One problem of this approach is that it does not present a quantitative measure of similarity. Another point that needs to be considered is the time consumed caused by calculating the Shape Matrix for every trajectory to be compared.

Frentzos et al. [9] propose a Dissimilarity Metric (DISSIM). The DISSIM between two trajectories Q and R is calculated by the integration of their Euclidean Distance over a definite time interval when both Q and R are valid. So it takes into account the time dimension in both trajectories. Moreover, DISSIM can be used for trajectories with different sampling rates, because non-recorded points are approximated by linear interpolation. Its main drawback is the high computation consumption.

Dodge et al. [10] developed a conceptual and methodological framework focused on the analysis of similarities in dynamic behavior of moving objects. They also proposed to pre-process the data, resampling the data to a regular time interval, using linear interpolation of fixed time intervals. It means that the authors concentrate on tracks, rather than sample points, and the method is limited to a fixed sampling period.

Liu and Schneider [11] proposed an approach to calculate the similarity of trajectories that not only considers the geographical issues, but also the semantic aspects of the trajectories' movement. For the geographical part, the authors consider the following aspects: bearing, distance between trajectories, center of mass, smaller distance between the initial and final point of the trajectory and angle cosine to find subtrajectories. The problem is that the method for calculating the semantic similarity depends on the similarities obtained in the geographical part.

Pelekis et al. [12] define a method that groups trajectories using various distance functions such as GenLIP, GenSTLIP and others. Based on motion properties such as spatial location, speed, acceleration and direction, the similarity is calculated. This work use a clustering approach to group trajectories.

Sankararaman et al.[13] present a framework to rate the trajectories according to their similarities based on distance. To calculate this similarity, they use the following algorithms: DTW (Dynamic Time Warping), euclidean distance and direction of the segment. Their main contribution is to find equal and not equal parts of the trajectories. The authors include the time as an extra dimension, allowing their model to be extended to spatio-temporal data.

Xie [14] proposes a metric to calculate the distance called Edit Distance on Segment (EDS). This metric is used to check the similarity in sub-trajectories using their segments. To calculate this similarity metric the authors define the cost of a segment-wise transformation, i.e., the cost of changing a segment to another one. The idea is that given two segments it is possible to transform them by displacing, stretching and rotating properly, in order to identify the similarity of sub-trajectories.

Concerning the reviewed works, it is possible to see that most of them also assume that the trajectories should have the same number of points to perform the shape comparison. Also, some of them replace points as well as estimate approximated points in order to perform the comparison. Besides that, other works produce qualitative results, instead of quantitative ones. Finally, even using bearing and azimuth, none of the reviewed works identifies shape similarity among trajectories in different directions and far from one another.

III. Basic Concepts and Definitions

This section details concepts and definitions about trajectories, angular measurements and statistical correlation, based on the following works [17], [18], [19], [20], [21].

A. Trajectories of moving objects

Below, we present the definitions concerning the trajectory of moving object.

Definition 1: A coordinate (c) is a tuple (x, y), where x is a latitude and y is a longitude. A coordinate defines a georeferenced position on the earth surface.

Definition 2: A point (p) is a tuple (c, t), where c is a coordinate and t is a time-stamp that represents the time when the coordinate c has been taken.

Definition 3: A trajectory (T) is composed of a sequence of points and can be defined as $T = [p_i, p_{i+1}, ..., p_n]$, where p_i is the start point, p_n is the end point, $p_i < p_{i+1}$ and n is the number of points.

Definition 4: A segment (S) is a sequence $S = [p_i, p_{i+1}, p_{i+2}, ..., p_f]$, where p_i is the initial point of the segment, p_f is the final point of the segment, $0 \leq p_i < p_f$, $p_i < p_f \leq p_n$ and $p_i < p_{i+1}$, n is the number of trajectory points. It means that $S \subset T$.

B. Angular measurements

Below, we present the definitions of angular measurements used in this work.

Definition 5: Bearing is the angle formed between the North-South meridian and a line to West or East. It varies from 0° to 90°. In order to represent the bearing direction, it is necessary to define in which quadrant it is placed: North-West (NW), North-East (NE), South-East (SE), South-West (SW) [20].

Definition 6: Azimuth is the angle that begins on North and turns clockwise until it reaches the desired line. It varies from 0° and 360°. Instead of bearing, azimuth does not indicate the direction of the line because this is implicit [20].

The azimuth can be obtained by (1).

$$AZ_i = \arctan 2 \left(\frac{\sin \Delta\lambda . \cos \phi_{i+1}}{\cos \phi_i . \sin \phi_{i+1} - \sin \phi_i . \cos \phi_{i+1} . \cos \Delta\lambda} \right) \quad (1)$$

Where $\Delta\lambda = (\lambda_i - \lambda_{i+1})$, and λ and ϕ indicate the longitude and latitude of a point p_i, respectively. An azimuth vector is a sequence of azimuths such as $AZV = [AZ_i, AZ_{i+1}, AZ_{i+2}, ..., AZ_j]$, where $0 < i < j$ and $i < j \leq n - 1$, n is the number of points in the trajectory.

Definition 7: Deflection angle is calculated by the difference between the azimuths of two consecutive lines. It varies from 0° to ±180°. It is positive if the azimuth of the first line was greater than the second one and vice-versa [20].

The deflection can be obtained by (2).

$$DF_i = (AZ_i - AZ_{i+1}) \quad (2)$$

Where AZ_i represents the azimuth of a point inside the trajectory and AZ_{i+1} is the azimuth of the subsequent point in the same trajectory. A deflection vector is a sequence of deflections such as $DFV = [DF_i, DF_{i+1}, DF_{i+2}, ..., DF_k]$, where $0 < i < k$ and $i < k \leq j - 1$, j is the number of azimuths in the trajectory.

C. Statistical Correlation Coefficients

Correlation is a bi-variate analysis that measures the statistical relationships between two variables. The value of the correlation coefficient may vary between $+1$ and -1, where 0 indicates no correlation, and $+1$ and -1 indicate positive and negative correlation, respectively [17]. Nevertheless, when the coefficient varies between ± 0.10 and ± 0.29, it indicates weak correlation; between ± 0.30 and ± 0.49, medium correlation; and between ± 0.50 and ± 1, strong correlation [21]. Among the coefficients found in the literature, Pearson (*r*) [22], Spearman (*rho*) [23] and Kendall (τ) [24] are the most remarkable ones.

In statistics, p-value (p) represents the probability of a statistical test be considered valid, instead of random. It defines the probability to reject the null hypothesis (H_0) [21]. The level of significance is represented by α and it has the following general rule: if $p > \alpha$ then H_0 is accepted, but if $p \leq \alpha$ then H_0 is rejected.

IV. Proposed Method

As mentioned before, this work assumes that the shape similarity between two trajectories can be identified through the application of statistical correlation coefficient in their deflection vectors. This assumption is based on the idea that if the trajectories have similar shapes, then they would have similar deflection angles (DF) between their segments.

The method for identifying trajectories with similar shape is composed of the following steps:

1) **Selecting the reference trajectory:** The first step is to select which trajectory (or trajectory segment) to use as reference for comparing its shape with the others;
2) **Selecting the set of compared trajectories:** After that, it is necessary to select the set of trajectories (or trajectories segments) to be used in shape comparison with the reference one.
3) **Compacting the compared trajectories:** As each selected trajectory (or segment) must have the same number of points as the reference one, it is necessary to compact them using a compression algorithm that maintains a predefined and fixed memory size of points in the compressed trajectory. Examples of algorithms that use this approach are Spatiotemporal Trace (STTrace) [25] and Spatio Quality Simplification Heuristic (SQUISH) [26].
4) **Segmenting the trajectories:** For computing the azimuths and deflections, every trajectory should be segmented into segments composed of two points. So, for each trajectory, an array $S = [s_1, s_2, ..., s_s]$ of segments will be generated, where $0 < s \leq n-1$ and n is the number of trajectory points.
5) **Computing azimuths for trajectories segments:** For every trajectories segment we calculate its azimuth using (1). The azimuth is used as requirement to compute the deflection that is the main value used to identify trajectories shape similarity. This information is stored in an array of j positions, where $j = s$ and s is the number of trajectory segments.
6) **Computing deflection between two consecutive trajectory segments:** After calculating the segments azimuth it is possible to compute the trajectory segments deflection using (2). In order to decrease their variance, the values of deflection are modularized. This information is stored in an array of k positions, where $k = j - 1$ and j is the number of trajectory azimuths.
7) **Applying statistical correlation coefficient:** Having the deflection vectors it is possible to calculate the correlation coefficient between the reference trajectory and each one presented in the compare set (two by two). In this work, we applied the following correlation methods: Pearson (*r*) [22] and Spearman (*rho*) [23].

V. Experiments

To test the proposed method, we created four scenarios, one with synthetic data manually made (Figure 1) and three with real data collected in the city of Joinville - Brazil (Figure 2, Figure 4 and Figure 6). The collected data were produced by Costa and Baldo [27] in a work aimed at generating digital road maps. Three scenarios are used to assess whether the method can identify trajectories similarity in shape and one is used to assess whether the method does not identify similarity when the trajectories are considerably different in shape. This last scenario was proposed to ensure that the method rejects the H_0.

Based on the literature review, the ideal scenario would reach -1 or +1 correlation, with a p-value (p) ≤ 0.05, and this value of p, is a possible value to discard the null hypothesis (H_0), since the chance of the α error is small. However in these experiments it has been assumed that two trajectories are considered similar when the *correlation coefficient* is ≥ 0.30 and the *p-value* is ≤ 0.10. The values for these parameters are more flexible, and this is why we consider several scenarios of different complexity for analysis. Looking at similarity based on the correlation, the medium values are between ± 0.30 and ± 0.49 and strong values are between ± 0.50 and ± 1 [21].

All compared trajectories have the same size and a minimum of 10 points. This size has been chosen in order to produce a valid statistical analysis, because this is a requirement for statistical correlation, where the analyzed variables must be the same size. At last, in order to reach better linear distribution among the deflection values, they were normalized as follows: $T = [|DF_1|, |DF_2|..., |DF_k|]$, where k is the number of trajectory deflections.

The first experiment, presented in Figure 1, uses data manually created and tries to depict the ideal case where two trajectories, even though in different orientation (one North-South and another West-East) and relatively far from one another, with similar segment deflections should have high correlation coefficient and thus high similarity using a statistical correlation approach. As seen in Table I, this experiment reaches 0.99 of Pearson correlation with 0.01 of p-value which means that they can be considered equal in shape.

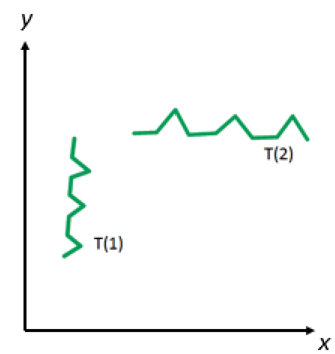

Figure 1. Experiment 1, same shapes and different directions, T_1 is the reference trajectory.

TABLE I. EXPERIMENT RESULTS.

Experiment	Points	Pearson	Spearman	p-value
1	10	0.99	0.83	0.01
2	11	0.38	0.63	0.07
3	32	0.08	0.24	0.19
4	33	-0.03	0.02	0.88

The second experiment presented in Figure 2 has been created based on trajectories extracted from the sample presented in Figure 3. This experiment tries to analyse if the method can be applicable and reaches high correlation value (similarity) comparing real trajectories collected by GPS receivers. As can

 ISBN: 978-1-61208-469-5

be seen in Table I, this experiment reaches 0.63 of Spearman correlation with a p-value of 0.07. It means that the trajectories have relatively high correlation, so it would be said that they have similar shapes too, as expected.

Figure 2. Experiment 2, same shapes and directions, T_1 is the reference trajectory.

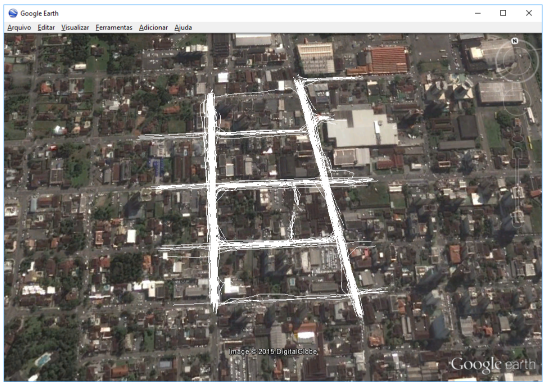

Figure 3. Trajectories collected in downtown.

The third experiment presented in Figure 4 has been created based on trajectories extracted from the sample presented in Figure 5. This experiment tries to analyse if the method can reach high correlation value (similarity) comparing trajectories with complex shapes as, for instance, a turn of $360°$. As can be seen in Table I, this experiment reaches only 0.08 of Pearson and 0.24 of Spearman, correlation with a p-value of 0.19. It means that not only they do not have correlation, but also that the H_0 can not be rejected, so this result can be considered aleatory. Analysing this results it is possible to see that the proposed method can not be applied to identify similarity to every kind of trajectory's shape. Depending on the shape complexity it would not reach the expected result, even with trajectories that have visually similar shapes.

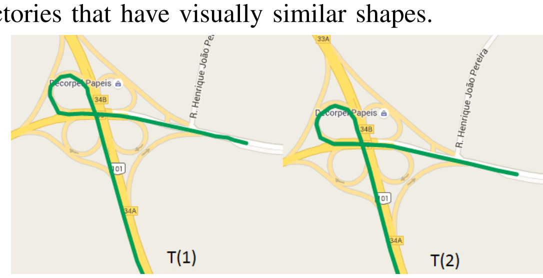

Figure 4. Experiment 3, same complex shapes and directions, T_1 is the reference trajectory.

The fourth experiment presented in Figure 6 has been created based on trajectories extracted from the sample presented in Figure 3. This experiment tries to analyse if the method can recognize when two trajectories do not have similarity in shape. This situation is represented by low correlation (value near to 0) and high p-value (value near to 1). As can be seen in Table I, this experiment results in a -0.03 Pearson value and a 0.02 Spearman value with a p-value of 0.88. It means that the trajectories do not have correlation among their deflection

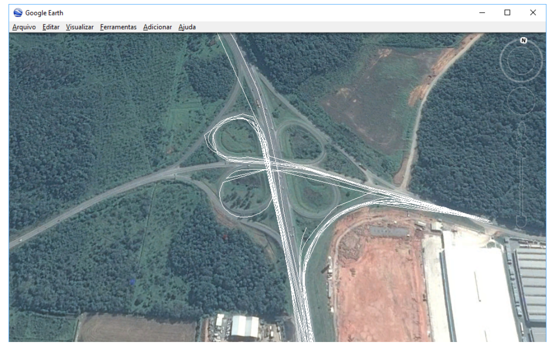

Figure 5. Trajectories collected in the Joinville Industrial District.

vectors which ensure that they are not similar in shape. Besides that, as the p-value is high, it is not possible to reject the H_0 (null hypothesis), which contribute to ensure their no similarity in shape.

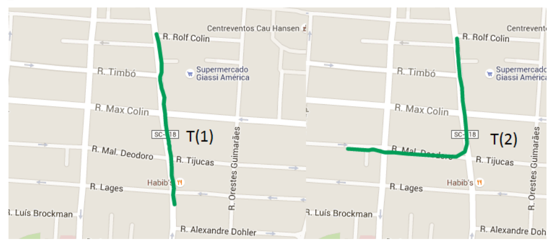

Figure 6. Experiment 4, different shapes and directions, trajectories T_1 is the reference trajectory.

VI. Result Analysis

Analysing the results showed in Table I, it is possible to see that the proposed method has potential to identify the similarity between trajectories. This can be answered by the experiments 1 and 2. However, it can not be applied in every trajectory shape with suitable results, as presented in experiment 3. This occurs because the array of trajectory deflections does not follow a linear distribution and this is a requirement for applying statistical correlation methods.

By nature, statistical correlation methods work with linear distributed variables. It means that if one variable is increasing the other one should have the same behavior in order to present high correlation coefficient. Concerning the two applied correlation methods, Pearson suffers more impact in its results than Spearman due to the fact that variables do not have linearity. This can be seen in Table I, experiments 2 and 3, where Spearman reaches high correlation values. This is explained because Spearman takes into account not only the variables linear distribution, but also the similarity of values in the same position in both variables. So, for Spearman even if the variables are not linearly distributed, if they have similar values in the same position they can be considered similar.

Although positive results have been obtained using Spearman, as this is an ongoing work, it was not decided which one to elect as the statistical method to be adopted. The next steps will include a bench of massive tests where it will be decided which method to adopt as well as which range of correlation values will be considered enough to ensure similarity in shape among trajectories.

 ISBN: 978-1-61208-469-5

VII. Conclusion

Trajectories are considered valuable sources of information for analysing and understanding of moving objects' behavior. Despite the considerable progress concerning the measurement of trajectories' shape similarity, the literature does not present a method able to measure similarity among trajectories in different directions and/or far from each other.

The main goal of this research is to develop a mechanism to calculate similarity among trajectories applying statistical correlation over their vectors of angular deflections. Another objective is to use correlation methods in order to calculate the level of similarity. Considering the preliminary results it is possible to affirm that the proposal is able to find similarity trajectories considering the angular deflexions. However, the developed work is not conclusive in order to identify the more adequated correlation method to analyze those sets of angular deflexions. By using the proposal to analyze complex shapes the method does not reach suitable results. This can be explained by the non-linearity of the deflections array.

As future work, we plan to start a bench of massive tests in order to decide which statistical method to adopt (Pearson or Spearman), as well as which range of correlation values to consider suitable to ensure similarity in shape among trajectories. A very important point of research is the improvement of data quality. Actually, current research does not check the relationship between data dispersion level and accuracy of the results of similarity. This investigation, considering several different compression algorithms in order to decrease the data dispersion, than those mentioned in this article, is an additional point to execute in the future works.

Furthermore, the presented approach does not consider other characteristics in order to calculate similarity. However, this proposal is just a piece of a method to find groups of similar trajectories. The method considers two additional characteristics: spent time and length of trajectories. Therefore, the proposal of the future method is to find groups of similar trajectories considering several gradients of shape, duration and length of the trajectories.

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 ISBN: 978-1-61208-469-5