

Time Geographic Network Modeling for Restraint Space of Transportation Network

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Abstract—Time geographic accessibility in heterogeneous space can be converted to the shortest path problem. Based on raster grids, current time geography theory analyzes the shortest path among grid cells by using 8-neighbor mode (network). However, due to the irregularity of transportation network, the network is not constantly included in 8-neighbor network. This means neighbor network paths are different from continuous-space paths in location and length, and as cause a time distance error. In order to address this problem, the paper focuses on the restraint space of irregular transportation network, and introduces Voronoi diagram grid. Then, we establish a neighbor network model based on Delaunay Triangulated Irregular Network to contain transportation network. This means neighbor network paths can converge to continuous-space paths under the same condition of start and end, and as decrease a time distance error. As a result, the shortest path between Thiessen polygons is calculated with less error of time distance. Thus, we have the approach to model time geographic network for restraint space of transportation network. This article also describes implementation of the models using ArcGIS 9.3 with a real transportation network as a way of validation.

Keywords—network mode; irregular triangle network; field; space-time mode; raster

I. INTRODUCTION

Time geography's main concern is measuring the accessibility region. Since the gradual space can be divided into raster grids, it enables the grid cell's accessibility problem to be converted to the shortest raster path problem [1]. The raster path consists of a number of adjacent grid cells, the sum of whose weights is the path length. The weight of a cell is set to be the shortest time for passing the cell. Assuming the shortest time for passing a cell c horizontally or vertically is actually t_h and t_v respectively. Without loss of generality, we assume $t_h > t_v$, thus the weight of c , $t_c = t_v$; in such a way, t_c would replace t_h to be the shortest time for crossing c horizontally in the shortest path algorithm analysis with a error: $t_h - t_v$. However, the error would be quite small in gradual space, and get larger in transportation network restraint space (TNRS). For example, if c only has one vertical road, the result of $t_h - t_v$ would be quite large. This means we expect time geography in TNRS.

Miller introduced a time geography field for TNRS [2]. It describes how the time geography network models can be established for the city space by adapting raster grid and its

8-neighbor mode or network (neighbor network, NN). In the city, Salt Lake City, urban roads crisscross. A raster unit of $500\text{m} \times 500\text{m}$ either has no road, or has a crisscross network of roads. Thus, t_h and t_v of the unit are pretty much the same, resulting in small error: $t_h - t_v$. According to the trace data of individuals, Miller verified the efficiency of raster time geography field model in regular-TNRS, such as Manhattan, Salt Lake City.

So far, however, the elementary theory of time geography in TNRS deals with regular transportation network only. Since raster grids are regular rectangle units, the subdivision of units can hardly allow for transportation irregularity. Some units would only contain a single route (for instance, the unit c only has a vertical road), which may have larger weight errors. We would study the mathematical basis of time geography in irregular-TNRS. As a result, we will have a complete time geography theory for TNRS.

Heterogeneous space's time geography is an extension of (classical) homogenous space's time geography based on fields. Raster time geography field is regular-based, and this article would introduce irregular Voronoi diagram field, to reflect the additional transportation network by the neighbor networks based on Delaunay Triangulated Irregular Network (D-TIN). Thus, we can measure the shortest path and spatio-temporal accessibility for TNRS.

The article is structured as follows. Section 2 summarizes the existing time geography. Then, we introduce time geography in TNRS (Section 3). Section 4 gives the approach to construct time geography networks for TRNS. The approach will be implemented and tested in ArcGIS in Section 5. Conclusions close the article (Section 6).

II. TIME GEOGRAPHY

The fundamental problem of time geography is measuring the accessibility region of an agent within the known time T . Given the agent's start point s and velocity distribution v , if the agent leaves from s and arrive at p within the shortest time $D(s, p)$, $D(s, p) \leq T$, p should be included in the reachable region. When the end point e is also known, another condition must be added: the agent can leave from p and reach e within the minimum time interval $D(p, e)$, $D(s, p) + D(p, e) \leq T$. In this way, the accessibility of p can be converted to the shortest-path problem [1]. $D(s, p)$ and $D(p, e)$ are corresponding to path $S(s, p)$ and $S(p, e)$ respectively.

In homogenous space, v is everywhere equal, so the shortest paths $S(s, p)$ and $S(p, e)$ are line segments. Classical time geography uses cones to show an agent's reachable area while moving freely at s (Fig. 1a), and uses prisms to represent that while proceeding directed movements from s to e (Fig. 2a) [3].

In heterogeneous space, v is not constantly the same, so $S(s, p)$ and $S(p, e)$ may not always be straight, but curves. The algorithms of finding the shortest path curves are almost network-based, such as single-source shortest path algorithm (Dijkstra). This means the accessibility problem, via the shortest-path problem, can be converted to the network-modeling problem, including space grid-modeling and its NN-modeling. Therefore, the process to measure accessibility region is 3 steps:

- 1) to construct the grid and its NN;
- 2) to analyze the shortest path $S(s, p)$ and $S(p, e)$;
- 3) to determine whether p is reachable, and to construct the accessible region including reachable point p or the discrete cell whose center is p .

Transportation network is heterogeneous space, and is a NN covering itself, and thereby only from step 3) Kwan [4] analyzed time-space accessibility, in which the time-space prism degrades into vertical sections based on transportation network [5], shown as Fig. 2b.

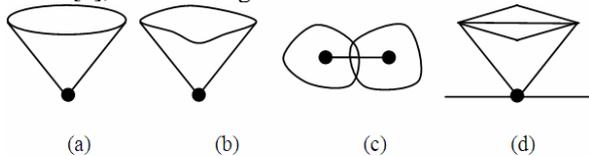


Figure 1. (a) time-space circular cone; (b) time-space cone; (c) intersection of two reachable regions; (d) the time-space cone of transportation network in restraint space

TNRS is also heterogeneous, and can be divided to raster grid [2], by which enabling us to investigate time-space accessibility [6, 7, 8, 9] to analyze and visualize the individual's reachable region in GIS [10]. The research on time geography application is beyond this paper; here, it is important to note that network modeling for TNRS has so far stayed in NN indirectly representing transportation network.

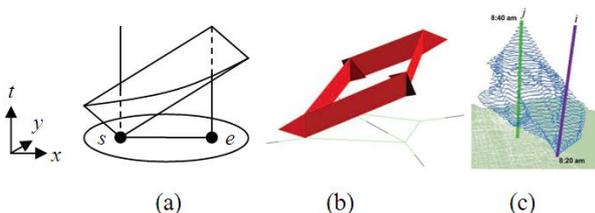


Figure 2. (a) the prism in homogenous space; (b) the prism of transportation network; (c) spatial prism of regular transportation network

It is simple to establish NN indirectly, representing transportation network. After TNRS is transformed to raster grids, we usually turn to an auxiliary NN to investigate the shortest-paths between raster cells [9]. The NN can be constructed by two restraints [6, 8]:

- 1) the center of each cell is a network node;

- 2) the lines connecting adjacent centers are the network edges.

In general, common adjacent relationship includes Rock mode (4-neighbor), Queen mode (8-neighbor) and Knight mode (16-neighbor and its extension), representing adjacent relations among discrete units of varied orders. In this paper, the auxiliary NN are called raster NN (some of them are called virtual network [6], planer network view [7], and implicit network [2]). The raster NN's weights can be dynamically generated by simple liner interpolation of raster cells' weights [8, 11]. For example, if both cells with weights t_a and t_b are horizontally adjacent, the edge connecting both cells has weight $(t_a + t_b)/2$. Since transportation network directly influences its restraint space' cell attributes, it affects the weights of NN. There exists a mapping relationship between transportation network and NN: transportation network \rightarrow cell \rightarrow NN. In regular-TNRS (like Fig. 3), the travel time in transportation network, via the weight of raster cell, can be transmitted to that of NN effectively. Armed with such a raster NN, Miller measured the space-time prism of Salt Lake City [2] (see Fig. 2c); of course, space-time cone can also be measured, shown as Fig. 1b. The question whether two agents would meet within T is answered by testing whether the agents' time-space volumes intersect (Fig. 1c).



Figure 3. Part of Salt Lake City

However, the way to establish NN directly representing transportation network has not been proposed till now. An exception may be YU et al. [8], who suggest 8-NN should add the edges connecting nonadjacent cells to stand for tunnel, but have not yet developed the algorithm for NN to cover plane transportation networks. Generally speaking, there are larger weight errors in the raster NN covering transportation network. The NN of the raster cells only containing a single route is belong to this category. Its reason is that the travel time in transportation net is transmitted to that of NN, via the weight of raster cell. A naive approach to construct NN is to build the mapping relation between transportation network and raster NN, and thus, transportation net' weights can be directly passed to raster NN rather than in an indirect way bypassing raster cells' weights, without cells' weight errors transmitting to the NN. But this approach is wrong, as we will explain later. Fig.1d describes the spatio-temporal reachable region in a single road restraint space.

III. TIME GEOGRAPHY IN TRNS

A. Voronoi Diagram and Delaunay Network

In the realm of GIS, Voronoi diagram is a significant field model. The field is a general conceptual space model constructing continuous spatial change; its logical model is usually described by specific field model of regular rectangles (such as raster), contours and irregular areas like Voronoi diagram and its dual graph D-TIN. The first two have been applied in raster time geography field [2] and isochrones (lines of equal travel time) [12]. In a multitude of geography spatial analysis, field model based on irregular area may be superior, suitable for the practical distribution of data.

There are two networks: raster grids and its NN in raster time geography field. After surface space is transformed to the raster grids, we can construct a NN to analyze the shortest path between raster cells. Similarly, we construct two networks: Voronoi diagram and its D-TIN, for irregular-TNRS (like Fig. 4). Since the nodes and edges of D-TIN stand for Thiessen polygons and their adjacent relationship respectively, D-TIN is the NN where a weight is the shortest time for passing an edge, and this diagram is the field model. Therefore, we can analyze $S(s, p)$ and $S(p, e)$ by D-TIN. Then, if p is accessible, it is accessible Thiessen polygon whose center is p .

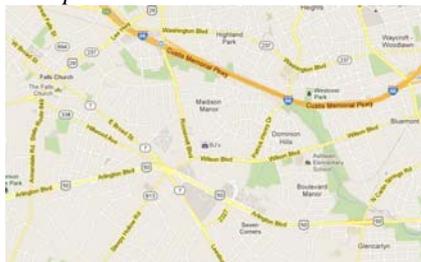


Figure 4. Part of Washington

At present, the ways to model D-TIN for various restraints are fairly mature, such as inserting restraint lines into D-TIN. Theoretically, D-TIN can converge to any transportation network. Consequently, transportation network's weights can be directly passed to D-TIN rather than in an indirect way bypassing Thiessen polygons' weights. That means it can avoid the deviation errors and weight errors, and can decrease the elongation errors, that D-TIN as NN is used to analyze $S(s, p)$ and $S(p, e)$. Below, we describe how to construct D-TIN and D-TIN's superiority to raster NN for irregular-TNRS.

B. Time Geography Networks based on Voronoi diagram and D-TIN

Let us take a crossroad for example (Fig. 5a), to picture the mapping procession from transportation net to NN. Let the point where road AB cuts road CD be called O (Fig. 5a). The weights (the shortest passing time) of AO, BO, CO and DO are all set to be 1. Supposing there are pathways for walk in other edges AD, AC, CB and BD, thus their weights are

all set to be 4. As a result, the shortest time from A to B is $D(A, B) = 2$.

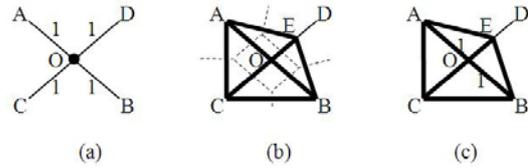


Figure 5. (a) crossroad; (b) Voronoi diagram and its TIN; (c) weighted TIN

Firstly, we picture the mapping from transportation net to D-TIN NN. Armed with the point set $\{A, B, C, E, O\}$, D-TIN and Voronoi diagram can be built (Fig. 5b), where E is the midpoint of OD. Again, Supposing there are pathways in edges AE and BE. According to Pythagorean Theorem, the weights of AE and BE are both 3.16. Through transportation' and pathway' net-to-NN maps, we can get the weights of AO, BO and CO with value of 1; that of OE is 1/2. On the other hand, the weights of AC and CB are both 4; that of AE and EB are 3.16. As a result, $D(A, B) = 2$, and $S(A, B) = \{AO, OB\}$, shown as Figure 5c.

Secondly, we picture the mapping from transportation net to raster NN. When this junction O is exactly at the raster corner (Fig. 6a), there are two kinds of projections of transportation network to 4- and 8-raster NN. (1) In 4-NN(Fig. 6b), edges AD, AC, BC and BD represent transportation net's four circuits, AOD, AOC, COB and BOD respectively with weights of 2, so $D(A, B) = 4$. When 4-NN edges AD, AC, BC and BD represent pathways, their weights are 4, and so $D(A, B) = 8$. (2) In 8-NN, edges AB and CD stand for the routes AOB and COD respectively, with weights of 2. However, AB and CD do not intersect at O, shown as Fig. 6c. This means, when raster NN cannot cover the transportation net, the gaps between them will cause differences between transportation net paths and raster NN paths in location and length. This, in turn, introduces elongation errors (NN path being longer than transportation network path) and deviation errors [2] in the shortest path algorithm analysis and accessibility analysis. It answers the question why transportation net's weights cannot be directly passed to raster NN.

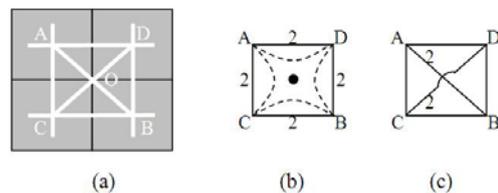


Figure 6. (a) transportation network and raster grid; (b) 4-neighbor network simulating transportation network; (c) 8-neighbor network simulating transportation network

Together, on condition that the transportation network is regular, that is, each cell has crisscross roads or no road, such as Manhattan net, raster NN based on Rock mode exactly seamlessly covers the transportation network, and avoids deviation error and reduces elongation error. The raster time geography field applied to Salt Lake City is belong to this

category. Also, that indicates the necessity of using NN to directly cover transportation network. However, when raster NN covers a single route in irregular transportation net, it introduces larger weight error; when does not cover the net, the gaps between raster NN and irregular transportation net introduce these errors of elongation, deviation, topology inconsistency. Hence, raster NN is unable to adapt to irregular-TNRS. This means, to enable the NN to cover irregular transportation net, we need introduce a kind of irregular NN and realize the mapping relation between NN and transportation net. The NN illustrating irregular transportation network is also irregular and D-TIN can seamlessly show the transportation network. In sum, the paper studies time geography theory for irregular-TNRS on condition that D-TIN is the prototype of NN and Voronoi dual graph is the field or grid model.

IV. CONSTRUCTION OF TIME GEOGRAPHY NETWORKS FOR TNRS

After studying on the theoretical issue of modeling NN and grid based on transportation network constraint, we would pay much attention to the mapping from transportation network to D-TIN NN, and to establish Voronoi diagram grid.

A. Time geography network modeling mechanics based on transportation network constraint

NN in TNRS, on the one hand, represents the centers of cells and their adjacent relations, so it is restrained by the spatial grid which subdivides TNRS into cells (grid → NN); on the other, have to contain the transportation net, so it is bound by transportation net (transportation net → NN). Governed by grid, the centroids of discrete units may not lay on the transportation net be located in the corresponding units. Here comes to the problem that how to build the NN on above both conditions. This directly influences the time geography network constructing for TNRS.

Since there generally exists a one-to-one relationship between the grid and its NN, hence, the one-way constraint relation: grid → NN, can be extended to a two-way relation: grid ↔ NN. This means that NN can also control the grid subdivision for TNRS; that is NN → grid. Due to the objectivity of transportation net, the subjectivity of grid and its NN, the priority of the constraint: transportation net → NN is higher than others, such as NN → grid. It is for such reasons that, we can reduce the double constraints on modeling NN to both relationships: transportation net → NN, and NN → grid, or transportation net → NN → grid (Fig. 7).

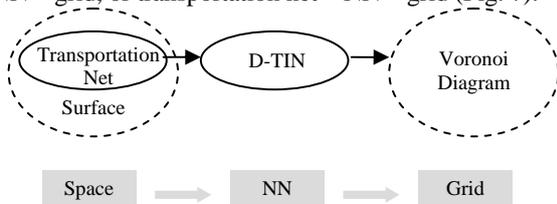


Figure 7. The model of constructing time geography network for TNRS

B. The mapping relation between transportation net and NN

On condition that TNRS is divided into transportation net and the restrained surface, we subdivide transportation net into plane transportation net on the flat surface and the non-planar three-dimensional transportation net on or under the surface, like a tunnel. This article only considers plane transportation net. The mapping from transportation net to NN includes discretizing the plane transportation net and the flat surface, and these discrete points based generating D-TIN, namely NN of TNRS. In this case, NN not only presents Thiessen polygons and their adjacency relations, but also contains transportation net, with the result that such double constraints of transportation net and grid on NN were fulfilled.

The D-TIN NN, allowing for the restraint of discrete points in transportation net, we find it difficult to avoid edges of D-TIN being crossed by transportation lines, which disobey the principle that NN completely contains transportation net. For instance, the D-TIN (Fig. 8b), generated from two discrete points A and B on one transportation line (Fig. 8a) and the surrounding points C and D, is crossed by the routine AB (Fig. 8c). If the transportation line is presented by TIN path, (e.g. AC-CB stands for AB), it would deviate the transportation line and cause related errors.

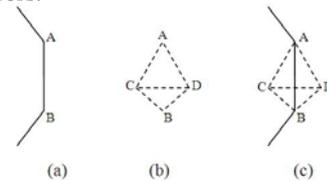


Figure 8. Transportation line crosses TIN (a) transportation routine; (b) TIN; (c) TIN edge intersecting transportation routine

As a result, constructing the D-TIN allowing for transportation line restraint is the key to construct NN. By adding new points, we may re-build D-TIN whose edges would not cross transportation lines. Thus, NN may completely contain transportation net.

C. The mapping relation between NN and grid

According to the theory of dual graph, D-TIN can be converted to the only Voronoi diagram. Since the centroid set of Voronoi diagram namely is the node set of D-TIN, which includes the discrete point set in transportation net. This means the discrete point set is contained in the centroid set. Because a Thiessen polygon has one and only one centroid, the centroid is a discrete point in transportation net, yes or no, Therefore, there is at most a transportation net' discrete point in a Thiessen polygon, and the discrete point must coincide with the centroid of the Thiessen polygon. As a result, the Voronoi diagram is the kind of space subdivision with constraint of transportation net, and it may become a kind of field model in TNRS. Hence, Voronoi diagram field cannot be replaced by raster field regardless of constraint of transportation net, in time geography for TNRS.

D. Calculation of NN's weights

To put it simple, we assume that the vector v_1 is constant in transportation net, which is the same to vector v_2 out of transportation net and $v_2 < v_1$. Thus, the weight of a NN edge representing transportation routine can be obtained by dividing routine length by v_1 ; as for a NN edge outside of transportation net, we gain its weight by dividing its length by v_2 . The nodes and edges representing transportation net in NN should be in correspondence with the real transportation network in node location, topological relation and weight; so it avoids the location deviation error, topological inconsistency error and path elongation error. Besides, since the weight calculations of transportation network and non-transportation network part in NN are relatively isolated, we can avoid weight error of non-transportation network caused by transportation network in NN.

V. EXAMPLE

This section illustrates the methodology introduced above. For this purpose, the methodology was implemented in ArcGIS 9.3. All figures in this section are computed with ArcGIS 9.3.

Taking part area of WuHan in China for example, we analyze the accessible domain in TNRS without considering 3-D transportation net such as tunnels and bridges. Firstly, according to the discrete points set of transportation net (Fig.9a), we can model the NN based on D-TIN for TNRS, and make sure that NN contains the transportation net completely. Let us suppose that v_2 is 60km/h, and v_1 is 10km/h. Thus, the weights of NN can be calculated, shown as Fig.9b.

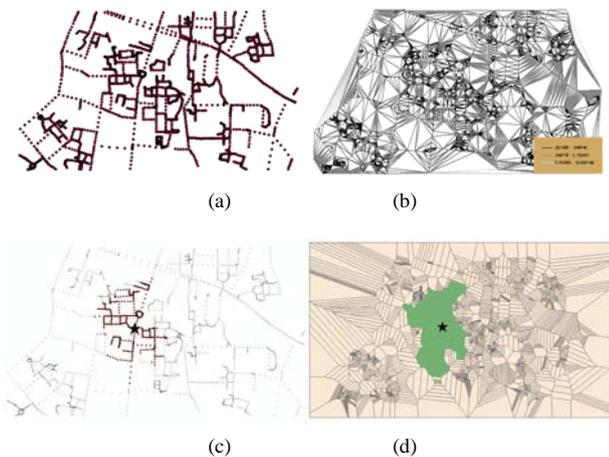


Figure 9. (a) discrete points; (b) D-TIN NN; (c) accessible points in NN; (d) accessible domain based on Thiessen polygons

Secondly, we construct Voronoi diagram field based on the discrete points set of transportation net. Finally, the accessible points in NN can be computed in known start point s and $T = 30$ seconds (Fig.9c), and these points Thiessen polygons comprise the accessible region (Fig.9d).

Similarly, using the principle of time geography field, we can also model the raster grid (Fig.10a), and can analyze the

weight of every cell (Fig.10b). After obtaining the weights of 8-NN (Fig.10c), we may measure the accessible points in NN, and their cells which form the accessible region (Fig.10d) under the same conditions of s, T, v_1 and v_2 .

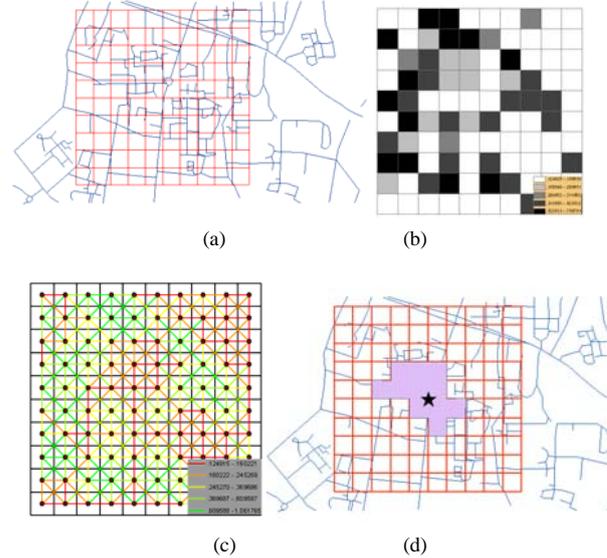


Figure 10. (a) raster grid; (b) grid weights; (c) 8-NN with weights; (d) accessible domain based on raster cells

Consequently, there are two kinds of different accessible domains for the same TNRS (Fig.11), where the solid line boundary is based on Thiessen polygons, and the dotted line boundary is based on raster cells.

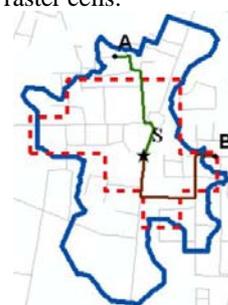


Figure 11. Two kinds of different accessible domains

As shown in the figure, two points A, B are located on the transportation net. It is easy for us to find the shortest path from the start s to A in transportation net, by the Dijkstra algorithm in ArcGIS. Obviously, the path lies also in transportation net, and its shortest passing time is 26.538 seconds. That means an agent can arrive at A from s within the given time T . Likewise, the shortest path from the start s to B, lies in transportation net, and its shortest passing time is 32.316 seconds, so in T the agent cannot arrive at B from s . Point A is contained within solid line boundary, and is out of dotted line boundary. On the contrary, point B is out of solid line boundary and is in dotted line boundary.

VI. CONCLUSIONS AND FUTURE WORK

Based on the summary of time geography theory such as that of raster time geography field, the article resolves spatio-temporal accessibility into time geography network modeling. In order to model time geographic network for TNRS, this paper introduces Voronoi diagram grid and NN based on D-TIN. After analyzing the relationship among transportation net, grid and NN, the paper approves a modeling strategy that transportation net is contained within NN, while NN controls grid. Finally, on the basis of NN weight calculation, we use instances to analyze time geography network modeling algorithm for TNRS. Future studies mainly consider about non-planar transportation net, anisotropy of transportation network, etc. It is necessary to validate the proposed solution in the paper in the large transportation net.

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