Electricity Price Forecasting in a Smart Grid

Md Mahbubur Rahman Khan, Md Minhaz Chowdhury and Kendall E. Nygard
Department of Computer Science
North Dakota State University
Fargo, ND, USA
Email: Mahbuburrahman.Khan@ndsu.edu, Md.Chowdhury@ndsu.edu, Kendall.Nygard@ndsu.edu

Abstract— **A smart grid refers to a digitized and intelligently controlled electrical power system. Intelligent monitoring and communication of digital information can support two-way between consumers and providers. The work of this paper concerns modeling of dynamic pricing, potentially helping to improve efficiency of electricity consumption and delivery. A simple variant of collaborative filtering is applied for dynamically predicting prices. Information on power consumption periods, and history of purchase levels and prices are used as input. The collaborative filtering approach is compared with a naïve forecasting method and the Winter method for incorporating seasonality. Actual price data is available for use in validating the models, which reveals that the collaborative filtering method provides the best results.**

Keywords- ***Collaborative Filtering; Winter Method; Dynamic Pricing; Smart Grid; Price Forecasting.***

I. INTRODUCTION

Collaborative filtering (CF) is becoming a popular technique of filtering for information or patterns using collaboration among multiple data sources, viewpoints, and agents, for large data sets. Collaborative filtering techniques have been applied in applications such as analyses of environmental data from multiple sensors, google news recommendations [1], Netflix movie recommendations, personalized pricing recommendations [2], financial data that integrates multiple financial sources, and electronic commerce user data. In this work, we apply a simple variant of collaborative filtering for prediction of electricity prices. The application environment is that of a fully instrumented and networked smart grid.

A smart grid is a digitally enabled electrical grid that gathers, distributes, and acts on information across energy suppliers and consumers within grid infrastructure. A smart grid holds promise to improve the efficiency, reliability and sustainability of electricity services [3]. A smart grid opens opportunity for changing the traditional electric pricing system that is typically based on peak and off peak hourly rates. The existing price model for electricity hides the temporal deviation in the cost of electricity depicted in the pattern of consumption of electricity during peak times and off peak times.

In this paper, collaborative filtering is applied to forecast prices of electricity consumed across regions. Our test day is from the ten regions of the New York Independent System Operator (NYISO). The effectiveness of applying collaborative filtering is compared with Winter Method of seasonality and a basic forecasting model.

The remainder of this paper is organized as follows: Section 2 describes the related works. Section 3 describes current electricity market. Section 4 describes steps related to data used in this work e.g. data source and data pre-processing steps. Section 5 describes the three approaches used in this work, for predicting the price of electricity. This section also describes dynamic pricing input output model. Section 6 describes the results of these predictions. Section 7 concludes this paper by mentioning limitations, suggests future use and possible improvement of collaboration filtering.

II. LITERATURE REVIEW

Collaborative filtering is based upon patterns that can be identified with limited details concerning the items or users under analyses. For the grid, the patterns can be such things as ratings, usage or purchases [4]. The idea is that a selective group of consumers of the same service shares a similar opinion with judgments based on their personal preferences.

Filtering proceeds by matching the available information from a domain in which information shares certain similarities in nature. For example, the movie recommendation system Netflix uses collaborative filtering that finds people with similar tastes in movies, called nearest-neighbors. Based on their history of movie ratings, the collaborative filtering approach recommends movies and predicts the rating for a movie. There are recursive application of the filtering for predicting such neighbors that is more effective in long term reusability [5].

Collaborative filtering falls into the general category of Recommender Systems [6]. Content-based information filtering is effective in identifying items similar to that a consumer preferred by analyzing textual similarity from user data [7]. However, the work presented in this paper focuses on finding similarities between customers pattern of electricity consumption that cannot be identified by keyword based searching [1]. Hence, the approach used for price forecasting applies collaborative filtering more broadly over content-based information.

There are several state-of-the-art works dealing with prediction of prices. For example, one recent study predicts hourly day-ahead electricity prices are using features like long memory, positive and negative price spikes, and seasonality [8]. However, this method is weak in capturing nonlinear patterns of price.

Other reported works uses approaches such as Auto-Regressive Fractionally Integrated Moving Averages with Feedforward Neural Networks [9], 3-Regime Markov Regime-Switching [10], and Hodrick–Prescott filters [11]. The work reported in [12] summarizes multiple methods. Concerning seasonality, there has been work reported on ways to identify and model seasonality [12]. As a benchmark seasonality method, the Winter method is applied in this work [13].

III. ELECTRICITY MARKET

Consumption levels for electricity increases during the working day, peaks in the late afternoon or early evening, and is at a low point by midnight [14]. The graph in Fig. 1 and Fig. 2, shows average consumption per hour, not instantaneous power. One interesting point is that the required maximum power is nearly twice as high as the average power consumption [15]. In much of North America, the problem is especially pronounced during the top 60 to 100 hours of the year, which may account for as much as 10–18 percent of the system peak load [16].

The yearly consumption increases in summer, when people use air conditioners. Figure 3 shows that in the summer, the power requirement doubles for several days [17]. To meet this critical peak load, expensive combustion turbines are purchased and installed, which raises rates for all customers. A prediction model predicting real time price of electricity can address this problem and increase economic efficiency.

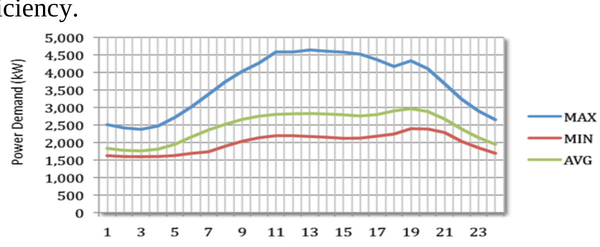

Figure 1. Daily electricity consumption pattern in San Diego

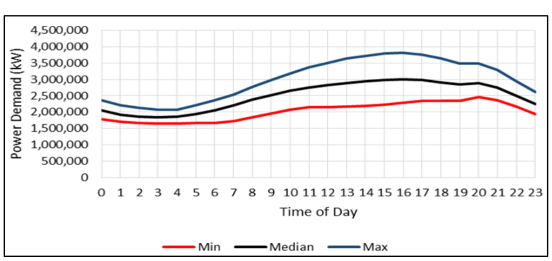

Figure 2. San Diego dynamic load profile from June to August 2014

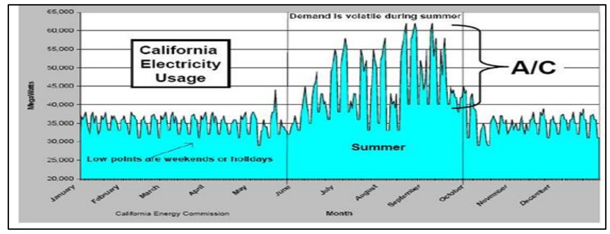

Figure 3. Yearly electricity consumption pattern in San Diego

Dynamic pricing that is aligned with demand response can reduce the maximum annual peak load to save investment in expensive large power plants. Large investments in generation capacity by power companies propagate to increase prices of electricity. Power plant reliability and the uncertainty of not achieving maximum utilization of resources increases supply side volatility and contributes to increasing the price of electricity. Conceptually, the dynamic pricing model for electricity is a mechanism that mitigates uncertainties in the electric grid by reacting to real-time fluctuations. The real time price reflects the capacity of the power generation system through price sensitive demand and supply. Design of a sustainable model that reflects consumer preferences, behavior and response is a challenge for researchers in modeling supply side uncertainties.

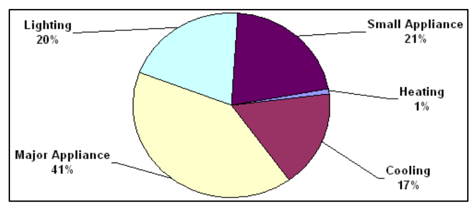

Figure 4. San Diego county average household electricity consumption: 6,300kw

These preferences, behavior and response of consumer can improve the use of electricity consumed. For example, Fig. 4 shows that a household consumes 62% of electricity for appliances and 20% for lighting. This 82% could be controlled and scheduled to use by making the consumer responsive to the price of the electricity [17].

IV. ELECTRICITY PRICE DATA

A. *Consumed electricity price Data*

NYISO is a not-for-profit organization based on New York's Capital Region to govern the New York's electricity market. It administers and monitors the wholesale electricity market, conducts planning, assesses long term projects and develops and deploys state-of-the-art technology for a sustainable and efficient power grid in the New York State. The NYISO publishes the wholesale price of consumed electricity every day on their website. The data (Table 1) used in this research is from that published data. The total area of New York is divided into 15 regions, each region is addressed as one node in this paper. For the analysis in this paper, data for 10 nodes are used. However, we could have done with 15 nodes as there are 15 regions in New York electricity market.

TABLE I. PRICE OF ELECTRICITY FOR 10 NODES IN NEW YORK

ID	Date	Node ID	HR00	HR01	HR02	HR03	HR04	……	HR23
000001	6/27/2011	61757	23.97	35.84	35.36	10.78	14.54	……	46.75
000002	6/27/2011	61754	22.96	34.34	33.87	10.35	13.86	……	45.28
000003	6/27/2011	61760	24.75	36.9	36.33	11.02	14.87	……	48.95
000004	6/27/2011	61753	22.32	33.2	32.71	10	13.22	……	43.68
000005	6/27/2011	61844	22.72	34.01	33.54	10.24	13.77	……	44.79
000006	6/26/2011	61757	37.64	35.56	35.07	27.83	12.96	……	31.91
000007	6/26/2011	61754	36.63	34.64	34.14	27.09	12.58	……	31.07
000008	6/26/2011	61760	42.27	44.38	46.79	46.68	35.5	……	98.26
……	……	……	……	……	……	……	……	……	……

B. *Fromatting extracted electricity price Data*

The price of electricity is published for each node every hour. A day is divided into K time slots within a range of 0 to 24.

V. THE DEVELOPMENT ENVIRONMENT

The simulation is developed by in Visual Studio 2010 using C# as a programming language. The reason for choosing C# as a programming language is to benefit from powerful .NET framework. The Visual Studio 2010 makes it simple and quick to develop and deploy a software project. Two Graphical User Interfaces (GUI) are used in this software. The Window Forms Designer provides the flexibility to control the layout that houses controls (textbox, label, list box, etc.). The Windows Presentation Foundation (WPF) helps to control the GUI by event driven programming and the Extensible Application Markup Language (XAML) file. For simplicity and better visualization, Microsoft Excel 2010 is used to hold the raw data. This provides quicker processing of data as the National Grid demand data is published in Microsoft Excel format.

VI. METHODS FOR PREDICTING PRICE OF ELECTRICITY

In this research, three methods for predicting the price of electricity are applied. They are based on history (basic or naïve forecasting method), collaborative filtering and the Winter method for seasonality. The Winter Method is considered to be a difficult competent of collaborative filtering. This is because of the capacity for capture the variation of price throughout the day.

A. *Basic or naïve forecasting method*

For time series data, naive forecasting is the simplest way to forecast by making forecasted value equal to the last observed value. It is easy to use Naive Forecast and it can handle seasonality effect. However, if there is an unusual change in the last period, this method will produce significantly inaccurate results.

B. *Collaborative Filtering as dynamic pricing model*

In dynamic pricing model, the price of electricity will depend on demand, and the demand will in turn depend on several qualitative and quantitative variables, such as temperature, number of appliances, user sleeping times, and user consumption preferences. Consumer behavior in each variable is not known and response could vary by clustering one or more variables together. One of the benefits of collaborative filtering is that it models the behavior of consumers based on their response without such complex details. By complex details it means dealing with the mentioned variables. Also, this collaborating filtering approach works well for predicting user recommendation for movie ratings [1]. Hence the use of collaborative filtering in predicting electricity price is beneficial in dynamic price domain.

Now, in the dynamic pricing model, consumers respond to the real-time price of electricity [18]. To calculate the dynamic price, the demand and supply must be forecasted, including information regarding generation capacity for fulfilling unexpected high demand (Fig. 5). The forecasted demand is based on user categories, such as household, commercial, and industrial. The user utility function is needed to provide a smooth estimate. The utility function considers the user level of satisfaction and behavioral patterns. By using collaborative filtering, user patterns are reflected in the choices they have made in their consumption of electricity. The output from the dynamic pricing model is the price of electricity for each group of users as well as load per power generator.

Classical time-window or instance-decay approaches are inappropriate in such scenario, as they lose signals when discarding data instances [19]. The factor and neighborhood models can be merged smoothly to predict more accurately [20]. Researchers at Yahoo applied collaborative filtering with bilinear predictive model for many of their predictions [21].

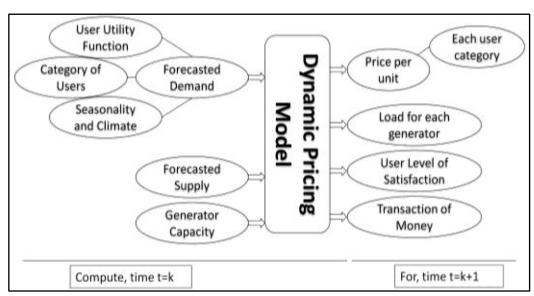

Figure 5. Dynamic pricing input output model

In the collaborative filtering approach a similar node is selected by nearest neighborhood search. The selection of nearest neighbor is done by calculating mean square deviation (MSD).

Each node in the previous time (yesterday) has its price for 24 hours (0….23). By applying the MSD calculation shown below, a Node is selected to have the closest behavior of the node for which price is need to be forecasted (Table 2).

TABLE II. MSD CALCULATION TO FIND THE BEST MATCH FOR EACH NODE

Node	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7
Node 1	-	0.707	983.944	5.224	28.275	6.043	5.378
Node 2	3.178	-	1027.595	2.098	26.340	10.811	9.924
Node 3	938.299	1027.595	-	1104.065	1101.345	867.937	873.540
Node 4	10.387	2.098	1104.065	-	26.946	22.311	21.032
Node 5	32.031	26.340	1101.345	26.946	-	41.974	41.010
Node 6	2.285	10.811	867.937	22.311	41.974	-	0.023
Node 7	1.891	9.924	873.540	21.032	41.010	0.023	-
Node 8	0.903	3.178	938.299	10.378	32.031	2.285	1.891
Node 9	1.907	9.949	873.232	21.058	41.130	0.023	0.00086
Node 10	57.256	76.715	684.736	96.495	116.437	45.648	46.483
Minimum	0.903	0.707	684.736	2.098	26.340	0.023	0.00086
Match Node	Node 8	Node 1	Node 10	Node 2	Node 2	Node 5	Node 9

The Price of the most similar node in the previous time is applied to provide the forecasted price for the current time.

C. The Winter Method for Seasonality

The Winter method of seasonality is applied to calculate the forecast, as shown below [16].

$$F(k) = \alpha \frac{A(k)}{C(k-K)} + (1-\alpha)[F(k-1) + T(k-1)]$$

$$T(k) = \beta[F(k) - F(k-1)] + (1-\beta)T(k-1)$$

$$C(k) = \delta \frac{A(k)}{F(k)} + (1-\delta)C(k-K)$$

$$f(k+\tau) = [F(k) + \tau T(k)]C(k+\tau-K)$$

This method updates a smoothed estimate F(k), a smoothed trend T(k), a seasonal factor C(k) and compares with actual demand A(k). The forecast period, τ is used to forecast more than one period in the future. The first and second equation calculates the smoothed estimate and the smoothed trend respectably by using exponential smoothing with a linear trend. These two equations capture the linear trend over recent days and the trend during the last couple of hours in consideration. The factor of seasonality is incorporated in the first equation above, to get the data about last time's demand as C(k-K).

For example, considering a day as a domain, then K =24 (24 hours a day) and considering a year, K=12 (12 months in a year). The parameters $\alpha, \beta, and\ \delta$ are smoothing constants between 0 and 1 either chosen or defined by the lowest mean square deviation (MSD) for the best performance in the test data. In this experiment, $\alpha = 0.10$, $\beta = 0.10,\ and\ \delta = 0.10$ are applied. Different combination of values of α, β and δ was applied to find the lowest RMS value and the 0.1 value for α, β and δ provides the lowest RMS. That's why 0.1 was uesed. Other combinations of α, β and δ could be used but they won't be optimized.

In this work, a machine learning algorithm, to determine smoothing constants dynamically, has been applied.

The following equations give the formula for MSD and Root Mean Square (RMS) values with k = 1, 2…K.

$$MSD = \frac{\sum_1^K [f(k) - A(k)]^2}{K}$$

$$RMS = \sqrt{\frac{\sum_1^K [f(k) - A(k)]^2}{K}}$$

As a benchmark, a ϵ A and A is set of actual price of electricity used to measure the effectiveness of the forecasted price. Since price could vary hence is an element of A e.g. a. Also, b ϵ B and B represents the price for the day before (Fig. 6). Again, n ϵ N and N represents number of nodes which are regions in the study area (in this case New York). M represents matched node with lowest MSD with the node for consideration and W is the forecasted price by applying the Winter Method for seasonality. R is the MSD values calculated while searching for the match node. For comparing the MSD values among three forecasting method s ϵ S is used. For applying the Winter Method for seasonality, e ϵ E represents the smoothed estimate, t ϵ T and T represents the smoothed trend which is the seasonality effect over the period of time, c ϵ C represents the seasonal factor and f ϵ F represents the forecasted price of electricity.

```
function bestForecastingMethod (α,ß,δ,B,A)
{
    S[0] ← getMSD(B,A); // basic forecasting
    M←getMatchNode(B); // CF
    S[1] ← getMSD(M,A);
    //Winter Mdthod
    W ← forecastByWinterMethod(α,ß,δ,B,A);
    S[2] ← getMSD (W,A);
    return forecasting method for lowest S;
}
function getMSD(A,B)
{
    sumOfSquare ← 0;
    for k:=0 to K
        sumOfSquare +=(A[k]-B[k])²;
    end for
    return sumOfSquare/K;
}
function getMatchNode(B)
{
    N ← getAllNodeExceptGivenNode(B);
    for each n in N do
        R ← getMSD(n,B);
    end for
    return n for min R;
}
function forecastByWinterMethod(α,ß,δ,B,A)
{
    E(K-1) ← ΣB/K;
    for k:=o to K
        C(k) ← B(k)/E(K-1);
    end for
    for k:=K to 2K
        E(k) ← αA(k-K)/C(k-K)+(1-α)[E(k-1)+T(k-
        T(k) ← ß[E(k)-E(k-1)]+(1-ß)T(k-1);
        C(k) ← δA(k-K)/E(k)+(1-δ)C(k-K);
        f(k+1) ← [E(k)+T(k)]*C(k+1-K)
    end for
    return f;
}
```

Figure 6. Algorithm for finding best forecasting method for forecasting the price of electricity.

 ISBN: 978-1-61208-530-2

In the algorithm for finding best forecasting method for forecasting the price of electricity (Fig. 6), best forecasting method is determined by finding the forecasting method with lowest MSD. Collaborative Filtering (CF) and the Winter Method is compared. For collaborative Filtering method, "getMatchNode" method by providing a Node to find out the best matching node with lowest MSD. Inside "getmatchNode" each node was compared except the provided node to find MSD by using "getMSD" method. The "getMSD" method takes two nodes calculate difference for all time period data available. The "forecastByWinterMethod" is used to forecast by using winter method.

VII. ANALYSIS OF RESULTS FROM FORECASTING METHODS

All three forecasting methods are applied in 10 Nodes. Table 3 shows results of calculation of MSD for three forecasting methods for node 1. The optimum column shows the optimum value of MSDs resulting from each of these methods. It shows that collaborative filtering (CF) provides lowest MSD for node 1. This table is summarized and then extended into Table 4.

TABLE III. CALCULATION OF MSD FOR FORECAST, CF AND WINTER METHOD FOR NODE 1

Method	HR00	HR01	HR02	HR03	……	HR22	HR23	MSD	Optimum
Actual	23.97	35.84	35.36	10.78	……	48.84	46.75	-	
Forecast	37.64	35.56	35.07	27.83	……	49.85	31.91	140.615	
CF	38.65	36.46	36.06	28.58	……	51.23	32.9	133.068	133.068
Winter	34.93	34.49	27.30	11.83	……	41.55	47.53	141.802	

Table 4 shows the listed MSDs of Table 3 for all 10 Nodes. This table 4 also shows that collaborative filtering gives the best results for 6 occurrences. The Winter Method shows best results in 3 out of 10 Nodes and for one Node the general forecasting method is best. The average MSD for CF is 148.99 (shown in the average row for CF MSD), which is about 39% of average MSD for the Winter Method. While calculating the Root Mean Square (RMS) value for CF, the average is 12.21 in a day. This means $0.51 deviation for each forecast price while applying collaborative filtering. The Winter Method, the deviation of each forecast is $0.815. Based on these results shown in Table 4, it can be concluded that collaborative filtering provides a better forecasting of the price of electricity than the Winter Method for Seasonality and basic forecasting.

TABLE IV. FINDING THE BEST FORECASTING METHOD FOR EACH NODE

Node	Best match	Matching MSD	Forecasting MSD	CF MSD	Winter Method	Optimum
Node 1	Node 8	0.9037	140.615	133.068	141.802	CF
Node 2	Node 1	0.7074	130.032	123.866	132.632	CF
Node 3	Node 10	684.736	631.198	173.702	401.333	CF
Node 4	Node 2	2.098	117.858	107.594	125.414	CF
Node 5	Node 2	26.345	121.381	93.238	53.802	Winter
Node 6	Node 5	0.0239	169.941	198.054	163.704	Winter
Node 7	Node 9	0.00086	3729.436	171.466	2191.85	CF
Node 8	Node 1	0.903	127.986	134.77	135.34	Forecast
Node 9	Node 7	0.00086	173.71	173.86	167.292	Winter
Node 10	Node 6	45.468	190.95	180.260	316.545	CF
Average		76.118	553.310	148.987	382.971	
Root Mean Square		8.724	23.522	12.206	19.569	

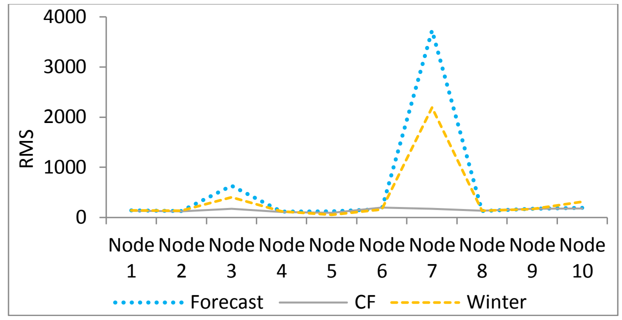

Figure 7. MSD for different forecasting method for each node

In Fig. 7, MSD for three forecasting methods are plotted. This figure shows that CF provides an excellent estimation of the price. Node 7 shows a significantly higher MSD. The data shows that the price of electricity in the day before was unusually high. By using collaborative filtering, such unusual behavior can be avoided.

To compare these methods, a program was developed, shown in fig. 6. Screen shots for the developed program are provided in Fig. 8, Figure 9, and Figure 10.

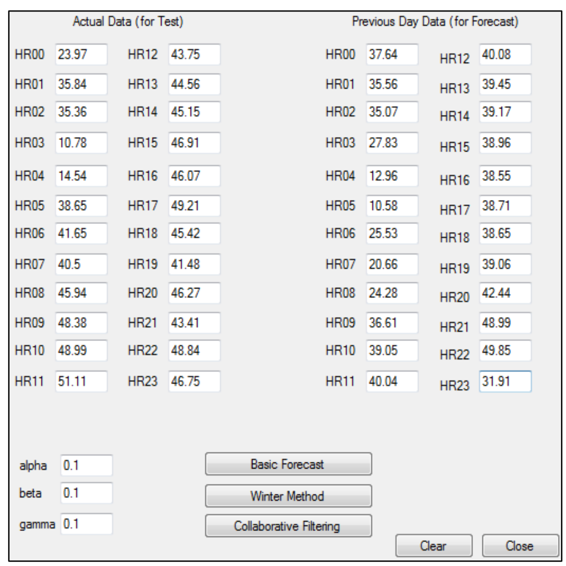

Figure 8. Data input for calculating forecast of the price of electricity

Winter Method

Period	Actual	Forecast
0	23.97	34.9366983699871
1	35.84	34.4996548873408
2	35.36	27.3044892018985
3	10.78	11.837202123417
4	14.54	9.78411151497416
5	38.65	30.8825877460063
6	41.65	26.4313893444546
7	40.5	33.5539688705311
8	45.94	53.7992231383779
9	48.38	58.2835204960329
10	48.99	60.4010807619692
11	51.11	61.0556849620902
12	43.75	59.725917949658
13	44.56	58.779649401727
14	45.15	58.0590293831749
15	46.91	57.0781899187859
16	46.07	57.0110375309788
17	49.21	56.7428166958921
18	45.42	56.4727621607123
19	41.48	60.1340939180185
20	46.27	68.4245200757151
21	43.41	66.7525905858175
22	48.84	41.5583941159769
23	46.75	47.5378825215703

RMS value is: 141.802034344981

Figure 9. Forecasted price by using winter method of seasonality

Forecast With Collaborative Filtering

Period	Actual	Forecast
0	23.97	38.65
1	35.84	36.46
2	35.36	36.06
3	10.78	28.58
4	14.54	13.33
5	38.65	10.87
6	41.65	26.08
7	40.5	21.13
8	45.94	24.85
9	48.38	37.39
10	48.99	39.85
11	51.11	40.94
12	43.75	41.07
13	44.56	40.51
14	45.15	40.18
15	46.91	40
16	46.07	39.55
17	49.21	39.81
18	45.42	39.76
19	41.48	40.04
20	46.27	43.55
21	43.41	50.56
22	48.84	51.23
23	46.75	32.9

MSD value is: 133.0682

Figure 10. Forecasting by using collaborative filtering method for forecasting the price of electricity

VIII. CONCLUSION AND FUTURE WORK

Collaborative filtering is an effective method of predicting prices. However, there may be limitations related to covering the full factorial set of other possible factors that may influence price. Further testing is expected to include much larger data sets. Another limitation is that it does not considered price changes due such things as natural disasters, power outage for maintenance, transmission device failure, and generator scarcity. Working with such effect of disasters or their combination in pricing is of interest for future research. In this research, only one neighbor is selected (best match), i.e. K=1 for K-nearest neighbor (k-NN). Higher values of k could be applied to obtain more than one match. Finally, user preferences on the source of power (e.g., coal, nuclear, hydro, solar, wind) or an open market with pay as you go can be considered for future work.

REFERENCES

[1] Das, A. M., Google news personalization: scalable online collaborative filtering: industrial practice and experience, 2007.

[2] T. Kamishima, S. Akaho, "Personalized pricing recommender system", The 2nd International Workshop on Information Heterogeneity and Fusion in Recommender Systems, pp 57-64, October 2011.

[3] W. Ketter, J. Collins, and C. Bloc, "Smart grid economics: policy guidance through competitive simulation", November 2010, ERIM Report Series Reference No. ERS-2010-043-LIS.

[4] Y. Koren, R. Bell, "Advances in collaborative filtering", recommender systems handbook, pp 77-118, 2015.

[5] J. Zhang, P. Pu, "A recursive prediction algorithm for collaborative filtering recommender system", The ACM conference on recommender systems, pp 57-64, 2007.

[6] F.O. Isinkaye, Y.O. Folajimi, and B.A. Ojokoh, "Recommendation systems: principles, methods and evaluation", Egyptian informatics journal, Volume 16, Issue 3, pp 261–273, November 2015.

[7] M. D. Ekstrand, J. T. Riedl, and J. A. Konstan, "Collaborative filtering recommender systems", Foundations and trends in human-computer interaction, Vloume 4, Issue 2, pp 81-173, 2011.

[8] E. Fanone, A. Gamba, and M. Prokopczuk, "The case of negative day-ahead electricity prices", Energy economics, Volume 35, pp 22-34, 2013.

[9] C. Najeh, "A hybrid ARFIMA and neural network model for electricity price prediction", International journal of electrical power & energy systems, Volume 55, pp 187–194, February 2014.

[10] J. Janczura, "Pricing electricity derivatives within a Markov regime-switching model: a risk premium approach", Mathematical methods of operations research, Volume 79, Issue 1, pp 1–30, February 2014.

[11] R. Weron, M. Zator , "A note on using the Hodrick–Prescott filter in electricity markets", Energy economics, Volume 48, pp 1–6, March 2015.

[12] R. J. Hyndman, G. Athanasopoulos, "Forecasting: principles and practice", OTexts, October 2013.

[13] W. J. Hopp, M. L. Spearman, "Factory Physics," Mcgraw-Hill Higher Education, third edition, August 2011.

[14] J. Smart, "What is the controllable electrical demand from residential EVSE in the San Diego region?", Idaho National Laboratory, April 2015.

[15] P. D., "San Diego gas & electric power demand (load) in Kw", San Diego, CA, 2009.

[16] A. Faruqui, R. Hledik, and J. Tsoukalis, "The power of dynamic pricing," The electricity journal, Volume 22, Issue 3, pp 42–56, April 2009.

[17] Bartley, T, "Effect of air conditioning", retrieved on December 11, 2011, from annual pattern of daily peak demand: http://www.mpoweruk.com/electricity_demand.htm

[18] M. Roozbehani, M. A. Dahleh, S. Mitter, "Dynamic pricing and stabilization of supply and demand in modern electric power grids", Smart grid communications (SmartGridComm), first ieee international conference, pp 543-548, 2010.

[19] Y. Koren "Collaborative filtering with temporal dynamics", Communications of the ACM magazine, Volume 53, Issue 4, pp 89-97, April 2010.

[20] Y. Koren, "Factorization meets the neighborhood: a multifaceted collaborative filtering model", The 14th ACM sigkdd international conference on knowledge discovery and data mining, pp 426-434, 2008.

[21] Chu, "Personalized recommendation on dynamic content using predictive bilinear models", The 18th international conference on World Wide Web, pp 691-700, 2009.